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ELEMENTAL GRAPH DATA MODEL (EGDM) SIGNIFICANCE: CONSTRUCTABILITY INFORMATION EXTRACTION

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Abstract: With the rapid increase in complexity in the building industry, project managers are facing more and more complex decision environments and problems. Therefore, it would be beneficial to aid project managers in making sound decisions regarding the intelligent knowledgeable selection of the optimal construction methodology for concrete skeletons while making use of Building Information Modelling (BIM). BIM models' usage fused with modelling, and simulation tools allow efficiently prototyping a building and examining its construction activities before breaking the ground. For this purpose, a semantic object-oriented building model that can support geometric-topological analysis of 3D building elements during design and construction was created. This research focused on the generation and implementation of a framework that extracts building elements, along with their topological relationships and geometrical properties, from an existing fully designed Building Information Model (BIM Model) to be mapped into a directed acyclic Elemental Graph Data Model (EGDM). Data extraction was done with the aid of AutoDesk Revit APIs in a C# platform. This paper demonstrates the significance of the EGDM through the extraction of useful information that can give insight into the construction of the project under study.

1 INTRODUCTION

Humans view buildings primarily as an aggregation of physical objects with well-defined geometry and specific spatial relations. In most cases, the architectural and/or structural function of a particular building component is closely related to its shape and its position in relation to other building components. For architects and engineers involved in designing buildings, geometric properties and spatial relations between building components, accordingly, play a major role in finding solutions for most of the design and engineering tasks. However, software tools that allow for a sophisticated spatial analysis of digital building models are not yet available. (Borrmann and Rank 2009)

Building Product Models

While geometric data of building elements can be manipulated and managed by a Computer-Aided Design (CAD) interface, their topological information are conventionally inserted into the model. However, the manual data representation is essentially a complex and challenging task as each constructed facility usually comprises of hundreds of elements with multiple types of topological relationship information, such as connectivity, adjacency, containment, and intersection (Nguyen, Oloufa and Nassar 2005). The lack of such building models that support geometric-topological analysis resulted in various research attempts in the field of computer-based representation of building elements over the last decade. These efforts have concentrated mainly on the development of a semantic object-oriented building model, also called Building Product Model or Building Information Model (BIM) (Borrmann and Rank 2009).

The adoption of Building Information Modelling (BIM) in construction has led to greater integration between project participants/stakeholders in the Architecture, Engineering, and Construction (AEC) industry at the project design phase; the result being the incorporation of new complex tasks into construction applications. However, these new complex tasks require enhanced methods to analyze, extract and present topological relationships in 3D, and, consequently, using such information for faster computation of topological queries.

2 EXISTING TOPOLOGICAL DATA MODELS

Several contributions were performed to represent geometric data and topological relationships among building elements through topological primitives: (Wei, Ping and Jun 1998, Tse and Gold 2003, Suter and Mahdavi 2004, Paul and Borrmann 2009, Grabska, Lachwa and Slusarczyk 2012, Plümer and Gröger 1996, Lamarche and Donikian 2004, Franz, Mallot and Wiener 2005, van Treeck and Rank 2007, Borrmann and Rank 2009), and (Lee and Kwan 2005). A comprehensive review of most of the existing models is thoroughly demonstrated by (Domínguez-Martin, García-Fernández and Feito-Higueruela 2011). Such models proved to be inefficient in handling various queries and complex network analysis, as they mostly store limited semantic information about rooms, openings and walls. Moreover, these models only deal with connectivity and adjacency neglecting any other type of topological relationship. However, BIM demands the availability of all the types of relationships (for example, connectivity, and containment) and their effective storage, in order to enhance the performance of spatial analysis such as construction sequencing, 4D simulation, energy simulation, emergency response and prefabrication optimization (Khalili and Chua 2014).

Subsequently, BIM was utilized to develop a reliable model to perform geometric-topological analysis of building elements that can support all types of spatial relationships. (Essawy and Nassar 2017)

3 ELEMENTAL GRAPH DATA MODEL (EGDM)

Performing geometric-topological analysis of building elements, that would aid professionals in the A/E/C industry in selecting an optimized construction methodology, is very difficult and in most cases impossible without the existence of a semantically rich representation of the building elements: a simple graph. Graphs are highly versatile models for analyzing a wide range of practical problems in which points (nodes or vertices) and connections (links or edges) between them have some physical or conceptual interpretation. Depending on the complexity of the problem, the appropriate kind of graph is employed and advanced graph algorithms, such as spanning trees, shortest path, Hamiltonian paths, search algorithms and topological sortings, are utilized, resulting in a significant reduction in the computational time and storage as compared to dealing with the real-world application directly.

