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Condition Prediction of Concrete Bridge Decks Using Markov Chain Monte Carlo-Based Method

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Abstract: In view of budget limitations and inadequate investment in civil infrastructure, concrete bridges are deteriorating; raising concern for public safety. This state of affairs necessitates the development of a smart and efficient integrated method for optimized bridge intervention plans at the project and network levels. The present study focuses on modelling deterioration of concrete bridge decks. A reliable deterioration model enables transportation agencies to optimize their maintenance, repair, and rehabilitation (MR&R) plans, and consequently address needed maintenance works effectively. This paper presents a hybrid Bayesian-optimization method to calibrate transition probabilities of the developed Markovian model. These probabilities are demonstrated in the form of posterior distributions, whereas the transition from a condition state to the next lower state is represented by a function that captures the severity of defects such as corrosion, delamination, cracking, spalling, and pop-out. The Bayesian belief network is utilized to investigate the severity of these defects. The proposed method incorporates Markov chain Monte Carlo (MCMC) Metropolis-Hastings algorithm to derive the posterior distributions of transition probabilities. Finally, a stochastic optimization model is designed to build a variable transition probability matrix for each five-year zone in an effort to speed up the computational effort.

1 INTRODUCTION

Infrastructure systems refer to systems that support the prevailing of the society. Infrastructure systems are divided into: bridges, highways, dams, waste water systems, sewer water systems, etc. Existing infrastructure is vulnerable to high levels of deterioration. Therefore, billions of dollars should be invested every year in order to maintain the desired levels of standards to the customers. The deterioration in Canada's infrastructure systems is mainly because of two main reasons: 1) the decrease in the investment of the infrastructure systems, and 2) most of the infrastructure systems were constructed relatively a long time ago. Bridges are subjected to aggressive influences such as overloading, chloride ingress, cycles of the freeze and thaw, earthquakes, etc. Thus, they are more likely to deteriorate significantly. The overall condition of the bridges and roads in Canada is "Good" where 57% of the bridges are in "Good" condition, and 22% of the bridges are in "Fair" condition (Felio 2016). The number of highway bridges in Canada is 75,000 where their average age is 24.5 years in 2007 compared to a mean service life of 43.3 years (Statistics Canada 2009a). This means that the bridges in Canada have passed 57% of their useful lifetime (Statistics Canada 2009a). Bridges in Quebec have the highest average age of 31 years followed by Nova Scotia with an average age of 28.6 years (Statistics Canada 2009b).

Bridge Management Systems (BMSs) have become a necessity nowadays in order to provide a tool for the government agencies to manage a large network of bridges under some constraints such as limited resources (budget). AASHTO defined Bridge Management System (BMS) as "a system designed to optimize the use of available resources for inspection, maintenance, rehabilitation and replacement of bridges". There are five main components of BMS which are (Czepiel 1995): 1) database for data storage, 2) condition rating model, 3) deterioration model, 4) cost model, and 5) optimization model for running the system. Deterioration model is one of the main pillars of the BMS because it enables the asset managers to forecast the future condition of bridge elements. A cost-effective MR&R activity is highly dependent on the capability of the deterioration model to predict the future time-dependent performance of the bridge element, whereas a reliable deterioration curve is needed in order to obtain information about the need and timing of maintenance activities for a certain planning horizon. This paper presented a hybrid Bayesian-based model that is capable of predicting the future performance of concrete bridge decks.

2 OVERVIEW OF DETERIORATION MODELS

A deterioration model can be defined as a relationship between the condition of the bridge element and a vector of explanatory variables, which represent a group of variables that affect the performance of the bridge element such as age, environmental conditions, applied loads, material properties, etc. The deterioration model can be divided into two categories which are: deterministic and stochastic models. Deterministic models assume that the future performance of the bridge elements is certain over time based on mathematical and statistical approaches such as linear and non-linear regression, artificial neural network, support vector machines, straight-line extrapolation, and curve-fitting.

