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LONG-TERM DEFLECTIONS OF REINFORCED CONCRETE BEAMS

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Abstract: This paper critically evaluates methods for computing long-term deflections due to creep and shrinkage described in the 4th Edition of the Cement Association of Canada Concrete Design Handbook. It also presents a mechanics-based approach for computing incremental deflections due to creep and shrinkage. The accuracy of both methods in predicting the total deflection of concrete beams under sustained loads is quantified by investigating test-to-predicted ratios. The method presented in the Concrete Design Handbook was found to be unconservative, with test-to-predicted ratios as large as 1.4, but can be improved by using more conservative creep coefficients and ultimate shrinkage strains. The mechanics-based method was found to yield accurate and slightly conservative test-to-predicted ratios of 0.82-0.92.

1 INTRODUCTION

The instantaneous and long-term deflections of concrete flexural members are heavily dependent on the effective moment of inertia, I_e. Two equations for calculating I_e are described in the Concrete Design Handbook (CAC 2016). The empirical equation proposed by Branson is based on an incorrect mechanical model that overestimates the effect of tension-stiffening in lightly reinforced members and therefore underestimates deflection (Bischoff, 2007). The equation proposed by Bischoff (2007) is based on a correct mechanical model. Scanlon and Bischoff (2008) recommended that the cracking moment, M_{cr}, be calculated based on two-thirds the modulus of rupture, f_r, when using the Bischoff Equation to account for the effect of restrained shrinkage. Using the Branson Equation with M_{cr} based on $0.5f_r$ yields similar results to using the Bischoff Equation with $0.67f_r$ (CAC 2016). Additionally, Mancuso and Bartlett (2016) showed that the instantaneous deflection computed using the Branson Equation with M_{cr} based on $1.0f_r$ yields test/predicted ratios greater than 1 (unconservative) with a high coefficient of variation. They also showed that using either the Branson Equation with M_{cr} based on $0.5f_r$ or the Bischoff Equation with M_{cr} based on $0.67f_r$ provides conservative results with a mean test-to-predicted ratio between 0.82 to 0.84 and a significantly lower coefficient of variation.

The current empirically based methods for computing long-term deflections due to creep and shrinkage, referenced in ACI318 (ACI 2014) and presented in the Concrete Design Handbook (CAC 2016), were originally proposed by Branson (1977). The creep deflection is proportional to the instantaneous deflection, and therefore inversely proportional to I_e. On the other hand, shrinkage deflection depends only on the ultimate shrinkage strain, the top and bottom reinforcement areas, and the overall depth of the member. Several researchers including Branson have developed analytical tools to calculate deflection due to shrinkage. However, these methods have been criticized due to uncertainties in quantifying the impact of creep on Young's Modulus of concrete and uncertainties in computing I_e (Branson 1977). Bischoff's Equation for computing I_e opens the door for calculating long-term deflections analytically. Moreover, Branson's methods for computing long-term deflections were derived based on empirical methods for computing I_e, and the compatibility of the Bischoff Equation with these methods must be revalidated. Therefore, the objectives of the research reported in this paper are to:

1. Critically evaluate existing methods for computing long-term deflection increments due to creep and shrinkage.
2. Propose mechanics-based methods for calculating long-term deflection increments due to creep and shrinkage.
3. Assess the accuracies of existing and proposed methods by investigating test-to-predicted ratios for the total (i.e. immediate plus long-term) deflections.
4. Outline any shortcomings with existing methods and propose improvements.

2 METHODOLOGIES FOR COMPUTING INCREMENTAL DEFLECTIONS DUE TO CREEP AND SHRINKAGE

2.1 Branson / Concrete Design Handbook

Methods for computing deflections due to creep and shrinkage presented in the Concrete Design Handbook (CAC 2016) and referenced in ACI318 (ACI-318 2014) were developed by Branson (1977). The equation for computing creep deflection is founded on the fundamental assumption that the increase in curvature due to creep is smaller than the increase in external fibre compressive strain due to creep. This assumption is appropriate because the depth of the compression region increases when creep occurs. It is formulated as

