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**MOMENT CAPACITY PREDICTION OF RECTANGULAR CONCRETE-FILLED FRP (CFFT) TUBE BEAMS FILLED WITH REINFORCED CONCRETE**

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**Abstract:** The concept of concrete-filled fiber-reinforced polymer (FRP) tubes (CFFTs) is promising for a variety of structural applications and a good alternative for innovative constructions because of numerous attractive features, including durability and concrete confinement. An extensive research work was conducted on circular (CFFT) with and without internal bars for beams and columns. However, much less attention has been given to rectangular CFFT especially those reinforced with internal reinforcement bars. This paper aims at developing simplified analytical design method and design formula to predicate the ultimate moment capacities of previously tested steel-reinforced rectangular CFFT beams at the University of Sherbrooke. A wide range of test parameters was considered in the experimental program such as the FRP tubes thickness, fibre laminates, and internal steel reinforcement. The analysis was conducted according to the equations derived from linear elastic analysis as well as a regression analysis of the test results. The proposed design procedures and empirical design formula were found to be acceptable and easy to use for predicting the ultimate moment capacities of the tested beams on average of 0.99 ±0.1 and 1.0±0.03, respectively. A strong correlation observed between the theoretical predictions and the test results for the ultimate moment capacity when the values of η (concrete neutral axis depth) are in the range of 0.28–0.45 H (beam height). More experimental tests on steel-reinforced rectangular CFFT beams are needed to cover a wide range of parameters to assess the confinement of concrete contribution under flexural with different cross-sectional aspect ratio, fiber laminate, and steel reinforced ratios.

1. **Introduction**

The construction industry is expressing a great demand for innovative and durable structural members. Concrete-filled fiber-reinforced-polymer (FRP) tube (CFFT) provides an innovative and effective system, suitable for several structural applications, including fender piles, girders, and bridge piers in marine and other harsh environments. The FRP tube has several advantages over steel reinforcement as it can offer the flexural, shear, and confinement reinforcement simultaneously, without the labour costs and time involved in building conventional rebar cages. Filling the FRP tube with concrete provides compressive resistance and prevents local buckling of the thin tube. The CFFT system may also include internal longitudinal steel reinforcement, as the tube will protect the steel from water infiltration and corrosion, while the steel reinforcement provides ductility and additional strength and stiffness as well as minimize slip of the tube relative to the concrete core, which can be a major factor in the loss of composite action. Therefore, utilization of FRP tubes filled with plain or reinforced concrete as a hybrid composite system can provide the proposed system many unique advantages, which impact on civil engineering infrastructures costs, strength and performance, and their sustainability for longer service life (Ahmed and Masmoudi 2016; 2017; Ahmed et al. 2018).

Rectangular sections are more favorable in construction due to erection speeds and easy feasibility to create composite action with other members such as bridge decks and columns. The growing popularity of using circular CFFT members has motivated researchers to expand the investigation of the system feasibility for rectangular sections in both axial and flexural applications. An extensive research project has been conducted at the University of Sherbrooke to investigate flexural performance of rectangular CFFT beams totally filled with reinforced concrete [(Abouzied and Masmoudi (2015, 2017); Masmoudi and Abouzied (2018)]. A total of 24 beams were fabricated and tested under a four-point bending moment. The tubes were fabricated filament-wound technique at the University of Shebrooke using glass FRP and [Vinyl Ester Resin](https://www.compositesone.com/product/polyester-vinyl-ester-resins/vinyl-ester/). The beams have a 3200 mm long and 305×406 mm2 cross section. The test matrix covered a wide range of test parameters such as tube thickness of 3.4 mm to 14.2 mm, fiber structure laminate, and internal reinforcement ratio. The tested beams were reinforced at the tension side with four steel bars 15 mm diameter (15 M) as flexural reinforcement with a concrete cover of 38 mm. The test results showed that the reinforced-rectangular CFFT beams experience significantly higher ductility, higher stiffness, and superior strength than the RC beams. Figure 1 shows the rectangular CFFT beams used in this study. All beams failed in a flexural failure with no signs of web buckling, or slippage between the concrete core and the tubes, or rupture at tubes corners. Reinforced-rectangular CFFT beams exhibited gradually in a sequential manner (yielding of steel, buckling of compressed tube flange, and finally rupture of the fibers). Even after the ultimate failure, the CFFT beams can maintain a residual strength because of the existence of the steel that withstands high strains and elongation. Also, they concluded that the flexural strength of the reinforced-rectangular CFFT beams increases with increasing the FRP tube thickness until a certain limit of reinforcement ratio (ρf = Af/Ac = 10%, where A*f* is area of FRP tube and Ac is the area of concrete core) that separates the under-reinforced (flexural tension) and over-reinforced (flexural compression) CFFT section. Furthermore, the study showed that the deflection behaviour can be modelled and predicted. A comparison between fully, partially filled CFFT, and conventional steel-RC beams have also made. The results indicated that, the partially-CFFT beams had an overall strength-to-weight ratio 370% higher than that of the RC beam, while their weight was 30% lighter than the RC beam. Moreover, providing inner hollow GFRP tubes changed the failure from tension to compression failure, which is desirable in FRP composite section design.