The main limitation of the deterministic models is that they fail to consider the randomness and uncertainty of the deterioration process of bridges due to the existence of un-observed explanatory variables and in-accurate inspection procedures (Agrawal, 2010). Stochastic models define the deterioration process in the form of one or more random variables, which can be modeled using probability density functions. Markov-based model is considered one of the most common stochastic models. Stochastic models can be classified into: state-based models and time-based models. State-based model is based on calculating the probability that an element will deteriorate to the next lower state over a unit of time. On the other hand, time-based model is based on defining the probability of the time taken by an element to deteriorate to the lower state (Morcous et al., 2010).

3 LITERATURE REVIEW

In the recent years, several studies have been conducted to model the deterioration of the concrete bridges. Zambon et al. (2017) compared between a group of stochastic models which are: Markov chain with exponentially-distributed and Weibull-distributed sojourn times, and gamma process. They concluded that the gamma process has better prediction capabilities when compared to the Markov chain models. Mašović and Hajdin (2014) utilized expectation maximization (EM) to estimate the transition probabilities of the Markov chain model. The developed model was applied to data from the Serbian Bridge Information Database to improve the deterioration of the bridge elements. They highlighted that the introduced developed model can be used when limited inspection records are available. Shim and Lee (2016) modeled the deterioration of the bridge decks based on stochastic Markov decision process. They estimated the transition probabilities as a function of the median duration years.

Le and Andrews (2015) modeled the deterioration of the bridge elements based on the two-parameter Weibull distribution. Anderson Darling test is used to compare between a group of probability distributions. The parameters of the Weibull distribution were defined based on the rank regression. Muñoz et al. (2016) presented a methodology to predict the deterioration of the bridges using both Markov chain and regression analysis in the case of small sample size. They illustrated that the proposed methodology provided conservative estimates for the future condition ratings as well as similar estimates to the traditional methods in calibrating the Markovian models and regression analysis. Son et al. (2010) incorporated both time delay neural network (TDNN) and backward prediction model (BPM) to predict the future bridge condition rating. The backward prediction model was used to determine the missing historical records, which subsequently improves the prediction accuracy. M.Abdelkader et al. (2019) modeled the deterioration of bridge decks using semi-Markov decision process and Latin hypercube sampling. The sojourn time of each condition state is fitted to a certain probability distribution based on some goodness of fit tests. The parameters of the probability distribution functions are obtained using maximum likelihood estimation. They highlighted that the semi-Markovian model outperformed the time-based Markovian model in addition to the weibull and gamma distribution models.

However, deterministic models such as artificial neural network and multiple regression, often fail to capture the uncertainty and randomness of the deterioration process, whereas there is no certainty associated with the condition state the bridge element will enter within the next period of time. Moreover, sate-based models do not consider the sojourn times (waiting times). Thus, it is more realistic to model the deterioration in terms of a function of the time spent in a certain condition state (Ravirala and Grivas, 1995).

4 PROPOSED METHOD

The proposed method has five modules 1) data processing, 2) conditional probabilities module, 3) Bayesian belief network (BBN) module, 4) Metropolis-Hastings module, and 5) stochastic optimization module (see Figure 1). The proposed method is a five-stage process, whereas it is divided into five main modules which are: The output of the data processing stage is a group of censored events. The purpose of the second module is to calculate the conditional probabilities. The conditional probabilities can be either known or unknown. For the known conditional probabilities, the Anderson-Darling test is performed for each probability distribution to select the best-fit distribution. The best-fit distribution is the one associated with the smallest Anderson-Darling statistic. Then, the parameters of the probability distribution are defined using maximum likelihood estimation (MLE). The unknown conditional probabilities are calculated based on the maximum entropy (ME) principle. The conditional probabilities are calculated based on a single objective function that maximizes entropy. The decision variables are the conditional probabilities, whereas the optimization problem is solved via genetic algorithm. Genetic algorithm is a method that is used to solve problems based on genetic processes of biological organisms, whereas it is based mainly on two operators which are: mutation and crossover. More details about the genetic algorithm can be found in M.Abdelkader et al. (2019).