[1] $\frac{\psi_{cr}}{\psi_i} = k_r \frac{\varepsilon_{cr}}{\varepsilon_i}$

where k_r is a dimensionless factor less than 1, ε_{cr} is the creep strain, ε_i is the instantaneous strain, ψ_{cr} is the curvature due to creep, and ψ_i is the instantaneous curvature. An equation for calculating k_r for partially prestressed beams was derived theoretically by Shaikh and Branson (1970) and was later modified to fit test data for non-prestressed beams (Branson 1977). This modified empirical equation for k_r is given as

[2] $k_r = \frac{0.85}{1+50\rho'}$

where ρ' is the compression steel reinforcement ratio. Moreover, the creep coefficient, C_t, is defined as the ratio of the creep strain ε_{cr} to the initial strain ε_i (ACI 2008). The ratio of creep deflection to initial deflection, Δ_{cr}/Δ_i, is given by

[3] $\frac{\Delta_{cr}}{\Delta_i} = k_r \frac{\varepsilon_{cr}}{\varepsilon_i}$

Thus, combining Equations [2] and [3] and accounting for C_t yields:

[4] $\Delta_{cr} = \left[\frac{0.85C_t}{1+50\rho'}\right] \Delta_i$

The Concrete Design Handbook recommends that C_t be computed as C_t=0.8S_t, where S_t, the long-term deflection factor under sustained loads specified in A23.3 (CSA 2014), has a maximum value of 2.0. The maximum value of C_t is therefore 1.6.

Branson (1977) recommends computing shrinkage deflection from the curvature due to shrinkage, which is assumed to be directly proportional to the free shrinkage strain and an inversely proportional to the overall member depth. Branson's method is as follows

[5] $\Delta_{sh} = K_{sh}\psi_{sh}\ell_n^2$

where Δ_{sh} is the deflection due to shrinkage, ℓ_n is the clear span length, K_{sh} is a coefficient that accounts for the displacement boundary conditions of the member, and ψ_{sh} is the curvature due to shrinkage, defined as

[6] $\psi_{sh} = \frac{A_{sh}\varepsilon_{sh}}{h}$

where ε_{sh} is the ultimate free shrinkage strain, A_{sh} is a factor to account for the ratio of top to bottom reinforcement, and h is the overall depth of the member.

Branson recommended that the free shrinkage strain be taken as ε_{sh}=400×10^{-6}με in the absence of information on free shrinkage under local conditions. The Concrete Design Handbook recommends that ε_{sh} be computed as

[7] $\varepsilon_{sh}=\frac{S_t}{2.0}400\times10^{-6}$

Since $S_t \leq 2.0$, ε_{sh} computed using Eq. [7] cannot exceed 400×10^{-6}.

2.2 Proposed Analytical Approach

Short-Term Deflection

Mechanics-based methods for computing the instantaneous deflection are well-established. The procedure involves calculating the depth of the neutral axis, kd, using conventional mechanically derived equations (e.g., CAC 2016) and subsequently calculating the effective moment of inertia, I_e. The deflection can then be computed using standard methods. The deflection of a simply supported concrete member carrying a uniformly distributed load is

[8] $\Delta_i=5M\ell_n^2/48E_cI_e$.

where M is the maximum midspan moment and E_c is Young's Modulus for concrete. If the member is subjected to uniform moment along its entire length, the deflection can be computed from the curvature, $\psi_i=M/EI_e$, which implies radius of curvature, R, of $1/\psi_i$, and so

[9] $\Delta_i=R-\sqrt{R^2-\left(\frac{\ell_n}{2}\right)^2}$

Equation [8] is valid when curvature varies along the length of the member, and so can be used to compute instantaneous and creep deflections. Equation [9] can be used to compute shrinkage deflection because the shrinkage curvature is constant along the length of the member.

Members that are exposed to prolonged drying experience shrinkage, and consequently develop tensile stresses in the concrete due to restraint of shrinkage by the reinforcing steel. These tensile stresses reduce the applied moment necessary to initiate flexural cracking. Scanlon and Bischoff (2007) showed that the effect of restrained shrinkage is most pronounced in members with reinforcement ratio less than 1%, and less pronounced in members with higher reinforcement ratios. As a result, A23.3-14 requires that I_e be calculated using the Branson Equation with M_{cr} computed using $0.5f_r$ to account for the reduction in cracking moment due to shrinkage restraint, and for the impact of the equation being derived using the wrong mechanical model.

