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# MULTI-PERIOD LOCATION OPTIMIZATION OF NEW PUBLIC FACILITIES TO MAXIMIZE EQUITY IN ACCESS AND CAPACITY-SATURATION

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**Abstract:** In this paper, the multi-period facility location problem is introduced where the goal is to determine the location of new facilities over a finite time horizon so as to maximize equity, while satisfying all the demand in each time period. In the introduced model, the selection of the location of the new facilities is limited to a number of potential sites. In addition, the number of new facilities that can be opened in each time period is bounded by a budget constraint. The proposed formulation aims to optimize coverage while balancing the excess flow at capacity-saturated facilities. We propose to minimize the sum of demand-weighted travel distance to schools and the total excess flow at supplying facilities. A deterministic analysis was performed to estimate the values of all the uncertainties, including the demand in each time period. An illustrative case study on Sydney's public school network is presented where the location of new schools is determined during a planning horizon extending over 4 years and split into 4 time periods. The proposed model provides decision makers with the needed tool to improve the provision and maximize equity in locating new public facilities over a multi-period planning horizon.

## 1 INTRODUCTION

Given the rapid urban growth and expansion in most cities around the world, the need to open new public facilities, such as schools and hospitals, is very substantial, while closing some of the existing facilities might be necessary in some zones with declining populations (Delmelle et al. 2014). Facility location models are essential tools for decision makers that are responsible to select the location, size and catchment area of a facility in order to serve the set targets and generate maximum value (Onio Antunes and Peeters 2001). Whether the problem is to locate a public or a private facility, the impact may be significant as it affects the flow, the efficiency and possibly network-wide performance (Li et al. 2018). Further, public facilities such as schools and hospitals require particular care as they are likely to induce an uptake travel demand to and from these locations, thus possibly impacting network congestion and quality of service (Guerriero et al. 2015) (Hammad et al. 2017).

Urban settings are extremely dynamic. Factors that are considered, such as demand and costs, change over the planning horizon, leading to a multi-period analysis problem. Multi-period facility location problems have been studied by many researchers, where the location, size, and time of opening new facilities is determined to optimize the network over the whole planning horizon. Decisions on where, when, and with which capacity are all dependent of each other (Delmelle et al. 2014). According to (Erlenkotter 1979), two main factors should be present to imply the existence of a dynamic case: the cost of allocation changes considerably throughout the planning horizon, and the relocation cost of a facility is high. If the first factor is missing, then the problem can be simply considered as single-period FLP and if the second factor is missing, then the problem can be formulated as a series of disconnected single-period FLP (Erlenkotter 1979). Different types of multi-period facility location problems have been studied by researchers to respond to the different situations and considerations. Marufuzzaman et al. (Marufuzzaman et al. 2016) formulated and solved an capacitated facility location problem DFLP to minimize the total costs incurred along the planning horizon including the costs of opening, closing or relocating facilities as well as the costs of transportation and operation of these facilities. Correia et al. (Correia and Melo 2017) presented an FLP model to account for two segments of customers with disparate sensitivity to delivery time, where one requires timely delivery and the other tolerate late deliveries. A penalty factor was introduced for demand served with delay. The problem of minimizing cost of locating the network of facilities throughout the time-periods was formulated using two alternative mixed-integer linear optimization. Azimi et al. (Azimi and Charmchi 2012) presented a dynamic facility location problem with budget constraints (DFLPB) as well as a heuristic algorithm to solve the proposed DFLPB using optimization through simulation techniques. Delmelle et al. (Delmelle et al. 2014) proposed a model to optimize the location of schools in an urban network subject to changes in configuration and population density. A capacitated multi-period median model was used in selecting the location of the schools where the objective was to minimize the transportation costs, while functional costs were under a budget constraint.