In the third module the likelihood of the in-state probabilities is calculated accounting for the uncertainties associated with the transition time, and uncertainty associated with the transition probability. Thus, the conditional probabilities and the marginal probabilities are expressed in the form of probability distributions rather than discrete values. The developed computerized tool enables users to select the number of samples, the type, and parameters of both conditional and marginal probabilities. The probability distributions are generated using stratified sampling technique called “Latin hypercube sampling” in order to overcome the limitations of Monte Carlo sampling. Bayesian belief network (BBN) is used to analyze the relationship between the extent of defect severity and the value of in-state probability.

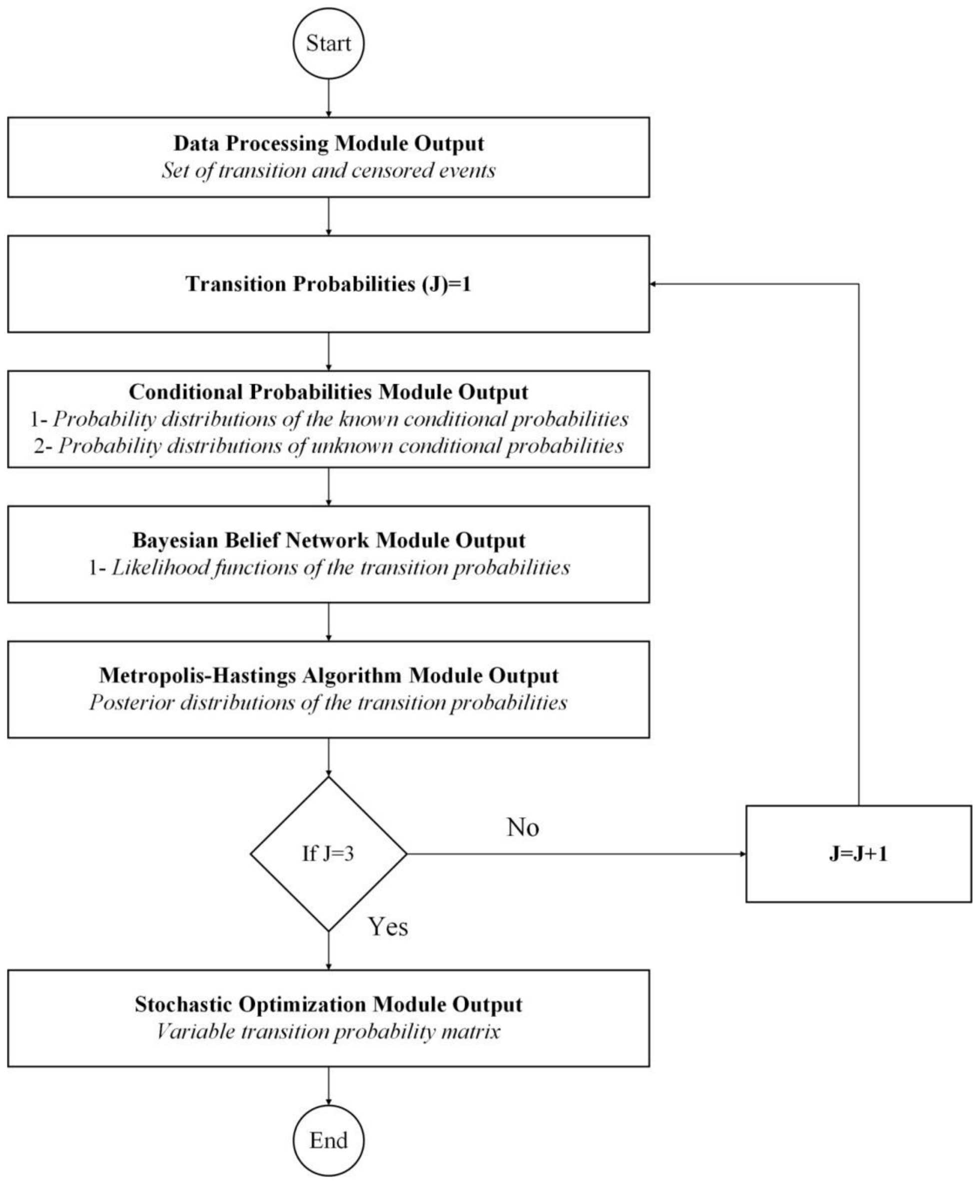

Figure 1: Framework of the overall research methodology

The term “in-state probability” refers to the probability that a certain element remains in a certain condition state i, varying from 1 to 4, for a certain period of time. The direct links denote the dependencies among the five bridge defects considered in the paper. In addition, it depicts the dependency between each defect and related in-state probability. The proposed model is concerned with five types of bridge defects which are: corrosion, delamination, cracking, spalling, and pop-out. The in-state probability include: P_{11}, P_{22}, and P_{33}. The next step is to define a set of mutually exclusive states for each node. For each one of the five bridge defects, there are four condition states which are: “Good”, “Medium”, “Poor”, and “Very poor”. For the in-state probability, there are two states which are: “Yes”, and “No”.

The fourth module is the Metropolis-Hastings algorithm. Metropolis-Hastings is employed to generate the posterior distribution of each in-state probability. The computerized tool enables the user to define the following parameters to calculate the posterior probabilities: 1) type and parameter of the prior distribution, 2) type and parameter of the proposal distribution, 3) number of samples, 4) number of burn-in samples, and 5) acceptance rate. The previous modules are repeated for each of the three transition events (TEs): $TE(1,2)$, $TE(2,3)$, and $TE(3,4)$, whereas the output of the four modules is the three posterior distributions P_{11}, P_{22}, and P_{33}. A stochastic optimization model is designed in order to address the variability associated with each decision