|  |  |  |
| --- | --- | --- |
|  | **10th International Conference on Short and Medium Span Bridges****Quebec City, Quebec, Canada,** **July 31 – August 3, 2018** |  |

* **MODELLING CONCENTRATED LOADS USING AN ARCHED STRUT**

Alexander, Scott1,3 and Lantsoght, Eva2

1 COWI North America Ltd., Canada

2 Universidad San Francisco de Quito, Ecuador

3 scal@cowi.com

**Abstract:** Strut-and-tie models (STM) are appropriate for analyzing and designing disturbed regions in a reinforced concrete structure. The arched strut is an addition to the STM tool kit. It models the combination of disturbed behavior in one direction with slender behavior in the perpendicular direction. The arched strut is part of the Strip Model, originally developed to model load transfer between a two-way slab and its supporting column. It is difficult to define the geometry of conventional STM in a slab. One end of the strut is connected to the concentrated load but there is no similar feature to define the position of the other end. The arched strut is a means of addressing this difficulty. The method does not model a failure criterion; rather, it defines an acceptable load path that meets static and material constraints.This paper summarizes the technique in the context of column-slab connections, develops the modifications needed to model conventional punching of an approach slab under a patch load, and proposes additional modifications to adapt the analysis to fatigue loading. The principal findings are that, while the analysis for two-way shear given in S6-14 is deficient, the punching strength of a typical approach slab under both static and fatigue loading from a CL-W truck should not be a concern.

1. **INTRODUCTION**

The Strip Model (Alexander 2017) is a general approach for the analysis of load transfer at column-slab connection. The key element of this model is an arched strut to transfer shear between the slab and column. The curvature of the strut is a result of its interaction with a transverse tension field. In the case of a column-supported slab, the magnitude of the tension field is limited by the one-way shear strength of the slab; however, the concept can be applied to other problems involving a concentrated transverse load supported on a reinforced concrete slab.

Here the concept of the arched strut is applied to the case of a concentrated patch load (wheel load) rather than a column. As is usually the case with any type of strut and tie modeling, the global statics of the region to be modeled should be established prior to developing the strut and tie model itself. The example of an approach slab is used here because of its relatively straight-forward statics.

The arched strut does not model a particular failure model. It provides a load path that is consistent with static constraints and does not exceed material capacities.

1. **MECHANICS OF ARCHED STRUT**
	1. **Arched Strut for Column-Slab Connection**

Figure 1 shows a possible arrangement of arch strips framing into a rectangular column. Each strip extends from the column to a position of zero shear at either a free edge or middle of span. The designer selects configuration of arch strips that is appropriate for the load case being considered. For example, the configuration in Fig. 1 would be appropriate for a concentrically loaded column with no moment transferred between slab and column.



Figure 1: Arch Strips at Interior Column

Figure 2 shows the assumed loading diagram at maximum load on a side view of one arch strip of width *b* (dimension in and out of page)*.* The arched strut acting within the strip is shown as a dashed line. The flexural support of the strip, *Ms*, is the sum of the negative and positive moments acting at the ends of the strip. *Ps* equilibrates the vertical component of the arched strut at the face of the support.



Figure 2: Arch Strip with Asymmetric Loading

The arch strip is loaded on both sides in shear. In the general case, the loads tributary to each side will not be equal. A factor, $χ$ (Alexander 1999), is defined as the ratio of smaller tributary load or area to the larger. For a symmetric case, $χ$ is equal to one. In the most extreme asymmetric case, occurring at edge and corner columns, the strip is loaded on one face only and $χ$ is equal to zero.