In this paper, first a simplified analytical procedure based on force equilibrium, strain compatibility, linear elastic behaviour of FRP and partial stress– strain confinement model for concrete is developed to predict the ultimate moment capacities of previously tested beams at the University of Sherbrooke [(Abouzied and Masmoudi 2015, 2017 and Masmoudi and Abouzied (2018)]. Second, a new empirical design expression is proposed to facilitate reasonable predication for the ultimate moment capacities of the tested beams failed under tension failure-controlled. The results from the theoretical analysis as well as the empirical formula are compared against the experimental results to determinate their accuracy. The simplified analytical procedure and empirical design formula are presented in following sections.

1. **simplified Proposed Analytical method**

In this section, a simplified analytical method is developed to predict the ultimate moment capacities corresponding to the failure mode of the tested RCFFT beams. The analytical procedure (as shown in Figure 2) involves the determination of the position of the neutral axis for a given strain of the extreme compression fibre by using the principles of strain compatibility and cross-sectional forces equilibrium. The assumptions in the analysis include: 1) Euler–Bernoulli beam theory (that plain sections remain plain after deformation), 2) strain distribution throughout the depth of the section is linear, 3) perfect bond between FRP tube section, steel bars and concrete core, 4) GFRP tube section behaves linear elastically until failure, 5) Confinement effect is considered, 6) concrete contribution after cracking below the neutral axial is neglected, 7) εbar = εy= εu = 0.0021 (yield strain of steel bars), and 7) stress–strain curve for the GFRP tube was based on full section behaviour (based on mechanical properties of coupon test). The flexural analysis procedure is straightforward. The parts of the FRP tube above and below the neutral axis are considered effective in resisting compression and tension forces, respectively. Figure 2 shows the cross section of the reinforced rectangular CFFT beam that was used in the analysis, where the neutral axis is located within the cross section at a depth c.



Figure 1: Rectangular CFFT beams used in this study: schematic and test setup (Abouzied and Masmoudi 2015)



Figure 2: Strain and stress profile of reinforced rectangular CFFT beam cross section

Referring to the equivalent stress and strain distribution diagram shown in Figure 2, the internal tensile forces in the FRP tube (bottom flange and web) and steel bars, T, can be expressed respectively, as follows:

[1] 

[2] 

[3] 

where, H = Hi + tFRP, Hi and tFRP are the internal height and thickness of FRP tube, respectively, and Etx tensile modulus in the axial direction. Also, the strains εflange,Bottom and εweb,Bottom can be computed in terms of εtop (the design ultimate axial strain obtained from the axial compression tests) as follows:

[4] 

[5] 

The internal compression forces in the FRP tube (top flange and web), Ctube, flange top andCtube web, top, can be expressed as follows, assuming εtop the ultimate compression strain based on coupon tests:

[6] 

[7] 

[8] 

