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Behavior and Strength Modeling of GFRP-Reinforced Concrete Deep Beams with Web Openings

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Abstract: This paper reports the testing of four full-scale GFRP-reinforced concrete deep beams with and without web openings. All deep beams tested had the same overall geometrical dimensions. Three deep beams were having web openings while one solid deep beam was used as a reference. Additional reinforcement around the openings were employed in two deep beams to investigate the benefit of using such reinforcement configuration. As expected, the presence of web openings in a location interrupting the main diagonal strut reduced the strength of the deep beam by 46% comparing to the solid deep beam, while providing extra reinforcement around the openings noticeably increased the shear strength of the deep beams. The crack propagation and failure mode of the tested deep beams were very similar to that of steel-reinforced deep beams found in the literature. The applicability of different analytical models developed for steel-reinforced deep beams to predict the strength of the tested deep beams was investigated. The model developed by Kong and Sharp (1977) produced safe and adequate estimations, while Yang et al. (2007) model overestimated the capacity of the deep beams. On the other hand, Tseng et al. (2017) model was found to be not applicable to GFRP-reinforced deep beams.

1 INTRODUCTION

Reinforced concrete deep beams are usually used when designing transfer girders or bridge bents. In deep beams, openings are placed in the web area to facilitate fundamental services, such as conduits, and network system access. The use of web openings often interrupts the load transfer by concrete struts forming between the loading point and the support. This interruption causes an acute decrease of strength and serviceability of the deep beams (Kong and Sharp 1973, Yang et al. 2006). Hence, the strength evaluation and reinforcement details around openings in deep beams are of essential consideration to mitigate the strength degradation caused by the openings (Yang et al. 2007, Garber et al. 2014).

Deep beams with web openings exhibit diagonal cracks developed above and below openings due to the high-stress concentration at the openings' corners and the abrupt change of the main loading path. These diagonal cracks decrease the effective concrete compressive strength in the diagonal strut caused by the high transverse tensile strains at the diagonal concrete plane. Researchers examined the use of additional steel reinforcement around the opening, which increased the ultimate strength compared to deep beams without extra steel bars around the opening. However, experimental investigations showed that the strength of steel-reinforced deep beams with web openings was controlled by the yielding of steel bars provided around the opening followed by failure along the main diagonal struts (Tan et al. 2004, Yang et al. 2007, Campione and Minafo 2012).

Fiber-reinforced-polymer (FRP) bars are emerging as a realistic and cost-effective alternative reinforcing material to prevent costly corrosion issues related to steel reinforcement. The high tensile strength in the longitudinal direction of FRP bars, with the no-yielding plateau, comparing to steel ones could be effective in resisting the stress concentrations around the web openings and increasing the strength of the deep beam. Additionally, the successful implementation of FRP bars as main reinforcement in reinforced concrete deep beams (Andermatt and Lubell 2013a, Farghaly and Benmokrane 2013), motivated the examination of using FRP reinforcement around the web opening. However, the relatively low elastic modulus of FRP bars could affect the behavior of FRP-reinforced deep beams with web openings. Therefore, the main objective of this research proposal is to assess the performance of FRP-reinforced deep beams with web opening.

Researchers developed several analytical models to predict the capacity of steel-reinforced deep beams. However, existing models focused almost exclusively on solid deep beams, with little studies on deep beams with web openings and even fewer publications considering the effect of web reinforcement. Furthermore, each model possesses several assumptions with different load carrying mechanism leading to a different level of safety. Hence, the current study aims also at assessing the applicability of available analytical modeling for steel-reinforced deep beams to predict the capacity of GFRP-reinforced deep beams with web openings.

2 Experimental Program

The experimental program included testing four full-scale deep beams totally reinforced with GFRP bars. Th deep beams were measuring 1200×300 mm in cross-section with a clear span of 3000 mm. Three deep beams were having web openings (GO1, GO2, and GO3) while one solid deep beam was used as a reference (GS). Table 1 presents a summary of the testing program. The openings locations were chosen such that the center of an opening was located at the midpoint of a centerline connecting the tips of the loading and support plates. Openings size of 304 mm high and 340 mm wide were chosen to reduce the capacity of the deep beam by approximately 50% of the capacity of the reference solid deep beam (Yang et al. 2007).