As shown above, the available literature on multi-period facility location problem tends to focus on minimizing the costs as the main objective of the problem (Azimi and Charmchi 2012) (Delmelle et al. 2014) (Zhao et al. 2011). However, apart from the costs, the decision to locate public facilities should be made by considering the impact of selected facility on equity indicators including access to facilities, saturation of facilities, etc. (Marsh 1992). While not considered in multi-period facility location problem, the importance of accounting for such equity factors has been widely emphasized in literature on single-period facility location problem and several models have been developed (Mumphrey et al. 1971). Zhao et al. (Zhao et al. 2011) discussed the importance of fairness in the distribution and access to public facility location under limited resources. In this paper, a bi-criteria model was formulated where the first objective is to maximize the efficiency of the facility's services, and the second objective is to minimize inequity between the different demand points using the Gini coefficient. Bertsimas (Bertsimas et al. 2011), studied the problem of decision makers allocating resources between multiple interested parties. Two notions of fairness were used: the proportional fairness and the max-min fairness. The price of fairness was studied through setting it equal to the efficiency loss resulting from the "fair" allocation of resources. The efficiency loss was compared with the fully efficient allocation which was the one that maximizes the total users' services.

The present paper aims to address the gap in the available literature in terms of lack of multi-period facility location optimization models that account for equity objectives. A multi-period facility location optimization model is developed to maximize coverage of facilities while balancing the excess flow at capacity-saturated facilities over multiple periods. The proposed model is applied to a case study aimed at identifying the optimal location of new schools in the network of public schools in the city of Sydney, Australia.

## MATHEMATICAL FORMULATION

In this section, we propose and discuss a model that incorporates equity objectives in a multi-period facility location problem.

The model is based on the following assumptions:

- Fairness is defined to embrace two different components: i) the travel distance between the demand nodes, which represent the students, and the schools, and ii) the over-capacity demand for all supplying facilities, which affects the quality of education provided.
- Prior to the network expansion design, a set of potential sites to locate new facilities have been defined.
- A planning horizon with a finite number of discrete time periods is considered. Strategic decisions related to opening new schools at new sites can be made at every time period.

- All existing and newly opened facilities will remain open during the planning horizon. The possibility of reducing and expanding the size of existing facilities is not modeled as part of this model.
- The size and establishment costs of new facilities is considered to be predetermined at the beginning of the planning horizon and imputed to the model.
- An available budget to open new schools is set for each time-period, which the model should abide by when opening new facilities.
- All the demand must be served in each time-period, as schools are considered to be critical facilities; hence, all the demand must be served.

## 1.1 Notations

Below, is the introduction of the mathematical notion that is used throughout the paper.

***Indices and Sets***

| | |
|---|---|
| $N_D$ | Set of Demand Nodes |
| $N_E$ | Set of all Existing Supply nodes |
| $N_P$ | Set of all potential sites for new Supply nodes |
| $N_S$ | Set of all Supply Nodes; $N_S = N_E \cup N_P$ |
| $T$ | Set of time periods |

***Travel and Facilities Costs***

| | |
|---|---|
| $d_{ij}$ | Distance between demand node $i \in N_D$ and supply node $j \in N_S$ |
| $TC$ | Constant; Travel cost per unit distance |
| $\bar{d}$ | Constant; Maximum allowable travel distance |
| $P$ | Constant; Cost associated with the percentage of demand allocated to supplying facilities above their optimum capacity |
| $f_{jt}$ | Opening cost of a new facility at supply node $j \in N_p$ in time period $t \in T$ |

***Capacities and Additional Parameters***

| | |
|---|---|
| $D_{it}$ | Demand at demand node $i \in N_D$ at time period $t \in T$ |
| $\hat{C}_j$ | Maximum capacity of supply node $j \in N_E$ |
| $C_j$ | Optimum capacity of supply node $j \in N_E$ |
| $B_t$ | Budget in time period $t \in T$ |
| $C$ | Constant; Capacity of the new facilities |
| $\alpha$ | Constant; weighing coefficient to be used for the weighted sum method of multi-objectives functions |

***Variables***

| | |
|---|---|
| $x_{ijt}$ | Demand allocated from node $i \in N_D$ to node $j \in N_S$ at time period $t \in T$ |
| $y_{jt}$ | Binary variable equal to 1 if a facility is open at potential site $j \in N_P$ at time period $t \in T$, 0 otherwise |
| $\Delta_{jt}$ | Variable denoting the difference between the allocated demand to a facility $j \in N_S$ and its optimum capacity at time period $t \in T$, when that allocation exceeds its capacity |