variable. A variable transition matrix is employed because it is not reasonable to assume the same deterioration pattern for the whole service life of the bridge deck. A transition probability matrix is calculated for each zone. The decision variables are the transition probabilities for each zone, whereas they are calculated based on a single objective function that maximizes the joint probability. The transition probabilities are computed using genetic algorithm by sampling from the posterior distributions. Via the transition probabilities, the future performance of the bridge element can be forecasted.

5 BAYESIAN INFERENCE

Statistical inference can be defined as “theory, methods, and practices of building judgments about parameters of a population commonly using random sampling, which helps in drawing conclusions from the data that are subjected to random variation (Garfield and Ben-Zvi, 2008). There are two broad categories of interpreting the probabilities in the statistical inference which are: Bayesian inference and frequentist inference. The main significant feature between the Bayesian inference and frequentist inference is the capability of the Bayesian inference to include additional information in the form of prior distribution (Rudas, 2008). In the Bayesian inference, there are two sources of information about the unknown parameter which are: prior distribution and likelihood function. The prior distribution is mainly based on previous studies. Consequently, the prior distribution and the likelihood function are used to generate the posterior distribution. Prior probability is the probability of the parameter of interest before the current data is examined while likelihood function represents the likelihood of the parameter of interest given the current data is observed. Finally, the posterior probability is the probability after the current data is examined. In other words, the Bayesian inference provides a compromise between the likelihood data and prior knowledge.

Assume $Y = \{Y_1, Y_2, Y_3, \dots \dots \dots Y_n\}$ represents a set of condition ratings for a group of bridge decks. Bayesian inference is used to estimate the unknown parameter θ of the probabilistic model M as follows (Micevski et al., 2002).

$$[1]\ \varphi(\theta_1, \theta_2 | Y, M) = \frac{f(Y \,\big|\, \theta_1, \theta_2, M) \times \pi(\theta_1, \theta_2, M)}{\int f(Y|\theta_1, \theta_2, M) \times \pi(\theta_1, \theta_2, M)\ d\theta} = \frac{f(Y \,\big|\, \theta_1, \theta_2, M) \times \pi(\theta_1, \theta_2, M)}{f(Y|M)}$$

Where;