Table 1: Properties of the BFRP and Steel reinforcement

Specimen	f'_c (MPa)	Longitudinal Reinforcement		Crack Control Reinforcement				Extra Reinforcement around Opening			
				Vertical		Horizontal		Vertical		Horizontal	
		Bar size	Number of bars	Bar size	Spacing (mm)	Bar size	Spacing (mm)	Bar size	Number of bars	Bar size	Number of bars
GS	37.0	#8	8	#4	200	#5	200	N/A			
GO1	37.4	#8	8	#4	200	#5	200	-	-	-	-
GO2	37.4	#8	8	#4	200	#5	200	#4	2	#4	2
GO3	44.6	#8	8	#4	200	#5	200	#4	4	#4	4

The amount of GFRP reinforcement in Table 1 was calculated according to the CSA S806 (2012) requirements. Different reinforcement details around openings were used to assess the benefit of using additional reinforcement. GO1 was reinforced with vertical and horizontal reinforcement only without using extra reinforcement around the openings, while additional vertical and horizontal reinforcement were provided around the openings of GO2 and GO3. Figure 1 shows the assembled GFRP reinforcement for GO1. All deep beams were casted using normal ready-mix concrete with a target 28-day compressive strength of 35 MPa. Five concrete cylinders (100×200 mm) were cast with each specimen and kept under the same environmental conditions. Table 1 gives the actual concrete compressive strengths. All deep beams were tested under two-point loading with a load-controlled rate of 20 kN/min. The deep beams were instrumented with different strain gauges, linear variable differential transducers, and potentiometers to monitor the behavior of the deep beams. Details of the testing setup and testing procedure were provided elsewhere (Mohamed et al. 2017a).

Figure 1: Assembled GFRP cage of GO1 specimen.

3 Summary of Experimental Results

The effect of providing web openings on the behavior of GFRP-reinforced deep beams was assessed by testing four GFRP-reinforced deep beams with and without web openings. Figure 2 shows the attained ultimate loads for the tested deep beams. It shall be mentioned that the *f'c* had much less impact on the capacity of deep beams with openings than on solid deep beams, even when concrete was the limiting factor (Yang et al. 2006). Therefore, the capacities of the deep beams were not normalized to consider the change in *f'c* as shown in Figure 2. Figure 2 shows that the presence of web openings reduced the capacity by 46% when comparing GO1 to GS. Adding reinforcement around the openings on GO2 and GO3 increased the capacity of the deep beam by 22% and 57%; respectively, comparing to GO1. These results clearly explain that the additional reinforcement around the openings of GFRP-reinforced deep beams is essential to retrieve the capacity of the deep beam and increasing the amount of additional reinforcement increases the capacity of the deep beam. This retrieved capacity in the case in GFRP-reinforced deep beams was found to be higher than that in case of steel-reinforced deep beams when adding approximately the same percentage of extra reinforcement around the openings (Campione and Minafò, 2012; Yang et al., 2007; Kong and Sharp, 1973).

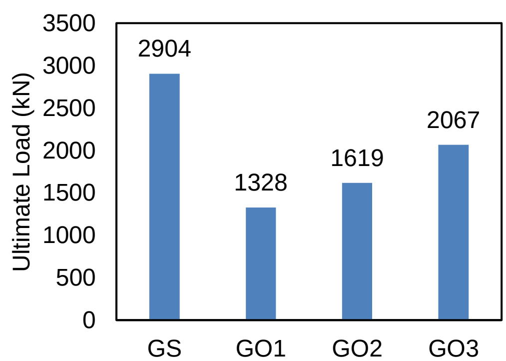

Figure 2: Ultimate capacity of tested deep beams

Similar crack propagation was observed for deep beams with web openings. The first crack was initiated at the corner openings and propagated toward the loading/support points with increasing the loads. The main diagonal crack was formed at the openings' corners opposite the loading points and extended rapidly toward the edges of the loading/support plates at approximately 60% of the ultimate load. Failure of the deep beams occurred along this main diagonal crack, which also the failure plane in steel-reinforced deep beams with web openings found in the literature. Figure 3 shows the typical failure of the deep beams with openings. The additional reinforcement around the openings in GO2 and GO3 induced a larger number of diagonal cracks at the openings' corners with smaller crack widths comparing to GO1. Further, failure of

GO1 and GO2 was associated with rupture of the vertical reinforcement at the bent portions, while adding extra reinforcement around GO3 web openings induced failure as concrete crushing without rupturing in GFRP stirrups.