At ultimate, the side shear is limited to the one-way shear capacity of the slab, *vc*. For static loading, where the position of the concentrated load does not move relative to the slab, a stable cracking regime can develop around the loaded area. There may be local yielding of reinforcement. Under these conditions, the distribution of side shear along the along the strip is reasonably modeled as rectangular and stepped (Afhami et al, 1998). From static equilibrium:

[1] $l\_{s}=\sqrt{\frac{2M\_{s}}{\left(1+χ^{2}\right)v\_{c}}}$

In the case of very wide arch strips or where *Ms* is small, $\left(1+χ\right)l\_{s}$ may be less than the width *b.* In this case it is assumed that the arched strut does not develop and the column face is loaded in one-way shear. Hence:

 [2] $P\_{s}=\left(1+χ\right)v\_{c}l\_{s} but not less than b×v\_{c}$

* 1. **Arched Strut for Patch Load**

An extension of the Strip Model (Lantsoght et al. 2017) examines a one-way slab bridge under a concentrated load applied through a stiff platen. The capacities given by the method are in good agreement with test results. There is a subtle difference between the case of a load applied through a hard plate (or column) and a true patch load. Figure 3 shows the loading diagram for an arch strip that is distributing a patch load applied over length *c*.



Figure 3: Strip with Patch Loading

As before, a dashed line indicates the arched strut acting within the strip. In this case, the arch is doubly curved in response to the distributed loads applied to the strip. From static equilibrium, the length *ls* is given as:

[3] $l\_{s}=\frac{c\left(1+χ\right)}{2\left(1+χ^{2}\right)}\left[-1+\sqrt{1+\frac{8M\_{s}\left(1+χ^{2}\right)}{\left(1+χ\right)^{2}c^{2}v\_{c}}} \right]$

Equation [3] reduces to equation [1] as the dimension *c* approaches zero. As before, the total load transferred, *Ps*, is given by equation [2].



 a) Partial Plan b) Section

Figure 4: Approach Slab

1. **APPLICATION TO STATIC LOADING OF AN APPROACH SLAB**
	1. **Description of Problem**

Figure 4 shows a partial plan and section of 6-meter approach slab, 300 mm thick. The approach slab is treated as having a span, *a*, that is ¾ of the length of the slab, *l*. The patch load (one wheel of axle 4 of the CL-W truck defined in S6-14) is located a distance $ξ$*a* from the abutment backwall. The specified compressive strength of concrete and yield strength of steel are 35 MPa and 400 MPa, respectively.

A detail plan of the patch load is shown in Fig. 5. The dimensions of the patch load are taken from the Canadian Highway Design Code; hence, $r=300 mm$ and $u=250 mm$. The shear span $l\_{v}$ is the distance between the near support and the side of the loaded area. Assuming the patch load is not too close to the free edge of the slab, axis 1 may be treated as a line of symmetry and, therefore, a line of zero shear. From static equilibrium of the overall slab, axis 2 is a line of zero shear.



Figure 5: Detail Plan of Patch Load

* 1. **Relevant Capacities**
		1. **One-Way Shear**

Consistent with the Canadian Bridge Design Code (CSA 2014), the one-way shear *vc* is taken as:

[4] $v\_{c}=2.5 β ϕ\_{c} f\_{cr} d\_{v}$ where $β= ^{230}/\_{(1000+d\_{v})}$

Taking $ϕ\_{c}=0.75$, $d\_{v}=0.72 h=216 mm$, and $f\_{cr}=0.4\sqrt{f\_{c}^{'}}=2.37 MPa$ results in $v\_{c}=181 kN/m$.

* + 1. **Local Bending Moments**

It is tempting to assume that the local bending moments are simply the resistances provided by flexural reinforcement in the vicinity of the applied load; however, this is not generally the case. In most instances, the flexural reinforcement for the entire approach slab span is proportioned on the basis of the maximum positive moment at mid-span. The design bending moments associated with a patch load that is applied away from the mid-span will be smaller.

The approach used here is to estimate the local elastic bending moments with a series solution (Timoshenko and Woinowsky-Krieger 1959) that models a rectangular patch load on a simply supported slab of infinite width. One could also use a simple linear finite element model. The local elastic bending moments are linear with applied load and independent of slab stiffness. Table 1 lists results for local bending moments under a patch load of 100 kN. Only positive bending moments are considered here. While not equal to zero, the negative moments associated with a patch load measuring 250 mm by 600 mm on a 4.5 m simple span are negligible.