### 1.2 Formulation of the Multi-period facility Location Problem Factoring the Equity Factor

[1] minimize $$F = \alpha\left(C\sum_{i\in N_D}\sum_{j\in N_S}\sum_{t\in T} x_{ijt}d_{ij}\right) + (1-\alpha)\left(\sum_{j\in N_E}\sum_{t\in T}\frac{\Delta_{jt}}{C_j}\right)$$

subject to

[2] $$\sum_{j\in N_S} x_{ijt} = D_{it} \qquad \forall i \in N_D, \forall t \in T$$

[3] $$\sum_{i\in N_D} x_{ijt} \leq \hat{C}_j \qquad \forall j \in N_E, \forall t \in T$$

[4] $$\sum_{i\in N_D} x_{ijt} \leq C \qquad \forall j \in N_P, \forall t \in T$$

[5] $$\sum_{j\in N_P} y_{jt}f_{jt} \leq B_t \qquad \forall t \in T$$

[6] $$y_{jt} \leq y_{jt+1} \qquad \forall j \in N_P, \forall t \in T : t \neq |T|$$

[7] $$\Delta_{jt} \geq \sum_{i\in N_D} x_{ijt} - C_j \qquad \forall j \in N_E, \forall t \in T$$

[8] $$\Delta_{jt} \geq 0 \qquad \forall j \in N_E, \forall t \in T$$

[9] $$x_{ijt} = 0 \qquad \forall i \in N_D, \forall j \in N_S, \forall t \in T : d_{ij} \geq \bar{d}$$

[10] $$x_{ijt} \geq 0 \qquad \forall i \in N_D, \forall j \in N_S, \forall t \in T$$

[11] $$y_{jt} \in \{0,1\} \qquad \forall j \in N_P, \forall t \in T$$

The objective function (Eq. 1) aims to minimize the total travel costs of the users and minimize the percentage of over-capacity in the schools. Travel costs are calculated by summing the total travel distance of all users by a unit travel cost. The excess flow costs are calculated by introducing a penalty factor and multiplying it by the sum of the percentage of over-capacity in all the schools. Since our approach entails multiple objectives, we have decided to use the weighted sum method in our formulation. This method allows the multi-objective optimization problem to be cast as a single-objective mathematical optimization problem wherein α is the weighing coefficient. Eq. 2 ensures that the sum of the supplied demand in all the supplying facilities is equal to the total demand, at each and every time-period. Since schools are considered to be critical facilities, all the demand must be served. Eq. 3 ensures that the sum of demand allocated to any facility j does not exceed the maximum capacity $C_j$ of that facility. Eq. 4 stipulates that the sum of demand allocated to any new facility $j \in N_E$, does not exceed the maximum capacity $C$ of the new facilities. Eq. 5 constraints that the sum of the opening costs of new facilities at potential sites, at each time period t, does not exceed the yearly allocated budget at that time period. Eq. 6 guarantees that when a new facility is open at any potential site $j \in N_P$, at any time period t, it remains open until the end of the planning horizon. Equations 7 and 8 are introduced to represent the over-capacity $\Delta_{jt}$ at any existing facility $j \in N_E$ to enable the penalization of the demand allocated to any of these schools over their optimum capacity. In the case where the supplying facility j has no excess flow, $\Delta_{jt}$ is null. Eq.9 sets the allocation of demand to any facility equal to zero, when the travel distance $d_{ij}$ exceeds the maximum allowable travel distance $\bar{d}$. Eq. 10 imposes that any demand allocated from any demand node i to any supply node j, at any time-period t, is non-negative. Eq. 11 defines the binary variable $y_{jt}$ used to select the potential site in which a new facility will be opened.

## 2 Illustrative Case Study: Sydney, Australia

In order to test the proposed model, an illustrative case study designed based on the public school network in the city of Sydney, Australia. Figure 1, presents the zoning, the location of existing facilities, and the nominated locations for potential new schools within the selected study area.