Figure 3: Typical Failure of deep beams with web openings (GO1)

Figure 4 shows the load-deflection response of the tested deep beams. All specimens exhibited the same stiffness prior to the initiation of the first crack followed by a reduction in stiffness with the formation of the first crack. Figure 4 also shows that specimens having additional reinforcement (GO2 and GO3) attained higher stiffness after cracking than specimens GO1. Therefore, adding reinforcement around the openings compensated for the reduced stiffness due to the openings. GO2 and GO3 exhibited approximately the same stiffness; however, doubling the number of bars around the openings in GO3 resulted in 50% more deformation before the failure comparing to GO2.

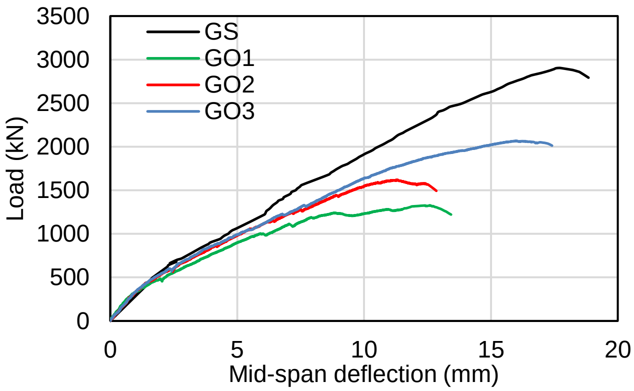

Figure 4: Load-deflection response of tested deep beams

4 Analytical Modeling

Design codes and standards recommend the use of the strut-and-tie model (STM) to account for the influence of discontinuity due to the presence of openings on the strength of the reinforced concrete members (ACI 318, 2014; CSA A23.3, 2014, CSA S6, 2014). Accordingly, researchers illustrated that the STM provides a great tool for predicting the ultimate strength of steel-reinforced deep beams with web openings (Maxwell and Breen, 2000; Tan et al., 2004; Brena and Morrison, 2007; Shaoo et al., 2012; Garber et al. 2014). The accuracy of STM in determining the strength of FRP-reinforced deep beams without web openings was examined and revealed adequate estimations (Andermatt and Lubell 2013b; Farghaly and

Benmokrane 2013; Mohamed et al. 2016). Further, Mohamed et al. (2016) proposed a strut-and-tie-based model for predicting the strength of FRP-reinforced deep beams.

The STM provides a conceptually simple design methodology and easy visualization of the flow of forces, however, its implementation is usually complicated for more complex truss models as in deep beams with web openings. The applicability of the model in the current study required performing iterative and time-consuming calculations that involved extensive graphical representations of struts and ties. Further, uncertainties in defining the strength, stiffness, and ductility capacities of STM components could also deviate engineers from using complex STMs. Therefore, researchers developed different analytical modeling to predict the shear strength of steel-reinforced concrete deep beams with web openings (Kong and Sharp, 1977; Tan et al., 2004; Yang et al., 2007; Tseng et al., 2017). Due to the complex flow of stresses in deep beams with web openings, very limited prediction models were found in the literature. The applicability of these models originally developed for steel-reinforced deep beams to be used in GFRP-reinforced deep beams was investigated in the current study.

4.1 Kong and Sharp (1977)

Kong and Sharp (1977) assumed a structural idealization based on two load paths to transfer the load from the loading points to the supports. Considering concrete struts joining the loading and reaction points, and tie action of longitudinal reinforcements, the authors gave the following equation to calculate the ultimate shear strength of deep beams with openings interrupting the main load path:

[1] $V_2 = C_1\left[1 - 0.35\frac{k_1 x}{k_2 h}\right] f'_t b k_2 h + \sum \lambda C_2 A \frac{y_1}{h} sin^2\ \alpha_1$

The left term of the equation presents the concrete contribution to the shear strength, while the right term presents the reinforcement contribution. In the left term, the coefficient C_1 is assumed equal to 1.40 for normal-weight concrete, and 1.35 for lightweight concrete. The distances $k_1.x$ and $k_2.h$ are the horizontal and vertical distance between the inner edge of the support plate to the corner of the opening; respectively, and b and h are the width and overall depth of beam; respectively.