Long-Term Deflection due to Creep

Creep of concrete at time t is usually quantified in terms of creep coefficient, $\emptyset(t,t_o)$, where t_o is the age at loading. Since the creep coefficient is defined as $\emptyset(t,t_o)=\varepsilon_{cr}/\varepsilon_i$, the creep strain at time t is given as

[10] $\varepsilon_{cr}(t,t_o)=\emptyset(t,t_o)\frac{\sigma_c(t_o)}{E_c(t_o)}$

where $E_c(t_o)$ is Young's Modulus and $\sigma_c(t_o)$ is the maximum compressive concrete stress at the t_o.

The effect of creep on a flexural member is analogous to a gradual and uniform change in Young's Modulus of concrete (Gilbert and Ranzi 2011) that causes a change in the modular ratio, n, and consequently a change in kd, I_e, and the maximum compressive stress in the concrete. Creep in cracked reinforced concrete beams causes a lowering of the neutral axis and therefore a reduction in the external fibre compressive stress (Gilbert and Ranzi 2011, Branson 1977). The time-dependent Young's Modulus, $\overline{E_c}$, is defined as

[11] $\overline{E_c} = \frac{E_c(t)}{[1+\chi(t,t_o)\emptyset(t,t_o)]}$

where $\chi(t,t_o)$ is an aging coefficient used to account for the age of concrete at the time of loading. It generally ranges between 0.4 and 1.0 and is commonly taken as 0.8 for most practical cases (e.g., Scanlon and Bischoff 2008, Gilbert 1988). Therefore, a value of 0.8 will be assumed for the remainder of this paper. Theoretically-derived equations for computing $\chi(t,t_o)$ are presented in Gilbert and Ranzi (2011) and Bazant (1972).

The age-adjusted modular ratio, $\bar{n}$, is therefore

[12] $\bar{n}=E_s/\overline{E_c} = n[1+\chi(t,t_o)\emptyset(t,t_o)]$

where the modular ratio for short-term loading, n, equals E_s/E_c

The depth of the neutral axis after creep, $\overline{kd}$, can be computed using the adjusted modular ratio, $\bar{n}$, in place of n in conventional equations for calculating kd found in the CAC Concrete Design Handbook. The change in the depth of the neural axis requires computing modified I_{cr} and I_e values, denoted as $\overline{I_{cr}}$ and $\overline{I_e}$ respectively. The stress in the concrete after creep has taken place can be calculated as

[13] $\sigma(t,t_o)=\frac{M\overline{kd}}{\overline{I_e}}$

The curvature due to creep at time t after loading can be obtained by substituting Equation [13] in Equation [10] to yield:

[14] $\psi_{cr}(t,t_o) = \frac{\varepsilon_{cr}(t,t_o)}{\overline{kd}}$

The deflection due to creep for a simply supported beam subjected to a uniformly distributed load can be calculated from Equation [8] as

[15] $\Delta_{cr}=\frac{5\psi_{cr}(t,t_o)}{48}$

Long-Term Deflection due to Shrinkage

Shrinkage-induced curvature in reinforced concrete beams is primarily due to shrinkage restraint by the reinforcing steel. As concrete shrinks, it imposes a compressive force on the reinforcing steel, which in turn imposes an equal and opposite tensile force on the concrete. In cases where the top and bottom reinforcement are not symmetrical the resulting non-uniform strain distribution causes warping and therefore deflection. Symmetrically-reinforced members such as columns have a uniform shrinkage strain distribution across the concrete cross section and therefore do not undergo shrinkage warping. Most available methods, including Branson's (1977) and Gilbert's (1999) empirical methods, compute shrinkage curvature assuming the section to be uncracked. This is justifiable because:

1. The curvature caused by shrinkage restraint depends on the size of the uncracked section (Gilbert and Ranzi 2011) since shrinkage shortening occurs only in the uncracked regions (Branson 1977). Therefore, the effect of restrained shrinkage may not be significantly influenced by the presence of cracks.
2. The majority of shrinkage occurs in the first few weeks after casting, before the application of the design live loads and before cracking (Branson 1977). However, this may be questionable because construction loading often exceeds twice the self-weight of the member (Zhou and Kokai 2010).