Subsequently, building elements along with their relationships and semantic data are mapped into a novel data representation model: a directed acyclic Elemental Graph Data Model (EGDM), in which vertices denote building elements; while edges represent the topological relations. The EGDM is enhanced by adding information to the vertices to be able to handle wide ranges of queries. (Essawy and Nassar 2017)

3.1 EGDM Graph Representation

Since the EGDM focusses on the representation of building elements and their interrelationships, the employed graph data structure employs the Primal Graph representation. In the EGDM, vertices represent building elements, and directed edges represent topological relations, whose direction identifies the interrelationships between the respective building elements. This simple directed acyclic graph is a logical network data model that can handle graph search algorithms, Depth-First Search (DFS) and topological sortings, to obtain possible elemental construction sequences. The building elements' semantic information required for such algorithms and a wide range of queries, are attached to their respective vertices by a data table, the DT. Hence, for an entire building, the EGDM defines the spatial relationships of its building elements. The types of topological relationships are represented through the direction of the edges, whereas the building elements' properties are included in the DT, which are labels assigned to the associated vertices. The Elemental Graph Data Model (EGDM) retrieval is thoroughly demonstrated in (Essawy and Nassar 2017), and (Essawy and Nassar 2017).

The graph data structure in its presentation preserves the physical and topological characteristics of building elements, as well as the topological consistency through mapping all the building elements along with their topological relationships into a set of vertices and edges.

3.2 Graph Theory and Adjacency Matrix

Graph theory can be used to represent topological relationships in any graph G through the stipulation of sets of edges and vertices, namely E and V respectively. Since most of the building projects comprise of a relatively large number of vertices, the Elemental Graph Data Model (EGDM) can be stored in the form of a matrix to facilitate computational analysis and storage purposes. The matrix is often referred to as the Adjacency Matrix (Adj). For any structure composed of N building elements, the dimensions of its adjacency matrix is N x N, and Adj is represented in Equation 1, as shown:

$$[1] Adj_{DAG} [v, v_h] = \begin{cases} 1 & \text{if there is an edge from } v \text{ and } v_h \\ 0 & \text{otherwise} \end{cases} \quad (\text{J. Gross 2013})$$

4 CONSTRUCTION-ABILITY INFORMATION EXTRACTION

One of the main substantial benefits beyond the graph representation of building elements, the EGDM, can be demonstrated through the ease of extracting useful information from the EGDM which could influence a project's construction sequence and methodology. Such information could be extracted by carrying out some graph measures and metrics and interpret the outcomes to show and illustrate its physical meaning in real life. Selected graph measures and metrics are categorized and explained hereinafter. Two (2) test cases, Figure 1 (3 rooms beside each other) and Figure 2 (3 rooms above each other), are used throughout this section to further illustrate the graph measures and metrics.