In the right term of the equation, Kong and Sharp (1977) presented the strength of the steel bars by an empirical coefficient, C_2, equal to 300 MPa for deformed bars and 130 MPa for plain round bars. A C_2 value of 130 MPa was implemented in the current study. The C_2 coefficient was multiplied by another coefficient, λ, to account for the pronounced effect of web reinforcement crossing the main diagonal load path. Hence, λ was equal to 1 for the longitudinal reinforcement and 1.5 for web reinforcement. The strength of the deep beams is very sensitive to the location and arrangement of web reinforcement. Hence, Kong and Sharp (1977) accounted for such effect by considering for the sum of the reinforcement crossing the diagonal crack in the right term of the equation, at which A is the area of the reinforcement located at a distance y_1 from the top of the deep beam and inclined with angle α to the main diagonal crack.

4.2 Yang et al. (2007)

Yang et al. (2007) predicted the shear strength of deep beams with web openings using upper-bound analysis by idealizing the deep beam as an assemblage of two rigid blocks separated by two yield lines presenting the main diagonal crack (Figure 5). The rigid block above the support was assumed to undergo a relative rotation around an instantaneous center having coordinates of X_{ic} and Y_{ic} relative to the global origin at the end supports. The same assumption was found to be valid for GFRP-reinforced deep beams (Mohamed et al. 2017a). Yang et al. (2007) also assumed that the concrete to be rigid perfectly plastic material with modified Coulomb failure criteria, which also can be applied to GFRP-reinforced deep beams.

Yang et al. (2007) proposed equation also considered total shear strength as the sum of the concrete and reinforcement contribution to the shear strength. However, Yang et al. (2007) accounted for the concrete stresses along the upper and lower chords of the main diagonal crack and the rotation of the concrete blocks around the instantaneous center. Accordingly, Yang et al. (2007) predicted the total shear strength of a deep beam with openings and having web reinforcement as:

[2] $V_n = \frac{bh}{2X_{ic}}\left[(f_c^*)_t r_t(1-\sin\alpha_t)\frac{1-k_2-m_2}{\sin\theta_t} + (f_c^*)_b r_b(1-\sin\alpha_b)\frac{k_2}{\sin\theta_b} + 2\sum_{i=1}^{m}\rho_{si} f_{yi} r_s \cos\psi_{si}\right]$

where *r* is the distance between the midpoint of the chord of the main diagonal crack and the instantaneous center, and *α* is the angle between the relative displacement at midpoints of the chord and main diagonal crack, subscripting by *t* and *b* indicate the upper crack formed at the top chord above opening and the lower crack formed at the bottom chord below opening, respectively. For the effect of the longitudinal and web reinforcement, ρ_{si} is calcualted as the reinforcement ratio *i* crossing the diagonal crack and equal to A_s/bh, f_{yi} is the yield strength of the reinforcement, r_{si} is the distance between the intersection point of the reinforcing bar *i* with the main diagonal crack and I.C. (Figure 5); and ψ_{si} is the angle between the relative displacement about I.C. and the reinforcing bar *i* crossing the yield line.

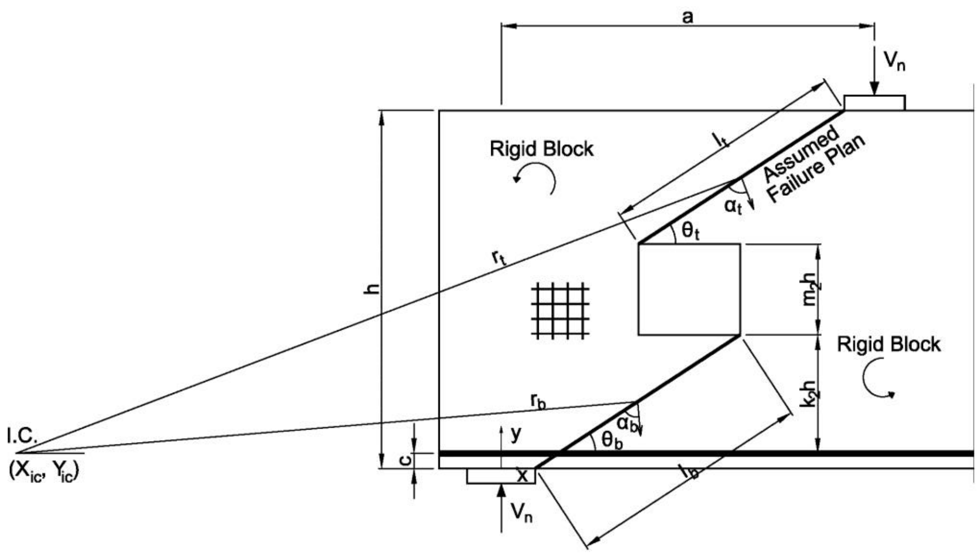

Figure 5: Idealized failure mode of deep beams with openings by Yang et al. (2007)