Table 1: Local Elastic Bending Moments\* under 100kN Wheel Load

|  |  |  |
| --- | --- | --- |
| $$ξ$$ | ms1 (kN⋅m/m) | ms2  (kN⋅m/m) |
| 1/15 | 30.79 | 8.52 |
| 0.075 | 33.46 | 9.49 |
| 0.1 | 40.44 | 11.75 |
| 0.15 | 49.81 | 14.88 |
| 0.2 | 56.04 | 16.96 |
| 0.35 | 66.02 | 20.28 |
| 0.5 | 68.79 | 21.21 |

\*4.5 meter simple span with *u* = 250mm and *r* = 300mm

* 1. **Layout of Arch Strips**

Figure 6 shows a plan view of the patch load described in Fig. 5, now including a layout of arch strips. The patch load is assumed to be sufficiently far from the free edge of the slab that edge effects are negligible. Axes 1 and 2 are lines of zero shear, by symmetry in the case of axis 1 and by statics in the case of axis 2. Strip 1 is parallel to the span of the approach slab. The dimension *u'* is defined as $\left(1-ξ\right)u$. Front, back, and side faces of the patch load are shown, with front referring to the face closest to the near support and back referring to the face farthest from the near support.



Figure 6: Plan of Patch Load showing Arch Strips

* + 1. **Strip 1**

Strip 1 is at the front face of the patch load. Figure 7a shows a loading diagram for Strip 1. By symmetry, $χ\_{1}$ is equal to 1, making the total load on the strip equal to 2*vc*. Substituting *u'* for *c* in equation [3] leads to:

 [5] $l\_{s1}=\frac{u^{'}}{2}\left[-1+\sqrt{1+\frac{4M\_{s1}}{\left(u^{'}\right)^{2}v\_{c}}}\right]$

The capacity of the strip, Ps1, is given by equation [2] but this holds only if *ls1* is less than the available shear span, *lv*, as defined in Fig. 6. Where equation [5] gives a length greater than *lv*, a fraction of the load will be carried by direct strutting from the applied load to the support. So:

[6] $P\_{s1}=2v\_{c}l\_{s1} but not less than 2r×v\_{c}$ (where $l\_{s1}\leq l\_{v}$) and

[7] $P\_{s1}=2v\_{c}l\_{v}+\frac{M\_{s1}-v\_{c}l\_{v}\left(l\_{v}+u^{'}\right)}{l\_{v}+^{u^{'}}/\_{2}}$ (where $l\_{s1}>l\_{v}$)

* + 1. **Strip 2**

The loading of strip 2, associated with the side face of the patch load, is shown in Fig. 7b. Statics of the simple span dictate the value of $χ\_{2}$. In general, $χ\_{2}=^{ξ}/\_{\left(1-ξ\right)}$ where $ξ\leq 0.5$.

With the appropriate substitutions, equation [3] becomes:

[8] $l\_{s2}=\frac{r\left(1+χ\_{2}\right)}{2\left(1+χ\_{2}^{2}\right)}\left[-1+\sqrt{1+\frac{8M\_{s2}\left(1+χ\_{2}^{2}\right)}{\left(1+χ\_{2}\right)^{2}r^{2}v\_{c}}}\right]$

From equation [2] the total load for the strip is:

[9] $P\_{s2}=\left(1+χ\right)v\_{c}l\_{s2} but not less than u×v\_{c}$

  

 a) Strip 1 b) Strip 2

Figure 7: Loading of Strips

* + 1. **Subcritical Strip**

Except for the particular case where a patch load is applied at mid-span ($ξ=0.5$ and $χ\_{2}=1$), stress conditions at the back face of the patch load can never govern. The load transfer associated with the back face of the patch load is constrained by statics to be $χ\_{2}×P\_{s1}$.

* 1. **System Capacity**

The total capacity is the sum of the individual strip contributions. Hence:

[10] $P\_{total}=\left(1+χ\_{2}\right)P\_{s1}+2P\_{s2}$

Table 2 summarizes the calculated results. An iterative process is used to find each value of static punching load, *Ptotal*, that is consistent with local bending moments and the calculated strip capacities for load applied at different positions. Two limits are imposed on local bending moments. An upper limit of 182 kN⋅m on *Ms1* comes from the estimated maximum moment resistance for c/d equal to 0.5 (CSA, 214). The second limit is based on *Ms2* being controlled by the greater of minimum shrinkage and temperature reinforcement (500 mm²/m from Clause 8.12.6) or minimum secondary moments ($m\_{s2} \geq 0.2m\_{s1}$ from Clause 5.6.5.3) in the non-spanning direction. This limits Ms2 to 15.2 kN⋅m. The values of W given in Table 2 are the total weights of the corresponding CL-W design truck, assuming axle 4 with a dynamic load allowance of 0.4 and a live load factor of 1.7.