The force in the concrete at the level of the steel due shrinkage strains in an uncracked section can be calculated using fundamental principles of mechanics (e.g., Scanlon and Bischoff 2008). For a rectangular section, the force at the level of the bottom reinforcement is, F_c,

$$[16]\ F_c = \frac{-E_s A_s \varepsilon_{sh}}{1+\bar{n}\rho(\frac{d}{h})(1+12\left(\frac{d}{h} - 0.5\right)^2)}$$

where ε_{sh} is the shrinkage strain, A_s is the area of the bottom steel, ρ is the reinforcement ratio, d is the effective depth, h is the overall depth of the member and $\bar{n}$ is as defined in Equation [12].

Similarly, the force at the level of the top reinforcement, $F_c^{'}$, is

$$[17]\ F_c^{'} = \frac{-E_s A_s^{'} \varepsilon_{sh}}{1+\bar{n}\rho^{'}(\frac{d}{h})(1+12\left(0.5 - \frac{d^{'}}{h}\right)^2)}$$

where $A_s^{'}$ is the area of the top steel, and $d^{'}$ is the depth of the top reinforcement from the top fibre.

The residual stress at the top and bottom fibres, $\sigma_{sh,T}$ and $\sigma_{sh,B}$, respectively, can be calculated as

$$[18]\ \sigma_{sh,T} = \frac{F_c}{A_g} + \frac{F_c(d\text{-}0.5h)(\text{-}0.5h)}{I_g} + \frac{F_c^{'}}{A_g} + \frac{A_g(d^{'}\text{-}0.5h)(\text{-}0.5h)}{I_g}$$

and

$$[19]\ \sigma_{sh,B} = \frac{F_c}{A_g} - \frac{F_c(d\text{-}0.5h)(0.5h)}{I_g} + \frac{F_c^{'}}{A_g} + \frac{A_g(\text{-}0.5h+d^{'})(0.5h)}{I_g}$$

The strain at the top fibre and bottom fibres at time t, $\varepsilon_{sh,T}$ and $\varepsilon_{sh,B}$, respectively, are therefore

$$[20]\ \varepsilon_{sh,T} = \frac{\sigma_{sh,T}}{\bar{E}_c}$$

and

$$[21]\ \varepsilon_{sh,B} = \frac{\sigma_{sh,B}}{\bar{E}_c}$$

and the net curvature (i.e., the curvature that causes warping) is

$$[22]\ \psi_{sh} = \frac{\varepsilon_{sh,B} - \varepsilon_{sh,T}}{h}$$

Finally, the deflection due to shrinkage, Δ_{sh}, can be calculated using Equation [9].

3 COMPARISON WITH EXPERIMENTAL DATA

Table 1 shows a comparison between deflections obtained from long-term tests on 30 simply supported beams and slabs, and deflections predicted using the proposed analytical method and methods reported in the Concrete Design Handbook. There is a lack of long-term tests on beams and slabs under sustained loading, and few studies report all data needed to carry out a comprehensive analysis (Kilpatrick and Gilbert 2017). Only tests on simply-supported rectangular beams made of normal weight concrete are considered in the present study.

Experimental values for creep coefficients and shrinkage strains were reported by Gilbert and Nejadi (2004) and can be deduced from data reported by Washa and Fluck (1952). Concrete properties such as the compressive strength at age of loading, $f_c^{'}(t_o)$, and Young's Modulus at age of loading, $E_c(t_o)$, were explicitly reported in both studies. The experimental program by Washa and Fluck spanned 926 days and involved curing the specimens for 5 days (t_c = 5 days) and loading them at 14 days after casting (t_o = 14 days), meaning that they were left to dry for nine days prior to loading. The load duration, t, was 912 days. Gilbert and Nejadi's specimens were not left to dry prior to loading (t_c=t_o), and the load was sustained for 380 days.