*Figure 1: Distribution of the existing facilities, potential sites, and demand nodes*

The selected urban area highlighted in Figure 1 encompasses the most populated areas of Sydney, within the region delimited by the black line. The selected areas are divided into different sub-areas delineated by the blue lines. The 11 sub-areas are symbolised from A1-A11. The orange pentagons represent the existing schools and the red triangles epitomize the potential sites. This case study was designed to illustrate the proposed formulation and does not intend to be a realistic representation of Sydney's public school network.

### 2.1 Data

Data used in this illustrative example on Sydney was synthesized to reflect a situation where supplying facilities at the initial state are already over-capacitated, i.e. have excess flow; and new facilities are needed to keep on serving the increasing volume of demand. The location decision is made over four time-periods; and the demand in each sub-area, the available budget, and the cost of establishing a new school at each potential site as well as all other input data to the proposed formulation are assumed known.

#### 2.1.1 Baseline situation

To reflect on the current characteristics of the public school network in Sydney and build our model starting from that state, a baseline situation is synthesized. Table 1 and Figure 1 show sub-areas where the existing facilities are located, as well as the location of potential sites. Table 1 also displays the optimum and maximum capacities of the existing facilities.

Table 1: Existing Facilities Locations and Capacities, and Potential Sites Locations

| Sub-Areas | Existing | Optimum | Max | Potential |
|---|---|---|---|---|

| | Facilities | Capacity | Capacity | Sites |
|---|---|---|---|---|
| **A1** | E1 | 230 | 350 | P1 |
| **A2** | E2 | 230 | 350 | P2 |
| **A3** | - | - | - | P3 |
| **A4** | - | - | - | P4 |
| **A5** | E5 | 230 | 350 | P5 |
| **A6** | E6 | 300 | 400 | P6 |
| **A7** | E7 | 300 | 400 | P7 |
| **A8** | E8 | 300 | 400 | P8 |
| **A9** | E9 | 300 | 400 | P9 |
| **A10** | - | - | - | P10 |
| **A11** | E11 | 300 | 500 | P11 |

### 2.1.2 Time-dependent Data

Table 2 presents the demand for each sub-area and for each of the four time-periods considered.

Table 2: Demand in the Different Sub-Areas at Each Time-Period

| Sub-Area | Demand in each Time Period | | | |
|---|---|---|---|---|
| | t:1 | t: 2 | t: 3 | t: 4 |
| **A1** | 200 | 220 | 250 | 290 |
| **A2** | 160 | 200 | 250 | 290 |
| **A3** | 270 | 275 | 260 | 280 |
| **A4** | 230 | 250 | 280 | 310 |
| **A5** | 210 | 260 | 300 | 320 |
| **A6** | 315 | 320 | 320 | 320 |
| **A7** | 323 | 330 | 330 | 330 |
| **A8** | 350 | 357 | 364 | 371 |
| **A9** | 230 | 237 | 244 | 251 |
| **A10** | 250 | 257 | 264 | 271 |
| **A11** | 290 | 297 | 304 | 311 |

The generated demand data in Table 2 reflects how the demand changes irregularly from one time-period to another and from one sub-area to another. Data regarding the opening cost of new facilities and the available budgets in each time-period were simplified and assumed to be all equal to 1 in each period. This means that the available budget, which was set to be equal to 1, would be sufficient to open only one new facility, as the cost of opening a new facility was set equal to 1 as well. Hence, the available budgets in all 4 time-periods would allow the opening of 4 new facilities in any potential site.

### 2.1.3 Sets and Parameters

Table 3 presents the distance data. Data is colour-coded to categorize the distances between the different nodes and to highlight the ones that exceed the maximum travel distance (in red). The following rules were used to colour-code the distances: i) 0-15 km (Green); ii) 15-30 km (Yellow), and iii) >30 km (Red).