Table 2: Static Punching Limit of an Approach Slab under a Patch Load\*

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$ξ$$ | $$χ\_{2}$$ | *lv*(mm) | *Ms1* (kN⋅m) | *Ms2*  (kN⋅m) | *ls1*(mm) | *ls2*(mm) | *Ps1*(kN) | *Ps2*(kN) | *Ptotal*(kN) | W(kN) |
| 1/15 | 0.071 | 175 | 92 | 8.5 | 604 | 184 | 379 | 45.3 | 497 | 1490 |
| 0.075 | 0.081 | 213 | 90 | 8.5 | 599 | 184 | 331 | 45.3 | 449 | 1347 |
| 0.1 | 0.111 | 325 | 88 | 8.5 | 594 | 181 | 245 | 45.3 | 363 | 1089 |
| 0.15 | 0.176 | 550 | 122 | 10.1 | 720 | 200 | 269 | 45.3 | 407 | 1222 |
| 0.2 | 0.250 | 775 | 174 | 14.5 | 884 | 250 | 323 | 56.6 | 517 | 1552 |
| 0.35 | 0.538 | 1450 | 182 | 15.2 | 924 | 223 | 335 | 62.3 | 640\*\* | 1379 |
| 0.5 | 1.000 | 2125 | 182 | 15.2 | 941 | 176 | 341 | 63.7 | 810\*\* | 1323 |

\*4.5 meter simple span with *u* = 250mm and *r* = 300mm

\*\*shear capacity based on maximum *Ms1*

For comparison, the two-way shear provision of S6-14 assume a critical shear stress acting on a critical shear perimeter that surrounds the patch load. For this example, the factored two-way shear resistance by code is approximately 700 kN. The shear is assumed to be uniformly distributed around the perimeter, regardless of the position of the patch load within the span.

Some observations regarding the results shown in Table 2 are:

* In all cases, the calculated total load is well in excess of any conventional design load. The punching capacity of 363 kN for a single wheel of axle 4 is consistent with a CL-1089 truck.
* The analysis shows that the shear capacity under a concentrated patch load depends upon where the load is placed in the span. For the approach slab considered, the shear capacity is minimum when the concentrated patch load is centred approximately 10% of the span away from the backwall support.
* Results of additional calculations (not shown in Table 2) show that, for a concentrated load centred 10% of the span from the backwall with a concrete strength of 30 MPa, shear demand should not exceed shear capacity as long as the slab thickness is at least 230 mm. For a slab thickness of 250 mm, shear demand should not exceed shear capacity as long as the concrete strength is at least 25 MPa. In both case it is assumed that adequate flexural reinforcement is provided.
* The two-way shear resistance given by S6-14 is in reasonable agreement with the maximum value for *Ptotal* in Table 2. However, the resistance given by S6 does not account for asymmetry of response when the patch load is not applied near the centre of the span. As a result, it may overestimate the capacity when the load is not applied near midspan.
* For a patch load near the centre of the span, the limit on maximum local bending moment defined by $\frac{c}{d}=0.5$ defines capacity. Compression failure in flexure is expected to occur before punching.
* Where static punching governs the capacity of the system, the contribution in the transverse direction (parallel to axis 2) is given by the one-way shear minimum. The arched strut in strip 2 does not develop.
1. **APPLICATION TO MOVING LOADS ON AN APPROACH SLAB**
	1. **Static Punching**

Based on the preceding analysis, static punching under pneumatic wheel loading, at least in the case of an approach slab, does not appear to be a practical concern for typical proportions and material properties. A load of 363 kN on a standard design wheel footprint of 0.25m by 0.6m gives an average pressure in excess of 2400 kPa, roughly three times the inflation pressure of highway tires.