Herein for simplicity, the internal compression force in the concrete block at the top is calculated based on an equivalent stress distribution assuming a rectangular stress block with a depth equal to some fraction of the neutral axis depth, where a = βc, and a magnitude equal to some fraction of the concrete compressive strength, fecu = α fcu, [CSA A23.3 (2014)]. α is the ratio of the assumed uniform stress in the rectangular compression block (*f* ecu) to the maximum partially confined concrete compressive strength (fcu), given by α = 0.85− 0.0015 f ′ c ≥ 0.6, β is the ratio of the depth of the rectangular compression block (a) to the depth to the neutral axis (c), where β = 0.97 − 0.0015 f ′c ≥ 0.6. In fact, these factors need more examination for rectangular CFFT beams with a wide range of concrete strength. Hence, the internal compression force in the concrete block can be expressed as follows:

[9a] 

Where A is the area of concrete in compression zone (= b\*a). It is assumed that the ratio of the partially confined concrete compressive strength fcu, to the confined concrete compressive strength *f* ’cc is equal to ξ (0.0 <ξ<1.0) or can be expressed as follows:

[9b] 

Substituting Eq. 9a in Eq. 9b can lead to:

[9c] 

The value of ξ presents the level of confinement gained from the interaction between FRP tube and concrete core of CFFT beam under flexural load. Also, this value (ξ) depends on different parameters such as thickness and fiber orientation of FRP tubes, type and reinforcement ratio of internal rebars and concrete compressive strength. Strength of confined concrete *f* ′ cc is related to its unconfined concrete strength and the confinement pressure from the FRP tubes. The *f* ′ cc values can be calculated from the proposed model by We and Wi (2010). The model considered for rectangular column, corner radius and aspect ratio.

[10a] 

Where *f* ’co unconfined concrete strength (MPa), r is the corner radius, (2r/b) is the corner radius ratio, h and b is large column length and width, respectively, *fl* is the lateral confinement pressure (), ffrpu is the ultimate hoop tensile strength of FRP tubes.

Therefore, the nominal moment strength of the beams can be determined by taking the moment of tensile and compression stress resultants of tube and bars about the compression stress resultant in concrete.

[11] 



In fact, in Eq. (11), there are three unknowns: strains, neutral axis depth c and ξ. As a result, the design procedure is iterative. Assuming compression failure of CFFT beams is triggered by failure of the tube, which is immediately followed by crushing of concrete as a secondary failure, where the hoop tensile stress (confinement effect is insignificant) (AASHTO 2012). Hence, the analytical procedure is performed by assuming a given strain value (εtop) at the level of the extreme level of the tube based on the compression coupon tests. Flexural analysis of reinforced and unreinforced CFFT beam suggests that the neutral axis is located within a region of 0.25–0.4Hi and 0.2-0.3H (Abouzied and Masmoudi 2015, 2017) and Fam et al (2006), respectively. Therefore, the analytical procedure for the reinforced rectangular CFFT beams is performed by assuming the neutral axis depth (c) equal to ηHi from the inner height, where η is taken in range between 0.20 and 0.45. The strain values for tube and concrete at any level are determined based on the assumption of a linear strain distribution along the depth of the beam between the extreme compression fibers and the tensile FRP tube reinforcement. The moment capacities of the CFFT beams are determined for all values of η. Then, the experimental bending moments of CFFT beams are compared with the theoretical moments. The optimum value of η is obtained and correlate the theoretical with experimental bending moments for the tested beams. Thereafter, by equating the compressive and tensile forces, the values of ξ (confinement efficiency) could be determined for the different values of η.