Table 3: Distances from Demand Nodes to Existing Facilities and Potential Sites

| Existing schools / Potential Sites | Demand Nodes | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | D5 | D6 | D7 | D8 | D9 | D10 | D11 |
| **P1** | 0 | 15 | 25 | 15 | 15 | 25 | 25 | 40 | 40 | 40 | 40 |
| **P2** | 15 | 0 | 15 | 15 | 25 | 35 | 40 | 40 | 40 | 45 | 60 |
| **P3** | 25 | 15 | 0 | 15 | 35 | 15 | 35 | 20 | 15 | 35 | 50 |
| **P4** | 15 | 15 | 15 | 0 | 15 | 15 | 35 | 35 | 40 | 35 | 50 |
| **P5** | 15 | 25 | 35 | 15 | 0 | 20 | 15 | 40 | 40 | 40 | 60 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **P6** | 35 | 29 | 15 | 15 | 20 | 0 | 15 | 15 | 15 | 15 | 40 |
| **P7** | 40 | 50 | 35 | 35 | 15 | 15 | 0 | 25 | 35 | 15 | 15 |
| **P8** | 50 | 45 | 20 | 35 | 45 | 15 | 29 | 0 | 15 | 15 | 29 |
| **P9** | 60 | 35 | 15 | 35 | 40 | 20 | 35 | 15 | 0 | 25 | 35 |
| **P10** | 45 | 45 | 29 | 29 | 35 | 15 | 15 | 15 | 25 | 0 | 15 |
| **P11** | 70 | 70 | 55 | 55 | 29 | 29 | 15 | 25 | 35 | 15 | 0 |
| **E1** | 0 | 15 | 25 | 15 | 15 | 25 | 25 | 45 | 45 | 45 | 45 |
| **E2** | 15 | 0 | 15 | 15 | 25 | 35 | 40 | 40 | 40 | 45 | 60 |
| **E5** | 15 | 25 | 35 | 15 | 0 | 20 | 15 | 40 | 40 | 40 | 60 |
| **E6** | 35 | 29 | 15 | 15 | 20 | 0 | 15 | 15 | 15 | 15 | 40 |
| **E7** | 40 | 50 | 35 | 35 | 15 | 15 | 0 | 25 | 35 | 15 | 15 |
| **E8** | 50 | 45 | 20 | 35 | 45 | 15 | 29 | 0 | 15 | 15 | 29 |
| **E9** | 60 | 35 | 15 | 35 | 40 | 20 | 35 | 15 | 0 | 25 | 35 |
| **E11** | 70 | 70 | 55 | 55 | 29 | 29 | 15 | 25 | 35 | 15 | 0 |

For all the cases where the distance between the school and the demand node exceed the maximum allowable travel distance, the cells are highlighted in red indicating that no allocation can be possible in this situation. The cells coloured in yellow indicate that the allocation of demand is possible in these situations; however, it is costly. The cells coloured in green indicate that these positions fall within a close proximity and are favourable from a travel distance perspective.

## 2.2 Numerical Results

The problem was modelled using AMPL and solved using CPLEX's mixed-integer linear programming solver. Table 4 summarizes the potential sites selected to open new schools.

Table 4: Selected Potential Sites to Open the New Facilities

| Potential Sites | Time-Periods t: 1 | t: 2 | t: 3 | t: 4 |
|---|---|---|---|---|
| **P1** | 0 | 0 | 0 | 0 |
| **P2** | 0 | 0 | 0 | 0 |
| **P3** | 0 | 0 | **1** | **1** |
| **P4** | 0 | **1** | **1** | **1** |
| **P5** | 0 | 0 | 0 | **1** |
| **P6** | 0 | 0 | 0 | 0 |
| **P7** | 0 | 0 | 0 | 0 |
| **P8** | 0 | 0 | 0 | 0 |
| **P9** | 0 | 0 | 0 | 0 |
| **P10** | **1** | **1** | **1** | **1** |
| **P11** | 0 | 0 | 0 | 0 |

Table 4 shows that, to reach the optimum solution, the model selected to open the first new facility, at time-period one (t:1), in potential site 10 (P10); the second new facility at t:2 in P4; the third new facility at t:3 in P3; and the fourth one at t:4 in P5.