Both top and bottom chords above and below openings in the deep beams tested were considered to be in a state of biaxial tension-compression. The presence of transverse tensile strains makes the compressive strength of cracked concrete greatly deteriorated as concluded in panel tests subjected to biaxial tension-compression carried out by Vecchio and Collins (1993). The same principles were also found to be valid for GFRP-reinforced deep beams (Mohamed et al. 2017b). Accordingly, Yang et al. (2007) considered the concrete tensile strength f^*_c as a function of the compressive strength and the ratio of the principal strains as follows:

[3] $f_c^* = \frac{\zeta}{1+K_c K_f} f'_c$

[4] $K_c = 0.35\left(-\frac{\varepsilon_1}{\varepsilon_3} - 0.28\right)^{0.8} \geq 1.0$

[5] $K_f = 0.1825\sqrt{f'_c} \geq 1.0$

[6] $\zeta = \frac{1}{\sqrt{1+\frac{d}{25d_a}}}$

where ε_1 and ε_3 are the principle tensile and compressive strains; respectively, and d_a is the aggregate size. Mohamed et al. (2017) found that the horizontal reinforcement was less effective than the vertical reinforcement in resisting the shear strength of the deep beams. Further, the horizontal web reinforcement would resist the rotation of the rigid concrete block as assumed by Yang et al. (2007) by means of dowel action, which found to be negligible for GFRP reinforcement. Therefore, the contribution of the horizontal web reinforcement was neglected when applying Yang et al. (2007) model to the GFRP-reinforced deep beams. Further, the tensile strength of the vertical bars was limited to the tensile strength at the bent portions (469 MPa), since vertical bars were ruptured at the bent portions at failure.

4.3 Tseng et al. (2017)

Tseng et al. (2017) model was based on a simplified strut-and-tie idealization of the deep beams with openings as shown in Figure 6. Loads are transferred by the load paths 134 and 124 in Figure 6. The resistance of nodes 1 to 4 needs to be assessed in order to evaluate the capacity of the member. Rupture to be inevitable whenever the occurrence of reinforcement yielding at node 4, concrete crushing at node 2, or concrete crushing at node 1. The lowest resistance from the four nodes verification is the governing resistance of the specimen. Nevertheless, the model allows for alternative load paths after yielding of steel bars at node 3, which could not be the case when failure controlled by rupturing of GFRP stirrups. Hence, the applicability of the model to the GFRP-reinforced deep beams could not be feasible.

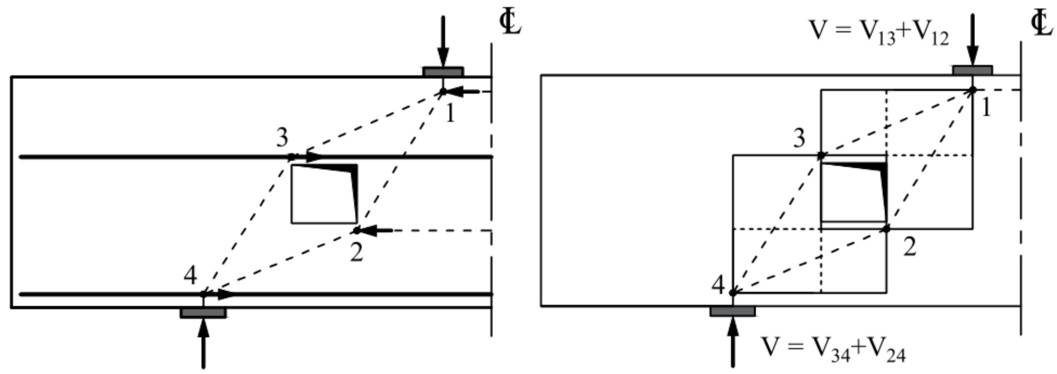

Figure 6: Load paths idealization adopted by Tseng et al. (2017)

4.4 Applicability of the models to the tested deep beams

Figure 7 shows the experimental to predicted capacities using Kong and Sharp (1977) and Yang et al. (2007) models. Kong and Sharp (1977) model adequately predicted the shear capacity of the deep beams with an average experimental-to-predicted value of 1.07 and a coefficient of variation of 8%. The model became more conservative when adding extra reinforcement around the openings. The experimental investigation showed that the extra GFRP reinforcement around openings could be more efficient than that in steel-reinforced deep. Hence, since Kong and Sharp (1977) model was preliminarily built for steel-reinforced deep beams, the model was less sensitive for the addition of extra reinforcement around the openings in case of GFRP-reinforced deep beams.