The code treatment of two-way shear comes from tests of column-slab connections. For these tests, a non-uniform distribution of shear is associated with a moment transferred between slab and column. There is no provision for a non-uniform distribution of shear in the absence of moment transfer. As a result, the method is not suitable for modeling shear around a concentrated load applied at any location within a span. As an aside, this is a problem for column-slab connections as well. The distribution of shear around the loaded perimeter can be non-uniform even if there is no moment transfer between slab and column.

It is debateable whether static punching shear is the appropriate model to be using for non-static, concentrated patch loads. The diagonal cracking associated with static punching occurs well before reaching the ultimate load (Regan and Braestrup 1985). The crack system is stable and can sustain repeated loading and unloading, but this is only because static loads not move around relative to the slab.

* 1. **Analysis for Moving Loads**

To adapt the Strip Model for repeated, moving loads, a number of changes are proposed. These are:

* A triangular loading diagram is used for strip 1.
* No arch struts develop on the side faces (strip 2) of the patch load.
* The one-way shear is reduced to account for fatigue

The stepped loading diagram shown in Figure 7, appropriate for static (stationary) loading, depends upon sufficient torsional moment developing on the side faces of the strip. Non-linear finite element analysis (Afhami 1998) shows that such torsional moments develop after cracking of the slab and local yielding of the flexural reinforcement. Cracking and local yielding would lead to an accumulation of damage and degradation of strength. Prior to cracking and yielding, the distribution of internal shear is approximately triangular. Test results (Alexander et al. 1995) are consistent with a triangular distribution of shear if the effect of torsion is not included.

Replacing the rectangular distribution of load shown in Figure 7a with a triangular distribution results in:

[11] $l\_{s1}=\frac{3u^{'}}{4}\left[-1+\sqrt{1+\frac{16M\_{s1}}{3\left(u^{'}\right)^{2}v\_{c}}} \right]$

Equations [6] and [7] become:

[12] $P\_{s1}=v\_{c}l\_{s1} but not less than 2r×v\_{c}$ (where $l\_{s1}\leq l\_{v}$) and

[13] $P\_{s1}=v\_{c}l\_{v}+\frac{M\_{s1}-v\_{c}l\_{v}\left(l\_{v}+u^{'}\right)}{l\_{v}+^{u^{'}}/\_{2}}$ (where $l\_{s1}>l\_{v}$)

With no strutting behaviour on the side faces, the lower limit of equation [9] applies; hence the load distributed at each side face is taken as $u×v\_{c}$.

Finally, a permissible one-way shear appropriate for repeated, moving load is needed. The one-way shear limit from equation [4] is associated with diagonal cracking of concrete. Repeated application of load, each following a slightly different track and generating this level of stress, would result in a widening zone of damaged concrete and a degradation of shear strength. It follows that a lower limit on one-way shear is needed. While fatigue data on concrete is both limited and scattered, 50% of the static shear strength is reasonable for unlimited loading cycles (Ruiz et al. 2015). The one-way shear limit for unlimited repetition of load is therefore:

[14] $v\_{c}=1.25 β ϕ\_{c} f\_{cr} d\_{v}$ where $β= ^{230}/\_{(1000+d\_{v})}$

Figure 8 shows an isometric view of the revised load distribution for a patch load on an approach slab, incorporating the modifications outlined above.



Figure 8: Proposed Internal Shear Distribution for Moving Loads

* 1. **Some Results**

Figure 9 shows some results for a 6 metre approach slab, considering the repeated application of axle 4 of a CL-800 truck. The basic wheel load is 112kN (from axle 4). A dynamic load allowance of 0.4 and a load factor 1.0 (Fatigue Limit State 1 under S6-14) results in a fatigue design wheel load of 156.8kN. Two positions of the wheel load are considered, one at 10% and the other at 15% of the span away from the backwall (i.e. Xi $(ξ)$ equal to 0.1 or 0.15).

a) $f\_{c}^{'}=35MPa$ b) $h=250mm$

Figure 9: Capacity/Demand

1. **DISCUSSION**

From a design perspective, the fatigue loading results shown in Fig. 9 suggest the approach slab are very similar to those presented earlier for static loading. An approach slab that is satisfactory under factored ultimate load will also have sufficient capacity under fatigue loading. For a concrete strength of 35 MPa, the fatigue shear in Fig. 9a is not critical so long as the slab thickness is about 200 mm. This is not too different from the value of 230 mm obtained for static loading. For a slab thickness of 250 mm, both the static and moving load analyses suggest the critical concrete strength is about 25 MPa.