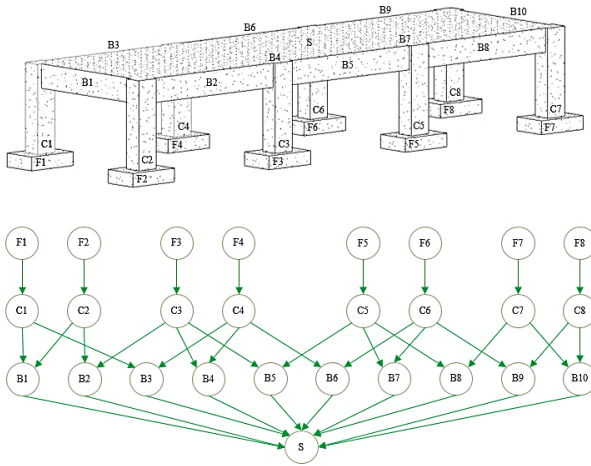


Figure 1: Test Case No.1

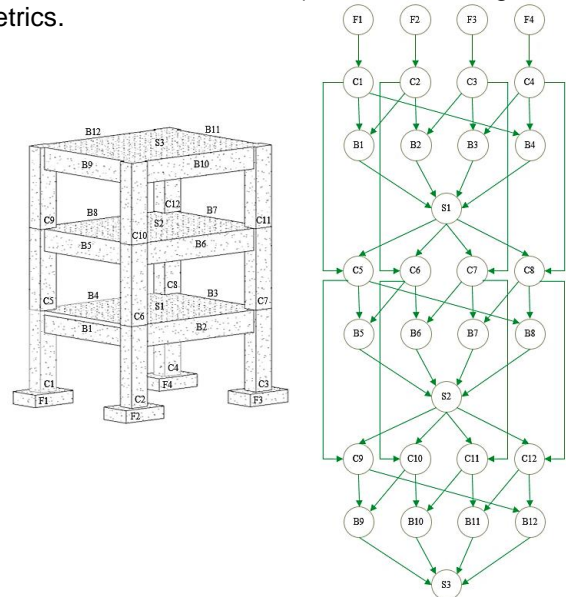


Figure 2: Test Case No.2

4.1 Basic Measures

- Vertex Count, as shown in Equation 2, demonstrates the graph size. When employing such a measure on the EGDM, it tells the number of building elements in the project under study, and, hence, giving insight into the size of the project.
- Edge Count, as shown in Equation 2, demonstrates the sparsity of the graph. When employing such a measure on the EGDM, it tells the number of interrelationships among building elements in the project under study, and, hence, giving insight into how dense and complex the project is.

$$[2] V(G) = \sum_{i=1}^n v_i \quad \text{and} \quad E(G) = \sum_{i=1}^n \sum_{j=1}^n E_{ij} \quad (\text{J. Gross 2013})$$

The Basic Measures for the projects presented in Figure 1 and Figure 2 are as shown in Table 1.

Table 1: Basic Measures of Test Cases 1 and 2

Test Case No.1		Test Case No.2	
Vertex Count:	27	Vertex Count:	31
Edge Count:	38	Edge Count:	56

4.2 Distance Measures

- Graph Distance: When employing such a measure on the EGDM, it tells the number of building elements' levels between the two building elements under study.
- Vertex Eccentricity: When employing such a measure on the EGDM, as shown in Equation 3, it tells the position of the building element under study in the project, by defining the minimum number of building elements between the building element under study and the source/sink vertex.

$$[3] e(v) = \max_{u \in V_G} \{ d(v, u) \} \quad (\text{Gross and Yellen 2006})$$

- Graph Radius: minimum eccentricity of the vertices in the graph G . When employing such a measure on the EGDM, it tells the level of the central building elements in the project under study.
- Graph Diameter: When employing such a measure on the EGDM, as shown in Equation 4, it tells the number of levels of building elements from the source to the sink for the project under study.

$$[4] \text{diam}(G) = \max_{u, v \in V(G)} \{ d(u, v) \} \quad (\text{Gross and Yellen 2006})$$

- Depth: max vertical distance in a graph G . When employing such a measure on the EGDM, it tells the number of levels of building elements for the project under study, hence, giving insight into the project's height.
- Width: When employing such a measure on the EGDM, it tells the number of building elements in the same level for the project under study, hence, giving insight into the project's foot print.
- Graph Path: A sequence of edges $((v_1, v_2), (v_2, v_3), \dots, (v_{n-1}, v_n))$ from a source vertex v_1 to a sink vertex v_n , thus having a length (distance or walk) of $n - 1$. When obtaining the number of Graph Paths in the EGDM, it gives insight into the depth, width and complexity of the project under study.

The Distance Measures for the projects presented in Figure 1 and Figure 2 are as shown in Table 2.

Table 2: Distance Measures of Test Cases 1 and 2

Test Case No.1		Test Case No.2	
Graph Distance: F3 → B5	2	Graph Distance: F3 → B5	5
Graph Distance: F4 → B5	0	Graph Distance: F4 → B5	5
Vertex Eccentricity: B5	2	Vertex Eccentricity: B5	5
	Max of: 2 (distance to source) & 1 (distance to sink)		Max of: 5 (distance to source) & 4 (distance to sink)
Graph Radius:	2	Graph Radius:	5
Graph Diameter:	3	Graph Diameter:	9
Depth:	4	Depth:	10
Width:	10	Width:	4
Number of Graph Paths:	20	Number of Graph Paths:	512

4.3 Connectivity Measures

- Vertex Connectivity: demonstrates the graph connectivity. When employing such a measure on the EGDM, it tells how strongly connected the building elements are.
- Edge Connectivity: demonstrates the graph connectivity. When employing such a measure on the EGDM, it tells how strongly connected the building elements are.

The Connectivity Measures for the projects presented in Figure 1 and Figure 2 are as shown in Table 3.