1. **A simplified Proposed Design experssion**

In this section, an empirical design formula is proposed. In order to establish an expression for the moment capacity of rectangular CFFTs fully filled with reinforced concrete, a regression analysis for the tested beams were considered in lieu of the rigorous equilibrium and strain compatibility approach. Since the tubes had different tube thickness and different laminate structures, a total reinforcement index parameter ω*t* is introduced to represent the different reinforcement ratios and material strengths. The total reinforcement index ω*t* is the sum of FRP and steel reinforcement indices ω*f* and ω*s* and is defined as given in Eq. 12:

[12] 

where the term (ρf) is the FRP reinforcement ratio, defined as the ratio of cross-sectional areas of the tube and concrete core. *ffut* is the tensile strength of the tube in the longitudinal direction (MPa), which is given in Table 1, and *f* c’ is the unconfined concrete compressive strength (MPa), (ρs) is the steel reinforcement ratio (=As/bd), b is cross-section width (mm), d is the effective depth (mm), and fy is the yield tensile stress of internal steel bars (MPa). Based on the best fitting of the experimental data (B1 to B4 in given Table 1), the following an empirical formula is proposed to predicate the moment capacity of rectangular CFFTs predominantly governed by tension failure. The new empirical design formula is similar design formula adopted by the AASHTO (2012) for circular CFFT members, however, the tube geometry and internal bars were considered. The FRP tube controls failure in tension. As such, the concrete longitudinal compressive strain at failure may exceed 0.003 [AASHTO (2012)].

[13] 

Where h is member total height (mm) (=*h*o+2*t*f).

1. **Verification of the propsoed analtical procedure and empirical model**

Table 1 shows the summary of the experimental test results conducted by (Abouzied and Masmoudi 2016). As seen in Table 2 a strong correlation is observed between the theoretical predictions and the test results for the ultimate moment capacity when the values of η are in the range 0.28–0.45. Similar findings have been found in (Abouzied and Masmoudi (2016). The corresponding Mu(exp)/Mu(theo) ratios and ξ values are on average in the range 0.92 ±0.1 to 1.16 ± 0.1, and 0.81 ±0.4 to 0.45 ± 0.14, respectively. It is interesting to mention that the concrete compressive strengths *f*cu at ultimate for the reinforced CFFT beams were on average 60% the confined concrete compressive strength *f* cc. These results confirm that the FRP tubes partially confined the concrete core of the tested CFFT beams. The value (ξ) depends on different parameters such as thickness and fiber orientation of FRP tubes, type and reinforcement ratio of internal rebars and concrete compressive strength. The analytical design procedures presented herein were found to be acceptable for predicting the ultimate moment capacities of reinforced rectangular CFFT beams with different tubes thickness and mode of failure; refer to Figure 3. The accuracy of the theoretical analysis procedures pertains good agreement with the measured values on average of 0.99 ±0.1.

On the other hand, comparisons between the theoretical predictions and the test results for the ultimate moment capacity using the empirical design formula are shown in Table 2. As seen in Table 2 a strong correlation is observed between the prediction values and the test results, particularly when the failure predominantly governed by tension failure. The Mu(exp)/Mu(theo) ratio value is on average of 1.0 ±0.031 for beams B1 to B4. However, the accuracy of the proposed formula slightly overestimates the moment capacity predications when other mode of failure was triggered such as balanced or compression failure. The empirical expression is simple and provide reasonable prediction to use by the designer. However, more experimental tests are needed to cover a wide range of parameters to assess the confinement concrete contribution for rectangular cross-section with different aspect ratio, fiber laminate, steel reinforced ratios.



Figure 3: Experimental to the theoretical moment capacity predications of steel-reinforced rectangular CFFT beams

Table 1: Summary of the experimental test results database (Abouzied and Masmoudi 2016)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Beam | (hi mm x bi mm | tf (mm) | f'c | Stacking  sequence | Mechanical properties | Axial direction | | | Transverse direction | | | Mode of failure | Mu  kN.m |
| Ef (GPa) | ff (MPa) | εf (mm/m) | Ef (GPa) | *Ff* (MPa) | εf (mm/m) |
| B1 | (406 x305) | 3.4 | 49.7 | [90o, ±30o, 90o] | Tens. test | 14.3 | 249.0 | 15.7 | 16.0 | 257 | 21.8 | Tension | 249 |
| B2 | Comp. test | 14.0 | -92.0 | -7.0 | 17.8 | -175 | -10.4 | 267 |
| B3 | 5.7 | 48.7 | [90o, ±30o, 90o,±30o, 90o] | Tens.test | 14.5 | 173 | 15.3 | 14.4 | 249 | 23.9 | Tension | 404 |
| B4 | Comp. test | 15.5 | -165 | -12.5 | 14.5 | -293 | -24.0 | 392 |
| B5 | 8.7 | 41.7 | [90o,±30o2, 90o,±30o2, 90o] | Tens.test | 16.2 | 19 | 18.9 | 13.7 | 168 | 19.2 | Balanced | 559 |
| B6 | Comp.test | 17.7 | -189 | -11.8 | 13.8 | -211 | -17.8 | 560 |
| B7 | 9.9 | 48.7 | [90o,±30o6, 90o] | Tens. test | 18.6 | 242 | 15.3 | 13.4 | 125 | 16.6 | Comp. | 581 |
| Comp. test | 20.1 | -17 | -9. | 12.2 | -217 | -24.0 |