For an approach slab, the conditions needed for punching to be a concern are not typical of current construction but they may be found in older structures. Slab thicknesses in the order of 230 mm or less were not uncommon in the past. Lower concrete strengths may also be found in older bridges. Other elements such as roof slabs and bridge decks will have different global static conditions that may or may not make them more susceptible to punching than the approach slab considered here.

While the focus of this work was on shear resistance, it should be noted that the demand for secondary moment in the one-way slab may be significant, especially where the secondary moment is sufficient to cause longitudinal (parallel to span) cracking of the slab. Such cracking would subject the secondary reinforcement to fatigue loading. Fracture of the secondary reinforcement would lead to loss of one-way shear transfer in a direction perpendicular to the span of the approach slab.

1. **CONCLUSIONS**

The arched strut of the Strip Model, originally developed to describe load transfer between a slab and column, is adapted to deal with a patch load applied to an approach slab. Variations to address both ultimate strength under static loading and resistance under fatigue loading are presented.

S6-14 assesses two-way shear using a design model meant for column-slab connections. This design model is incorrect when applied to patch loads at some arbitrary position in the span as it does not account for asymmetry of stress distribution around the critical perimeter of the patch load when the position of the load approaches a support.

While the treatment of two-way shear in S6-14 is questionable, the results presented here show punching of an approach slab meeting current design standards is not a concern.

**References**

Afhami, S., Alexander, S.D.B., and Simmonds, S.H. 1998. Strip Model for Capacity of Slab-Column Connections. *Structural Engineering Report No. 223.* Department of Civil and Environmental Engineering, University of Alberta, Edmonton. 231 pp.

Alexander, S.D.B. 1999. Strip Design for Punching Shear. *ACI SP-183: The Design of Two-Way Slabs*. ACI, Farmington Hills, Michigan, USA. 183 pp.

Alexander, S.D.B. 2017. Shear and Moment Transfer at Column-Slab Connections. *ACI SP-315: ACI-fib International Symposium - Punching Shear of Structural Concrete Slabs*. ACI, Philadelphia, PA, USA. pp. 1-22.

Alexander, S.D.B., Xilin Lu, and Simmonds, S.H. 1995. Mechanism of Shear Transfer in a Column-Slab Connection. *Proceedings of the CSCE Annual Conference,* Ottawa, Ontario. pp. 207-216.

CSA (Canadian Standards Association). 2014. *Canadian Highway Bridge Design Code*. S6-14 – Reprinted July 2017. CSA Group, Mississauga, Ontario, Canada. 875 pp.

Lantsoght, E.O.L., van der Veen, C., de Boer, A., and Alexander, S.D.B. 2017. Bridging the Gap between One-Way and Two-Way Shear in Slabs. *ACI SP-315 - ACI-fib International Symposium - Punching Shear of Structural Concrete Slabs*. ACI, Philadelphia, PA, USA. pp. 187-214.

Regan, P.E. and Braestrup, M.W. 1985. Punching Shear in Reinforced Concrete: A State of the Art Report. Bulletin d'Imformation No. 168. Comité Euro-International du Béton. Lausanne, Switzerland.

Ruiz, M.F., Zanuy, C., Natário, F., Gallego, J.M., Albajar, L. and Muttoni, A. 2015. Influence of Fatigue Loading in Shear Failures of Reinforced Concrete Members without Transverse Reinforcement. *Journal of Advanced Concrete Technology*, Japan Concrete Institute, **13:** May: 263-274.

Timoshenko, S. and Woinowsky-Krieger, S. 1959. *Theory of Plates and Shells*. Second edition. Engineering Societies Monographs. McGraw-Hill, New York. 580 pp.