Table 3: Connectivity Measures of Test Cases 1 and 2

Test Case No.1		Test Case No.2	
Vertex Connectivity:	8	Vertex Connectivity:	4
Edge Connectivity:	20	Edge Connectivity:	8

4.4 Degrees Measures

- Vertex Degree: demonstrates how locally well-connected each vertex is. Ordering the vertices based on their vertex degrees shows degree sequence of the graph under study. The minimum vertex degree, $\delta(G)$, indicates the vertex least connected, while the maximum vertex degree, $\Delta(G)$, indicates the vertex most connected. The vertex degree of a vertex v in a graph G , denoted $\rho(v)$, satisfies Equation 5:

$$[5] \sum_{i=1}^n \rho(v_i) = 2E \quad \text{where } E \text{ is the total number of graph edges} \quad (\text{Gross and Yellen 2006})$$

- Vertex In-Degree: The number of inward directed edges to a given graph vertex v in a directed graph G .
- Vertex Out-Degree: The number of outward directed edges from a given graph vertex v in a directed graph G .

Vertex In-Degree and Out-Degree demonstrates the connectivity of a vertex in a graph. When employing such measures on the EGDM, they determine the building element's immediate predecessor and successor element(s) respectively. For any DAG, the summation of vertex in-degrees for all the vertices v in a directed graph G is equal to the summation of vertex out-degrees for all the vertices which are equal to the total number of graph edges E , as shown in Euler's Degree-Sum Theorem in Equation 6.

$$[6] \sum_{i=1}^n \rho_{in}(v_i) = \sum_{i=1}^n \rho_{out}(v_i) = E \quad (\text{Gross and Yellen 2006})$$

- Vertex In-Neighborhood: The cumulative number of inward directed edges to a given graph vertex v in a directed graph G .
- Vertex Out-Neighborhood: The cumulative number of outward directed edges from a given graph vertex v in a directed graph G .

Vertex In-Neighborhood and Out-Neighborhood demonstrates the significance of a vertex in a graph. When employing such measures on the EGDM, the Vertex In-Neighborhood determines the building element's cumulative predecessors up to the source vertex (predecessor set), while the Vertex Out-Neighborhood determines the building element's cumulative successors up to the sink vertex (successor set). The in-neighborhood and out-neighborhood are important graph measures, as they reveal the relative significance and importance of the building element with respect to the whole project.

The Degree Measures of a number of selected building elements for the projects presented in Figure 1 and Figure 2 are as shown in Table 4.

Table 4: Degree Measures of Test Cases 1 and 2

Test Case No.1						Test Case No.2					
	F1	C2	C7	B5	S		F1	C2	C7	B5	S
Vertex Degree:	1	3	3	3	10	Vertex Degree:	1	3	5	3	4
Vertex In-Degree:	0	1	1	2	10	Vertex In-Degree:	0	1	2	2	4
Vertex Out-Degree:	1	2	2	1	0	Vertex Out-Degree:	1	2	3	1	0
Vertex In-Neighborhood:	0	1	1	4	26	Vertex In-Neighborhood:	0	1	13	24	30
Vertex Out- Neighborhood:	4	3	3	1	0	Vertex Out- Neighborhood:	22	21	12	1	0

Figure 3 shows the building elements' out-neighborhood for the test cases presented in Figure 1 and Figure 2. This indicates that the building elements with the higher out-neighborhood are essentially more important, as these elements have higher number of successors, i.e. more elements are dependent on them. Figure 4 shows the building elements distribution based on their respective out-neighborhood.