Figure 7 also shows that Yang et al. (2007) overestimated the capacity of the deep beams leading to unsafe estimations with an average experimental-to-predicted value of 0.87 and coefficient of variation of 12%. The strength of cracked concrete in Yang et al. (2007) model greatly depends on the amount of transverse tensile strain along the failure plans as well as the concrete aggregate interlock. GFRP reinforcement possesses higher strains comparing to steel bars at the same load level, hence, inducing higher concrete softening comparing to steel-reinforced deep beams. Additionally, GFRP-reinforced deep beams exhibit wider crack widths at ultimate comparing to steel-reinforced ones, resulting in less shear strength carried by aggregate interlock. Therefore, the model was not found to be adequate in predicting the strength of the GFRP-reinforced deep beams with web openings.

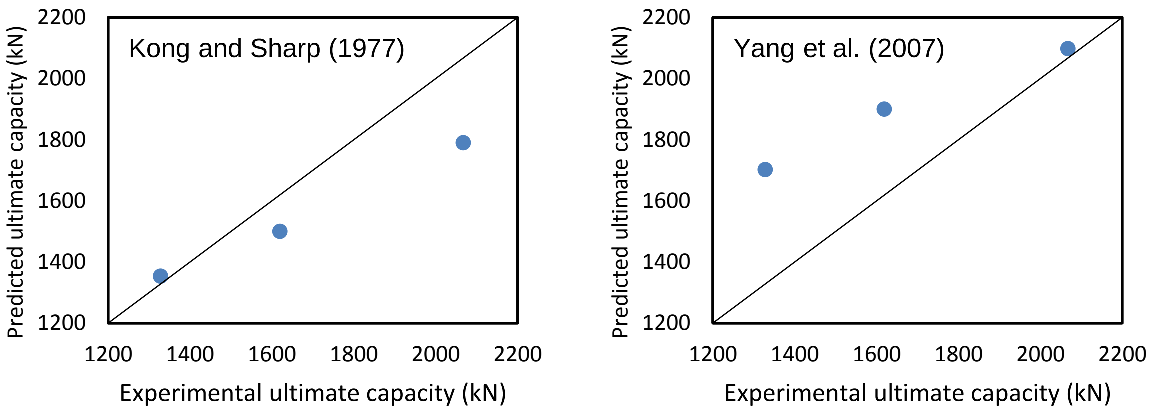

Figure 7: Comparison of test results and predictions

5 Conclusions

Based on the experimental and analytical results presented in the current study, the following conclusions can be drawn:

1. The presence of web openings in GFRP-reinforced concrete deep beam at a location interrupting the load transfer by compression strut with the sizes taken in the current study reduced the capacity by 46% comparing to solid deep beam with the same geometry and reinforcement detail. Adding reinforcement around the openings was capable of increased the capacity of the deep beam and doubling the amount of additional reinforcement further increased the capacity by 28%.
2. Failure of GFRP-reinforced deep beams with web openings occurred along a main diagonal crack connecting the edges of the load plates and openings corners opposite to the load points, which is the typical failure for steel-reinforced deep beams with openings.
3. Failure GFRP-reinforced deep beams with web openings was associated with rupture at the bent portion of the vertical reinforcement around the openings, except for specimen with the highest amount of extra reinforcement around the openings in the current study.
4. The developed model by Kong and Sharp (1977) was found appropriate to predict the capacity of the GFRP-reinforced deep beams with web openings. On the other hand, the proposed model by Yang et al. (2007) overestimated the capacity of the deep beams due to underestimating the influence of the GFRP strains on the strength of the surrounded concrete and overestimating the contribution of aggregate interlock. Additionally, Tseng et al. (2017) model was found to be inapplicable for GFRP-reinforced deep beams with web openings.,

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