Table 2: Theoretical results of moment capacities predications (Muexp./Mutheo) for the analytical design procedures and empirical design expression and η values

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Label | η=c/Hi | | | | | | | | | | | | | | Eq13 |
| 0.20 | 0.22 | 0.24 | 0.26 | 0.28 | 0.3 | 0.32 | 0.34 | 0.36 | 0.38 | 0.4 | 0.42 | 0.44 | 0.45 |
| B1 | 0.85 | 0.89 | 0.93 | 0.96 | 0.99 | 1.02 | 1.05 | 1.08 | 1.10 | 1.12 | 1.15 | 1.17 | 1.19 | 1.20 | **0.97** |
| B2 | 0.91 | 0.96 | 1.00 | 1.03 | 1.07 | 1.10 | 1.13 | 1.16 | 1.18 | 1.21 | 1.23 | 1.25 | 1.27 | 1.28 | **1.04** |
| B3 | 0.77 | 0.82 | 0.87 | 0.92 | 0.97 | 1.01 | 1.05 | 1.09 | 1.12 | 1.16 | 1.18 | 1.21 | 1.24 | 1.25 | **1.02** |
| B4 | 0.74 | 0.80 | 0.85 | 0.90 | 0.94 | 0.98 | 1.02 | 1.06 | 1.09 | 1.12 | 1.15 | 1.18 | 1.20 | 1.21 | **0.99** |
| B5 | 0.66 | 0.70 | 0.75 | 0.79 | 0.83 | 0.87 | 0.90 | 0.93 | 0.96 | 0.99 | 1.01 | 1.03 | 1.06 | 1.06 | 0.87 |
| B6 | 0.66 | 0.71 | 0.75 | 0.79 | 0.83 | 0.87 | 0.90 | 0.93 | 0.96 | 0.99 | 1.01 | 1.04 | 1.06 | 1.07 | 0.88 |
| B7 | 0.65 | 0.70 | 0.74 | 0.78 | 0.82 | 0.86 | 0.89 | 0.92 | 0.95 | 0.98 | 1.00 | 1.03 | 1.05 | 1.06 | 0.66 |
| Aver. | 0.75 | 0.80 | 0.84 | **0.88** | **0.92** | **0.96** | **0.99** | **1.02** | **1.05** | **1.08** | **1.10** | **1.13** | **1.15** | **1.16** | **1.00\*** |
| SD | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 | 0.10 | 0.09 | 0.09 | 0.10 | 0.03 |

\* average is calculated from B1 to B4 (failure dominated by tension failure).

1. **conclusions**

Using conventional beam theory for the analysis and design can be used for the reinforced rectangular CFFT beams. The present theoretical analysis procedures and empirical design formula found to be acceptable and easy to use for predicting the ultimate moment capacities of steel reinforced rectangular CFFT beams with an average of 0.99±0.1 and 1.0±0.03, respectively. A strong correlation observed between the theoretical predictions and the test results for the ultimate moment capacity when the values of η are in the range 0.28–0.45. However, more experimental tests are needed to cover a wide range of parameters to assess the confinement of concrete contribution for rectangular cross-section under flexural with different aspect ratio, fiber laminate, steel reinforced ratios.

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