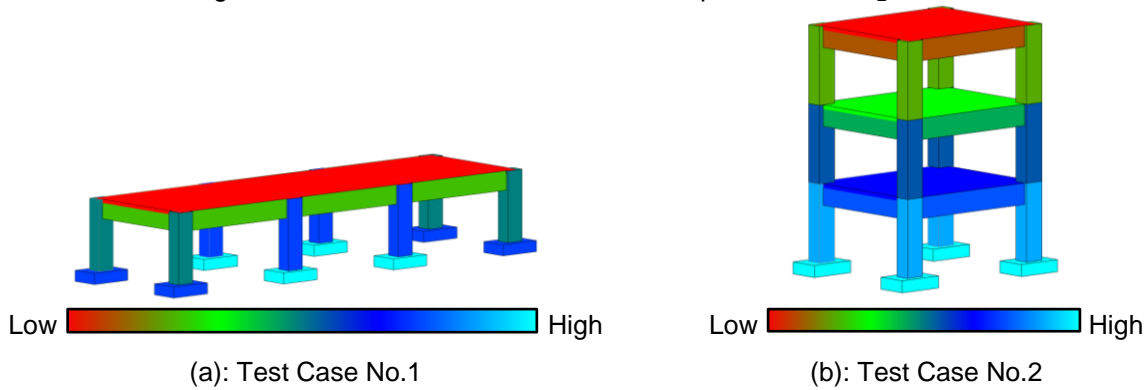


Figure 3: Elements' Out-neighborhood

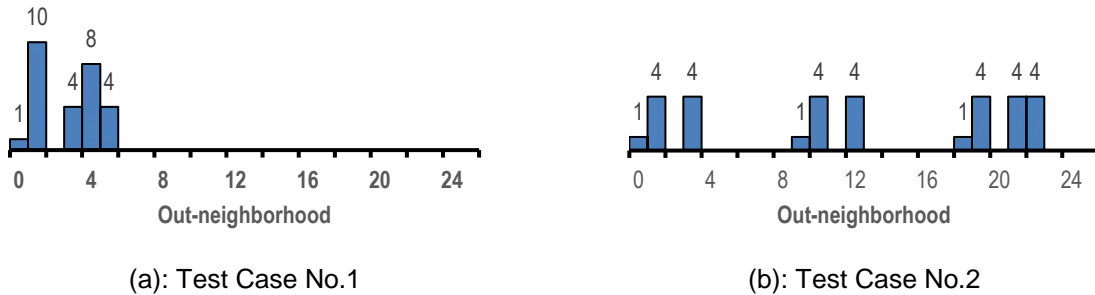


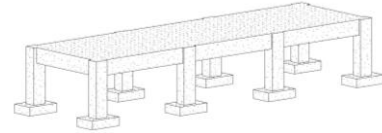
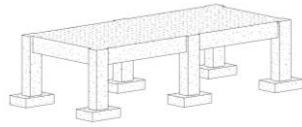
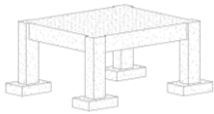
Figure 4: Out-neighborhood Distribution (Dependency-Moment)

4.5 Building Elements' Importance Measures

Based on the aforementioned measures, additional building elements' importance measures can be deduced, such as: Slenderness Ratio, Average Element Importance, Variation in Element Importance, and Degree of Variation in Element Importance.

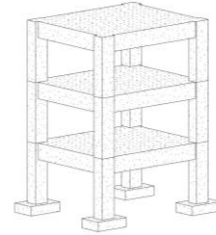
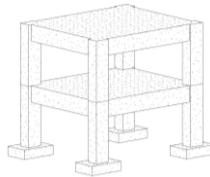
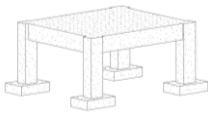
- Slenderness Ratio: determines the graph's depth to width ratio, as shown in Equation 7. When employing such a measure on the EGDM, the Slenderness Ratio determines the tallness and flatness of the project under study. For instance, the Slenderness Ratio of the EGDM for some illustrative cases are demonstrated in Figure 5.

$$[7] \lambda = \frac{\text{Graph Depth}}{\text{Graph Width}}$$



No. of Elements: 13	No. of Elements: 20	No. of Elements: 27
Depth: 4	Depth: 4	Depth: 4
Width: 4	Width: 7	Width: 10
Slenderness Ratio: 1	Slenderness Ratio: 0.57	Slenderness Ratio: 0.4

(a): Illustrative Cases No.1



No. of Elements: 13	No. of Elements: 22	No. of Elements: 31
Depth: 4	Depth: 7	Depth: 10
Width: 4	Width: 4	Width: 4
Slenderness Ratio: 1	Slenderness Ratio: 1.75	Slenderness Ratio: 2.5

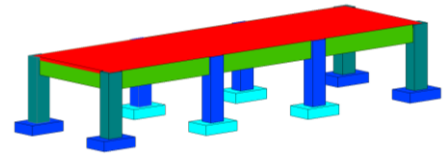
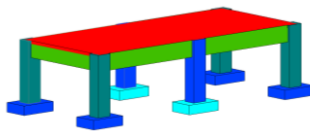
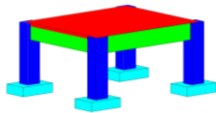
(b): Illustrative Cases No.2

Figure 5: Illustrative Cases – Slenderness Ratio

Hence, the slenderness ratio decreases as the project extends and expands horizontally, as shown in Figure 5(a), and increases as the project extends vertically, as shown in Figure 5(b).