Instantaneous deflections were computed using the Branson Equation with M_{cr} based on $0.5f_r$ to maintain consistency with the requirements of CSA A23.3-14 and the Concrete Design Handbook (CAC 2016). Analytical predictions were carried out using experimental creep coefficients and shrinkage strains, while predictions based on the Concrete Design Handbook were conducted using values suggested therein.

Table 1: Comparison between Test and Predicted Deflections

Spec.	Test			Predicted: Analytical Method								Predicted: Concerte Design Handbook							
	Δ_i (mm)	Δ_{LT} (mm)	Δ_{Total} (mm)	ε_{sh} ($\times 10^{-6}$)	$\phi(t,t_o)$	Δ_i (mm)	Δ_{cr} (mm)	Δ_{sh} (mm)	Δ_{LT} (mm)	Δ_{Total} (mm)	Test/ Predic	ε_{sh} ($\times 10^{-6}$)	C_t	Δ_i+ Δ_{cr} (mm)	Δ_{cr} (mm)	Δ_{sh} (mm)	Δ_{LT} (mm)	Δ_{Total} (mm)	Test/ Predic
Washa & Fluck (1952)																			
B3	26.4	60.0	86.4	720	4.45	26.7	54.0	16.7	70.7	97.4	0.89	349	1.40	58.3	31.6	6.5	38.1	64.8	1.33
B6	26.4	60.0	86.4	720	4.45	25.8	51.5	16.1	67.6	93.4	0.93	349	1.40	56.3	30.6	6.5	37.1	62.8	1.37
C3	47.8	92.9	140.7	720	4.40	43.6	91.4	30.3	121.7	165.3	0.85	349	1.40	95.4	51.7	11.3	63.0	106.6	1.32
C6	47.8	92.9	140.7	720	4.40	42.8	88.0	29.7	117.8	160.5	0.88	349	1.40	93.5	50.7	11.3	62.0	104.8	1.34
E3	63.0	121.9	184.9	720	4.35	49.3	95.3	32.7	128.0	177.3	1.04	349	1.40	107.8	58.5	13.0	71.5	120.8	1.53
E6	63.0	121.9	184.9	720	4.35	51.6	102.2	34.2	136.4	188.0	0.98	349	1.40	112.7	61.1	13.0	74.2	125.7	1.47
B2	24.9	40.1	65.0	720	4.45	25.4	52.0	4.5	56.5	81.9	0.79	349	1.40	47.0	21.6	4.0	25.6	51.0	1.27
B5	24.9	40.1	65.0	720	4.45	24.8	48.8	4.5	53.3	78.0	0.83	349	1.40	45.8	21.0	4.0	25.0	49.7	1.31
C2	43.4	57.2	100.6	720	4.40	41.9	91.9	9.5	101.4	143.3	0.70	349	1.40	77.7	35.7	6.9	42.6	84.6	1.19
C5	43.4	57.2	100.6	720	4.40	40.9	86.3	9.5	95.9	136.8	0.74	349	1.40	75.7	34.8	6.9	41.7	82.6	1.22
E2	55.9	72.9	128.8	720	4.35	47.2	92.3	9.3	101.6	148.8	0.87	349	1.40	88.1	40.9	9.3	50.2	97.4	1.32