Table 5: Demand Allocation to the Existing and New Facilities at Time-Period 1

| Existing Facilities / Potential Sites | Demand Nodes D1 | D2 | D3 | D4 | D5 | D6 | D7 | D8 | D9 | D10 | D11 | $\Delta_{jt}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **E1** | 200 | 0 | 0 | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **120** |
| **E2** | 0 | 160 | 130 | 60 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **120** |
| **E5** | 0 | 0 | 0 | 20 | 210 | 0 | 0 | 0 | 0 | 0 | 0 | **0** |
| **E6** | 0 | 0 | 0 | 0 | 0 | 315 | 0 | 0 | 0 | 0 | 0 | **15** |
| **E7** | 0 | 0 | 0 | 0 | 0 | 0 | 313 | 0 | 0 | 0 | 0 | **13** |
| **E8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 350 | 0 | 0 | 0 | **50** |
| **E9** | 0 | 0 | 140 | 0 | 0 | 0 | 0 | 0 | 230 | 0 | 0 | **70** |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **E11** | 0 | 0 | 0 | 0 | 0 | 0 | 10 | 0 | 0 | 0 | 290 | **0** |
| **P1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **-** |
| **P2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **-** |
| **P3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **-** |
| **P7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **-** |
| **P8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **-** |
| **P9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **-** |
| **P10** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 250 | 0 | **0** |
| **P11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **-** |

Table 5 presents the allocation of the demand to the existing and new facilities, at time-period one, that served the set objective function. Based on the generated results, it is clear that the model allocated the demand in any sub-area to an existing facility located in the same sub-area or within a close vicinity of it. For the demand nodes D1 and D2, the whole demand was directly allocated to the existing supplying facility located in the same sub-area, as it was possible in this case. For the demand nodes D3 and D4, which are located in sub-are where there are no supplying facilities, the demand was allocated to the closest sub-areas trying to reduce the travel costs and the over-capacities, while abiding by the maximum allowable travel distance. The new facility was opened at the potential site P10 located in a sub-area with no existing facilities and having the highest demand volume between all the sub-areas that does not have any of the existing facilities located in them. It is noticeable that E1 and E2 are highly over-capacitated during this time-period resulting significant additional costs due to the penalties incurred. Given the limitation of allocating demand to distant facilities due to the maximum travel distance constraint, the model had no other choice but to allocate a large volume of demand from D3 and D4 to E1 and E2, resulting in this over-capacity.

Table 6: Demand Allocation to the Existing and New Facilities at Time-Period 2

| **Existing Facilities / Potential Sites** | **Demand Nodes** | | | | | | | | | | | $\Delta_{jt}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **D1** | **D2** | **D3** | **D4** | **D5** | **D6** | **D7** | **D8** | **D9** | **D10** | **D11** | |
| **E1** | 220 | 0 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **E2** | 0 | 200 | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 120 |
| **E5** | 0 | 0 | 0 | 0 | 260 | 0 | 0 | 0 | 0 | 0 | 0 | 30 |
| **E6** | 0 | 0 | 52 | 0 | 0 | 320 | 0 | 0 | 0 | 0 | 0 | 72 |
| **E7** | 0 | 0 | 0 | 0 | 0 | 0 | 330 | 0 | 0 | 0 | 0 | 30 |
| **E8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 357 | 0 | 0 | 0 | 57 |
| **E9** | 0 | 0 | 63 | 0 | 0 | 0 | 0 | 0 | 237 | 0 | 0 | 0 |
| **E11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 297 | 4 |
| **P1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P4** | 0 | 0 | 0 | 250 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **P5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P10** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 250 | 0 | 0 |
| **P11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |

As represented in Table 6, in time-period t:2, the new facility was opened at potential site P4 supplying the demand in sub-area D4. The opened facility expunged the over-capacity in the existing facility E1, which was previously supplying some 150 students from D4. This brought down the over-capacity in E1 from 120 to 0. Moreover, this addition reduced the travel costs, since the demand in D4, which was previously allocated to nearby sub-areas, is now being allocated in P4. In time-period t:3, a new facility is opened at potential site P3, supplying demand in sub-areas D3 and D4. Observe that D3 did not initially have an existing school. This addition not only reduced the travel costs incurred from the demand in D3, but also reduced the over-capacity in E2 which was supplying 150 students from D3. It is noticeable from Table 7 that P3 does not fully supply the demand in D3, but supplied demand from D4, driving 40 students from D3 to be allocated to E9.