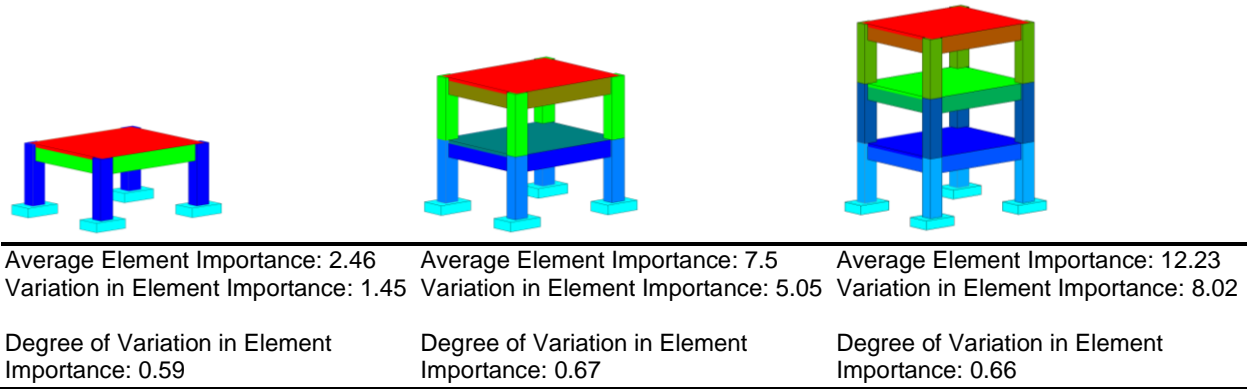
- Average Element Importance: determines the average out-neighborhood for building elements for the project under study. When compared against the maximum element importance, it gives insight into the building's height.
- Variation in Element Importance: determines the standard deviation of the out-neighborhood for all building elements for the project under study and gives insight into the building's footprint area.
- Degree of Variation in Element Importance: determines the coefficient of variation of the out-neighborhood for all building elements for the project under study. It is obtained by dividing the standard deviation by the mean.

When employing such a measure on the EGDM to compare between projects, the Building Elements' Importance Measures are determined, as shown in Figure 6.



Average Element Importance: 2.46	Average Element Importance: 2.65	Average Element Importance: 2.74
Variation in Element Importance: 1.45	Variation in Element Importance: 1.60	Variation in Element Importance: 1.65
Degree of Variation in Element Importance: 0.59	Degree of Variation in Element Importance: 0.60	Degree of Variation in Element Importance: 0.60

(a): Illustrative Cases No.1



(b): Illustrative Cases No.2

Figure 6: Illustrative Cases – Building Elements' Importance Measures

Hence, as the project extends and expands horizontally, as shown in Figure 6(a), the Average Element Importance and Variation in Element Importance slightly increases, while the Degree of Variation in Element Importance stays almost the same (projects of similar depth). And as the project extends vertically, as shown in Figure 6(b), the Average Element Importance and Variation in Element Importance increases, while the Degree of Variation in Element Importance slightly increases.

4.6 Construction Complexity Measures

Based on the aforementioned measures, additional construction complexity measures can be deduced, such as: Dependency Moment, Maximum Dependency Moment and Category-Out-neighborhood.

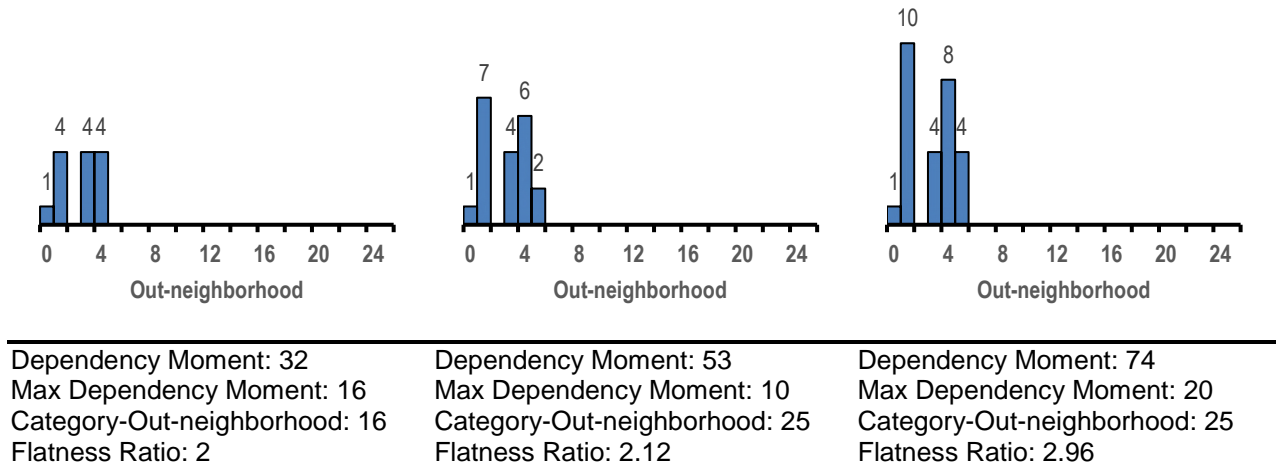
- **Dependency-Moment:** determines the vertices out-neighborhood distribution moment. When employing such a measure on the EGDM, the Dependency-Moment determines the summation of the product of the out-neighborhood categories and the number of elements for each category. As shown in Equation 8, it can, also, be expressed as the summation of all the building elements' out-neighborhood.