E5	55.9	72.9	128.8	720	4.35	48.9	100.8	9.3	110.0	158.9	0.81	349	1.40	91.3	42.4	9.3	51.7	100.6	1.28
B1	23.4	27.6	51.0	720	4.45	24.5	49.0	0.0	49.0	73.5	0.69	349	1.40	40.6	16.2	0.0	16.2	40.6	1.26
B4	23.4	27.6	51.0	720	4.45	23.9	46.2	0.0	46.2	70.1	0.73	349	1.40	39.6	15.8	0.0	15.8	39.6	1.29
C1	40.1	39.9	80.0	720	4.40	40.6	87.8	0.0	88.8	129.4	0.62	349	1.40	67.6	27.0	0.0	27.0	67.6	1.18
C4	40.1	39.9	80.0	720	4.40	40.0	84.8	0.0	85.8	125.8	0.64	349	1.40	66.6	26.6	0.0	26.6	66.6	1.20
E1	59.4	64.6	124.0	720	4.35	45.3	87.3	0.0	87.3	132.6	0.93	349	1.40	76.3	31.0	0.0	31.0	76.3	1.62
E4	59.4	64.6	124.0	720	4.35	46.8	94.7	0.0	94.7	141.4	0.88	349	1.40	78.8	32.0	0.0	32.0	78.8	1.57
										Mean	**0.82**								**1.34**
										Coefficient of Variation (%)	**14.32**								**9.50**
Gilbert & Nejadi (2004)																			
B1a	4.9	7.2	12.1	825	1.71	6.2	5.7	1.2	6.9	13.0	0.93	287	1.15	12.2	6.0	0.7	6.7	12.9	0.94
B1b	2.0	5.4	7.4	825	1.71	3.7	3.7	1.2	4.9	8.5	0.87	287	1.15	7.3	3.6	0.7	4.3	8.0	0.93
B2a	5.0	7.4	12.4	825	1.71	6.2	5.7	1.4	7.1	13.3	0.93	287	1.15	12.3	6.1	0.7	6.8	13.0	0.95
B2b	2.0	5.9	7.9	825	1.71	3.8	3.7	1.4	5.1	8.9	0.88	287	1.15	7.6	3.7	0.7	4.5	8.3	0.95
B3a	5.8	7.5	13.3	825	1.71	6.4	5.9	1.9	7.8	14.2	0.94	287	1.15	12.5	6.2	0.9	7.1	13.4	0.99
B3b	2.0	5.9	7.9	825	1.71	3.7	3.6	1.9	5.5	9.2	0.86	287	1.15	7.4	3.7	0.9	4.5	8.3	0.96
S1a	7.1	18.0	25.1	825	1.71	13.7	13.1	1.8	14.9	28.7	0.88	287	1.15	27.1	13.4	1.4	14.8	28.5	0.88
S1b	3.7	16.2	19.9	825	1.71	9.0	9.4	1.8	11.2	20.1	0.99	287	1.15	17.7	8.7	1.4	10.1	19.1	1.04
S2a	10.6	19.2	29.8	825	1.71	16.3	15.0	2.6	17.6	33.9	0.88	287	1.15	32.2	15.9	1.5	17.4	33.7	0.88
S2b	4.4	17.5	21.9	825	1.71	10.3	10.0	2.6	12.6	22.9	0.96	287	1.15	20.4	10.1	1.5	11.6	21.9	1.00
S3a	11.8	20.7	32.5	825	1.71	15.3	14.4	3.2	17.6	33.0	0.99	287	1.15	30.3	15.0	1.8	16.8	32.1	1.01
S3b	5.0	17.9	22.9	825	1.71	10.8	10.4	3.2	13.6	24.5	0.94	287	1.15	21.4	10.6	1.8	12.4	23.2	0.99
										Mean	**0.92**								**0.97**
										Coefficient of Variation (%)	**4.98**								**7.12**