Table 7: Demand Allocation to the Existing and New Facilities at Time Period 3

| Existing Facilities / Potential Sites | Demand Nodes D1 | D2 | D3 | D4 | D5 | D6 | D7 | D8 | D9 | D10 | D11 | ∆jt |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **E1** | 220 | 0 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **E2** | 0 | 200 | 150 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 120 |
| **E5** | 0 | 0 | 0 | 0 | 260 | 0 | 0 | 0 | 0 | 0 | 0 | 30 |
| **E6** | 0 | 0 | 52 | 0 | 0 | 320 | 0 | 0 | 0 | 0 | 0 | 72 |
| **E7** | 0 | 0 | 0 | 0 | 0 | 0 | 330 | 0 | 0 | 0 | 0 | 30 |
| **E8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 357 | 0 | 0 | 0 | 57 |
| **E9** | 0 | 0 | 63 | 0 | 0 | 0 | 0 | 0 | 237 | 0 | 0 | 0 |
| **E11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 297 | 4 |
| **P1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P4** | 0 | 0 | 0 | 250 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **P5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P10** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 250 | 0 | 0 |
| **P11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |

Table 8: Demand Allocation to the Existing and New Facilities at Time Period 4

| Existing Facilities / Potential Sites | Demand Nodes D1 | D2 | D3 | D4 | D5 | D6 | D7 | D8 | D9 | D10 | D11 | ∆jt |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **E1** | 250 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20 |
| **E2** | 0 | 250 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20 |
| **E5** | 0 | 0 | 0 | 0 | 300 | 0 | 0 | 0 | 0 | 0 | 0 | 70 |
| **E6** | 0 | 0 | 0 | 0 | 0 | 320 | 0 | 0 | 0 | 0 | 0 | 20 |
| **E7** | 0 | 0 | 0 | 0 | 0 | 0 | 330 | 0 | 0 | 0 | 0 | 30 |
| **E8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 362 | 0 | 0 | 0 | 62 |
| **E9** | 0 | 0 | 40 | 0 | 0 | 0 | 0 | 2 | 244 | 14 | 0 | 0 |
| **E11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 304 | 4 |
| **P1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P3** | 0 | 0 | 220 | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **P4** | 0 | 0 | 0 | 250 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **P5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **P9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| **P10** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 250 | 0 | 0 |
| **P11** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |

Table 8 shows that, in time period t:4, the new facility was opened in potential site P5, supplying demand from D5 and D1. The model selected to open a new facility in the sub-area where the existing facility E5 was the most over-capacitated between all the other existing supplying facilities. The results presented in column $\Delta_{jt}$, in Table 8, show that, at time period t:4, almost none of the existing facilities is significantly over-capacitated when compared to time-period t:1.

## 3 Conclusion

In this paper, we developed a multi-period facility location model aiming at maximizing equity in access and capacity-saturation, by locating new facilities among a series of potential sites. To satisfy this objective, we proposed to minimize the sum of demand-weighted travel distance to schools and the total excess flow at supplying facilities. A series of constraints were introduced in the model, including a budget constraint at each time period, a maximum travel distance, a full satisfaction of all the demand at each time-period, and maximum capacities for the existing facilities. An illustrative case study on Sydney's public school network with four time periods was presented to test the proposed model.

The proposed model is based on a series of assumptions that constitute limitations and present opportunities for future enhancements. These assumptions included: i) defining the notion of equity to being only related to travel distance and over-capacity of the facilities, ii) assuming that strategic decision of opening new facilities can be made at the beginning of each time-period, iii) requiring that all the existing and newly opened facilities remain open during the planning horizon, and iv) predetermining the size and establishment costs of new facilities. Future research will be focused on enhancing the model by catering for the abovementioned assumption and including them in our formulation.

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