$$[8] \text{ Dependency Moment} = \sum_{i=1}^n \rho_{out-neighborhood} (v_i)$$

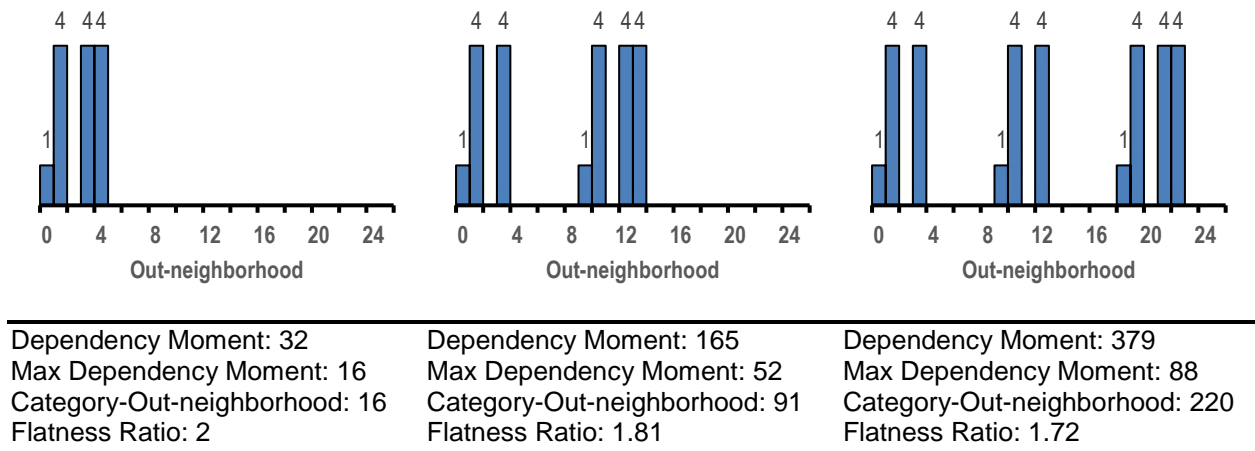
- **Maximum Dependency Moment:** determines the moment caused by the vertices with maximum out-neighborhood. When employing such a measure on the EGDM, the Maximum Dependency Moment reveals whether the building under study is flat or high-rise. It can be expressed as the product of the maximum out-neighborhood and the number of building elements with the maximum out-neighborhood.
- **Category-Out-neighborhood:** When employing such a measure on the EGDM, the Category-Out-neighborhood reveals the flatness and size of the building under study. It can be expressed as the product of the maximum out-neighborhood and the maximum number of building elements in any out-neighborhood category.
- **Flatness Ratio:** When employing such a measure on the EGDM, the Flatness Ratio reveals the flatness and size of the building under study. It can be expressed as the ratio between the Dependency-Moment and the Category-Out-neighborhood.

When employing such measures on the EGDM to compare between projects, additional construction complexity measures can be determined, as shown in Figure 7.

Hence, as the project extends horizontally, as shown in Figure 7(a), the out-neighborhood categories don't change, but their frequencies increases. Moreover, the Dependency Moment, Max-Dependency Moment, and Category-Out-neighborhood stays almost the same with a slight increase, while the Flatness Ratio significantly increases. And, as the project extends vertically, as shown in Figure 7(b), the out-neighborhood categories increase, but their frequencies stays the same. Moreover, the Dependency Moment, Max-Dependency Moment, and Category-Out-neighborhood significantly increases, while the Flatness Ratio decreases.



(a): Illustrative Cases No.1



(b): Illustrative Cases No.2

Figure 7: Illustrative Cases – Construction Complexity Measures

To conclude, the out-neighborhood distribution of 9 test cases, as presented in Figure 8, clearly demonstrates the effect of extending a building horizontally and vertically.

As the building extends vertically, the out-neighborhood categories increase, but their frequencies stay the same. The out-neighborhood distribution is nearly uniform and is clustered (each cluster represents a floor level). The average out-neighborhood is close to the maximum value, and as the building extends vertically, it becomes closer.