4 DISCUSSION

4.1 Branson / Concrete Design Handbook

The method presented in the Concrete Design Handbook was found to yield a highly unconservative test-to-predicted ratio of 1.34 for the results reported by Washa and Fluck (1952), but a significantly more accurate and slightly conservative ratio of 0.97 for the results reported by Gilbert and Nejadi (2004). The discrepancy can be attributed, in varying degrees, to:

1. Degree of conservatism of I_e computed based on a reduced modulus of rupture.
2. Oversimplification of Branson's creep coefficient, C_t, in the Concrete Design Handbook method.
3. Underestimation of ultimate shrinkage strains, ε_{sh}, in the Concrete Design Handbook method.

Instantaneous deflections predicted using I_e based on a reduced modulus of rupture were found to overestimate Gilbert and Nejadi's (2004) observed deflections, with a mean test-predicted ratio of 0.67. On the other hand, the same method provided significantly more accurate, yet slightly unconservative ratios for the results of Washa and Fluck (1952), where the mean test-to-predicted ratio is 1.09. This inconsistency may be due to Gilbert and Nejadi's specimens not being exposed to drying, and consequently not shrinking significantly before loading (t_o=t_c). In this case, using one-half the modulus of rupture to account for reduction of the cracking moment due to restrained shrinkage is an overly conservative assumption. Conversely, Washa and Fluck's (1952) test specimens were left to dry for nine days prior to loading and therefore were likely to have experienced significant shrinkage. Using a reduced modulus of rupture is clearly appropriate in this case. The overestimation of instantaneous deflection for Gilbert and Nejadi's specimens contributed to the overall accuracy in predicting the total deflection through compensating for shortcoming in methods for predicting creep and shrinkage deflections. On the other hand, the reasonable accuracy in predicting instantaneous deflections for Washa and Fluck's specimens, emphasized the deficiencies in methods for computing incremental deflections.

Table 2 shows values recommended by Branson (1977) for computing creep and shrinkage deflections. For a typical concrete member in the interior of a building where the age at loading is between 3 and 14 days and relative humidity is approximately 50%, C_t has a minimum value of 2.00. Similarly, ε_{sh} varies between 506 and 795$\times10^{-6}$. However, the Concrete Design Handbook suggests a simplified method for computing C_t and ε_{sh} where their respective maximum values are 1.6 and 400$\times10^{-6}$. C_t and ε_{sh} were computed, using this technique, to be 1.4 and 349 $\times10^{-6}$ for Washa and Fluck's test specimens and 1.15 and 287 $\times10^{-6}$ for Gilbert and Nejadi's specimens respectively. These values appear to be smaller than those obtained from Table 2, and the creep and shrinkage deflections are therefore likely to be underestimated.

Using creep coefficients and shrinkage strains presented in Table 2 is likely to yield more conservative creep and shrinkage deflections and therefore more accurate test-to-predicted ratios for Washa and Fluck's specimens, and more conservative results for Gilbert and Nejadi's specimens. Preliminary calculations show that a mean test-to-predicted ratio of 1.11 for Washa and Fluck's (1952) specimens and 0.69 for Gilbert and Nejadi's (2004) specimens can be achieved using this approach. These ratios can be further improved (i.e. made to approach 1.0) if more accurate methods for predicting I_e based on the drying period before loading are developed and the creep deflection is uncoupled from instantaneous deflection. However, these topics are beyond the scope of this study and will not be explored further in this paper. Moreover, Table 2 does not provide creep coefficient values for concretes loaded at ages less than 7 days, while current construction practice may cause concrete members to be loaded at 3 days (Zhou and Kokai 2010).

Table 2: Creep Coefficients and Shrinkage Strains Suggested by Branson (1977)

Age at Loading (days)	Average Relative Humidity, Ultimate Creep Coefficient or Shrinkage Strain											
	≥90%		80%		70%		60%		50%		≤40%	
	C_t	ε_{sh} ($\times 10^{-6}$)	C_t	ε_{sh} ($\times 10^{-6}$)	C_t	ε_{sh} ($\times 10^{-6}$)	C_t	ε_{sh} ($\times 10^{-6}$)	C_t	ε_{sh} ($\times 10^{-6}$)	C_t	ε_{sh} ($\times 10^{-6}$)
1	-	281	-	562	-	655	-	749	-	842	-	936
7	1.57	234	1.72	468	1.88	546	2.04	624	2.21	702	2.35	780
10	1.50	182	1.63	364	1.79	425	1.94	485	2.10	546	2.23	607
20	1.37	149	1.49	298	1.64	347	1.78	397	1.92	447	2.05	496
28	1.32	130	1.44	260	1.58	303	1.72	347	1.86	390	1.97	433
60	1.21	86	1.32	172	1.45	201	1.57	230	1.70	259	1.81	287
90	1.17	66	1.27	131	1.39	153	1.51	175	1.63	197	1.74	218

4.2 Consideration of Proposed Analytical Approach by Others

The proposed analytical approach for computing long-term deflections due to creep is inspired by "the increased 'n'" approach described in numerous references, including Pauw and Meyers (1964) and Branson (1977), where creep deflections are obtained by computing a neutral axis depth based on an age-adjusted modular ratio. Branson pursued the "increased 'n'" approach with great interest but was unsuccessful in obtaining satisfactory agreement with experimental data, including that reported by Washa and Fluck (1952), likely due to uncertainties in computing kd and I_e. Methods available for computing kd for doubly reinforced beams were iterative and fairly complex, which created room for uncertainties (e.g., Pauw and Meyers (1964)). Additionally, Branson computed I_e using an empirical equation based on an incorrect mechanical model and the impact of I_{cr} based on an increased 'n' may have been misrepresented. The "increased 'n'" approach when implemented using appropriate, mechanics-based methods for computing kd and I_e was found to yield satisfactory test-to-predicted ratios, and the effect of compression reinforcement, creep coefficients, and shrinkage strains on the total deflection can be accurately represented.

Final creep deflections were computed based on the concrete stress at time t, after creep has taken place. Since creep causes a lowering of the neutral axis (i.e., $\overline{kd}$ > kd), the concrete stress at time t is smaller in magnitude than the instantaneous stress at time t_o. More conservative creep deflections (approximately 1.5 times those presented in Table 1) could be obtained by computing creep deflections based on the instantaneous concrete stress. This might be a practical consideration if creep deflections at time $t_1 < t$ are of interest but appears to be overly-conservative when only the final deflection is of interest.

Deflections due to shrinkage were computed for an uncracked section due to reasons outlined in Section 2.2. The net curvature was computed based on residual strains at the top and bottom fibres caused by shrinkage restraint by the top and bottom steel areas. This reflects the efficiency of compression reinforcement in markedly reducing the deflection due to shrinkage warping.

5 CONCLUSIONS

This study presents an overview of the method described in the CAC Concrete Design Handbook for computing incremental deflections due to creep and shrinkage, which is based on empirical methods proposed by Branson (1977). It also presents a mechanics-based approach for computing long-term deflections due to creep and shrinkage, while outlining discrepancies between the two methods. The accuracy of both methods in predicting long-term deflections obtained from test data was investigated. The conclusions of this study are as follows:

1. Instantaneous deflections computed using I_e based on a reduced modulus of rupture were overestimated for Gilbert and Nejadi's (2004) test specimens because specimens were not exposed to drying before loading and the effects of restrained shrinkage were likely slight. On the other hand, instantaneous deflections were more accurately predicted for Washa and Fluck's (1952) test specimens that were exposed to drying prior to loading and so experienced restrained shrinkage.
2. Methods for computing incremental deflections due to creep and shrinkage described in the CAC Concrete Design Handbook are based on empirical methods proposed by Branson (1977). However, the Concrete Design Handbook provisions for computing ultimate shrinkage strains and creep coefficients are simplifications of the values tabulated by Branson (1977). The CAC Concrete Design Handbook method yielded an unconservative mean test-to-predicted ratio of 1.34 for Washa and Fluck's (1952) test specimens. More accurate and slightly conservative test-to-predicted ratios were obtained for Gilbert and Nejadi's (2004) test specimens, and the mean test-to-predicted ratio was 0.97. This is due to the overestimation of the instantaneous deflection for Gilbert and Nejadi's test specimens, which compensated for the underestimation of incremental deflections due to creep and shrinkage.
3. Incremental deflections due to creep and shrinkage can be more accurately predicted using the method described in the Concrete Design Handbook using creep coefficients and ultimate shrinkage strains recommended by Branson (1977).
4. A more detailed analytical approach for computing creep deflection based on an increased modular ratio and a lowered neutral axis, and shrinkage deflection based on strains due to forces imposed by the top and bottom reinforcing steel on the concrete, was found to yield accurate and slightly conservative test-to-predicted ratios with Washa and Fluck (1952) and Gilbert and Nejadi's (2004) test specimens. The mean test-to-predicted ratio computed using this approach is 0.82 for Washa and Fluck's (1952) specimens and 0.92 for Gilbert and Nejadi's (2004) specimens. The coefficient of variation of test-to-predicted ratios are 4.98% for Gilbert and Nejadi's (2004) specimens and 14.32% for Washa and Fluck's (1952) specimens.

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