As the building extends horizontally, the out-neighborhood categories slightly increase, but their frequencies significantly increase. The out-neighborhood distribution stays clustered (each cluster represents a floor level), but as the building extends horizontally, each cluster becomes more left skewed. The average out-neighborhood is less than the maximum value, and as the building extends horizontally, it becomes smaller.

Finally, these measures can be utilized on the EGDM of more complex buildings in order to determine the graph complexity, and, hence, provide insight into the project under study, as demonstrated in Chapter 5.

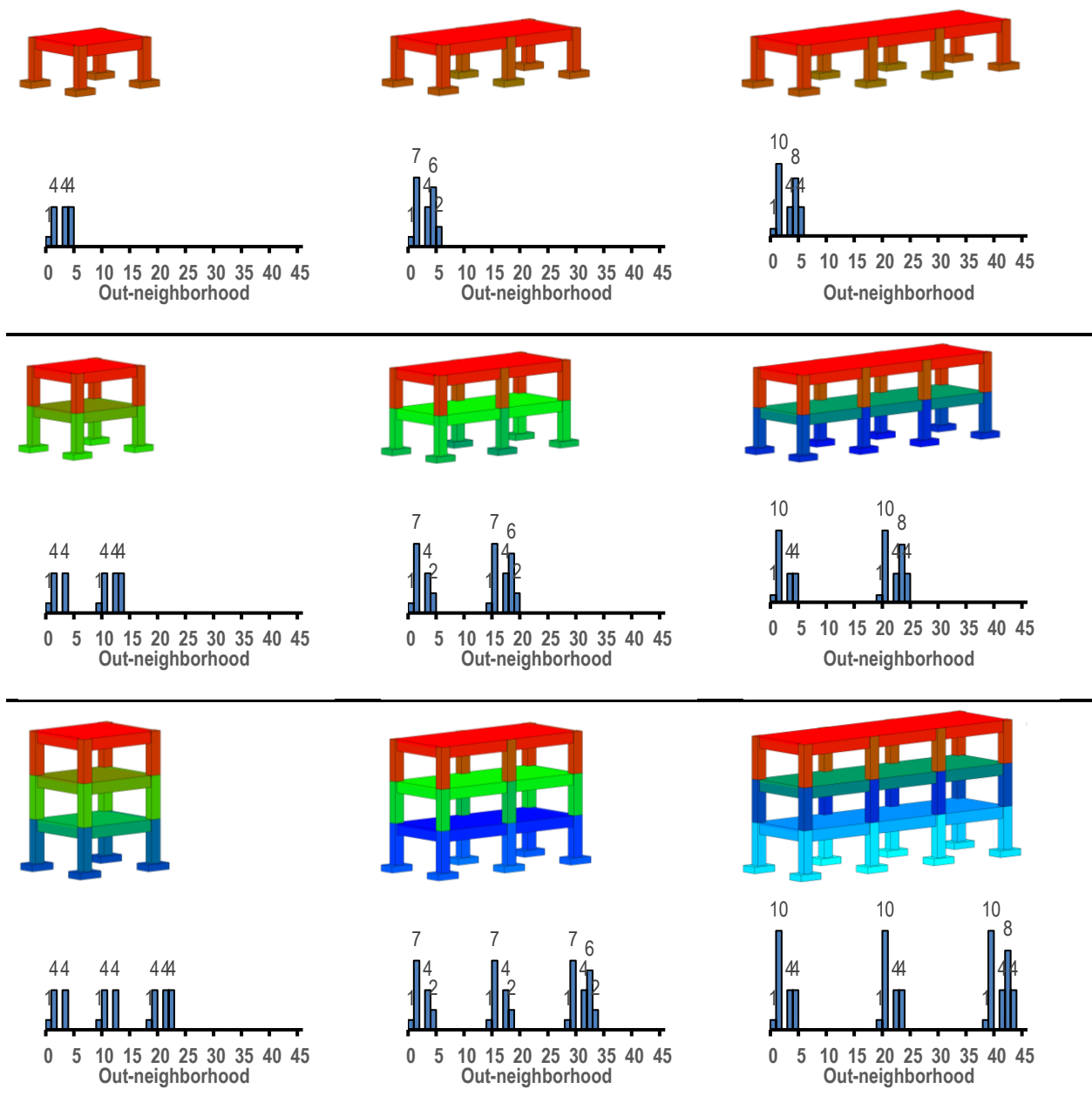


Figure 8: Out-neighborhood Distribution of 9 Test Cases

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