θ_1, θ_2 denote the unknown parameters of the model which are the transition probabilities in the present study. $\varphi(\theta_1, \theta_2|Y, M)$ denotes the posterior distribution. $f(Y \,\big|\, \theta_1, \theta_2, M)$ indicates the likelihood function and it is obtained as a result of a combination of BBN, Latin hypercube sampling, and genetic algorithm. $f(Y|M)$ is the marginal likelihood function, whereas it is independent of θ. Therefore, the posterior distribution can be expressed as follows. $f(Y|M)$ is a normalizing constant to ensure that the posterior distribution is integrated to one over all its possible values.

$$[2]\ \varphi(\theta_1, \theta_2|Y, M) \propto f(Y \,\big|\, \theta_1, \theta_2, M) \times \pi(\theta_1, \theta_2\theta_3, M))$$

Where;

The posterior distribution is proportional to the likelihood function multiplied by the prior distribution. The definition of prior probabilities requires special attention, whereas it is based on knowledge and feedback of the experts. Prior distribution is assumed non-informative distribution. Consequently, the posterior distribution is proportional to the likelihood distribution. Non-informative distribution is chosen because there is no available previous information about the unknown parameters, whereas all the values of the unknown parameters are assumed to be of equal probability. The prior distribution is assumed to be a uniform distribution within the interval $[0, 1]$. As the prior distribution becomes more informative, it will have a greater influence on the posterior distribution.

6 PERFORMANCE METRICS

The proposed model utilizes three performance indicators to compare between the three deterioration models. The three performance indicators are: root-mean square error ($RMSE$), mean absolute error (MAE), chi-squared statistic (x^2). $RMSE$, MAE, and x^2 can be calculated using Equations 3, 4 and 5, respectively (Nazari et al., 2015; Ranjith et al., 2013). For the chi-squared statistic, the model is based on the null hypothesis that the observed condition rating of the bridge deck is consistent with the predicted condition rating of the bridge deck. The chi-squared test is applied to the three deterioration models at a confidence level 95%. If the chi-squared test statistic is larger than the critical chi-squared test statistic at K degrees of freedom and α (significance level) equals to 0.05 (confidence level is 95%). Then, the null hypothesis is rejected, otherwise the null hypothesis is accepted.

$$[3]\ RMSE = \sqrt{\frac{1}{K}\sum_{i=1}^{K}(O_i - P_i)^2}$$

$$[4]\ MAE = \frac{1}{K}\sum_{i=1}^{K}|(O_i - P_i)|$$

$$[5]\ x^2 = \sum_{i=1}^{K}\frac{(O_i - P_i)^2}{P_i}$$

Where;

O_i indicates the observed condition of the bridge deck. P_i indicates the predicted condition of the bridge deck. K represents the number of observations (bridge decks).

7 MODEL IMPLEMENTATION

The proposed methodology utilizes 181 inspection records from the Ministry of Transportation in Quebec (MTQ), Canada. One hundred fifty six are used for training the model, while the remaining twenty five records are used for testing the model. Out of the 156 inspection records, there are 104 transition events and 52 censored events, whereas the number of events for the $TE(1,2)$, $TE(2,3)$, and $TE(3,4)$ are 55, 19 and 30, respectively. There are four condition states covering the three transition events. The four condition states are: Good (1), medium (2), poor (3), and very poor (4).The posterior distributions for each of the three in-state probabilities are computed using the Metropolis-Hastings algorithm interface, whereas the user is able to define the number of samples, number of burn-in samples, optimum acceptance rate, parameters of the proposal distribution, and the lag of the autocorrelation function. The proposed model utilizes a multi-variate normal distribution as a proposal distribution and a uniform distribution as a prior distribution. The model output is the posterior probabilities of the in-state probabilities, values of the convergence diagnostics, and a set of plots. The interface of the Metropolis-

Hastings algorithm module is shown in Figure 2. The posterior distributions of P_{11} is shown in Figure 3. The values of the mean of three posterior distributions of P_{11}, P_{22}, and P_{33} are: 0.9552, 0.9597 and 0.9211, respectively. The values of the standard deviation of the three posterior distributions are small, whereas the values of the standard deviation of the three posterior distributions of P_{11}, P_{22}, and P_{33} are: 0.01348, 0.01328 and 0.01361, respectively.

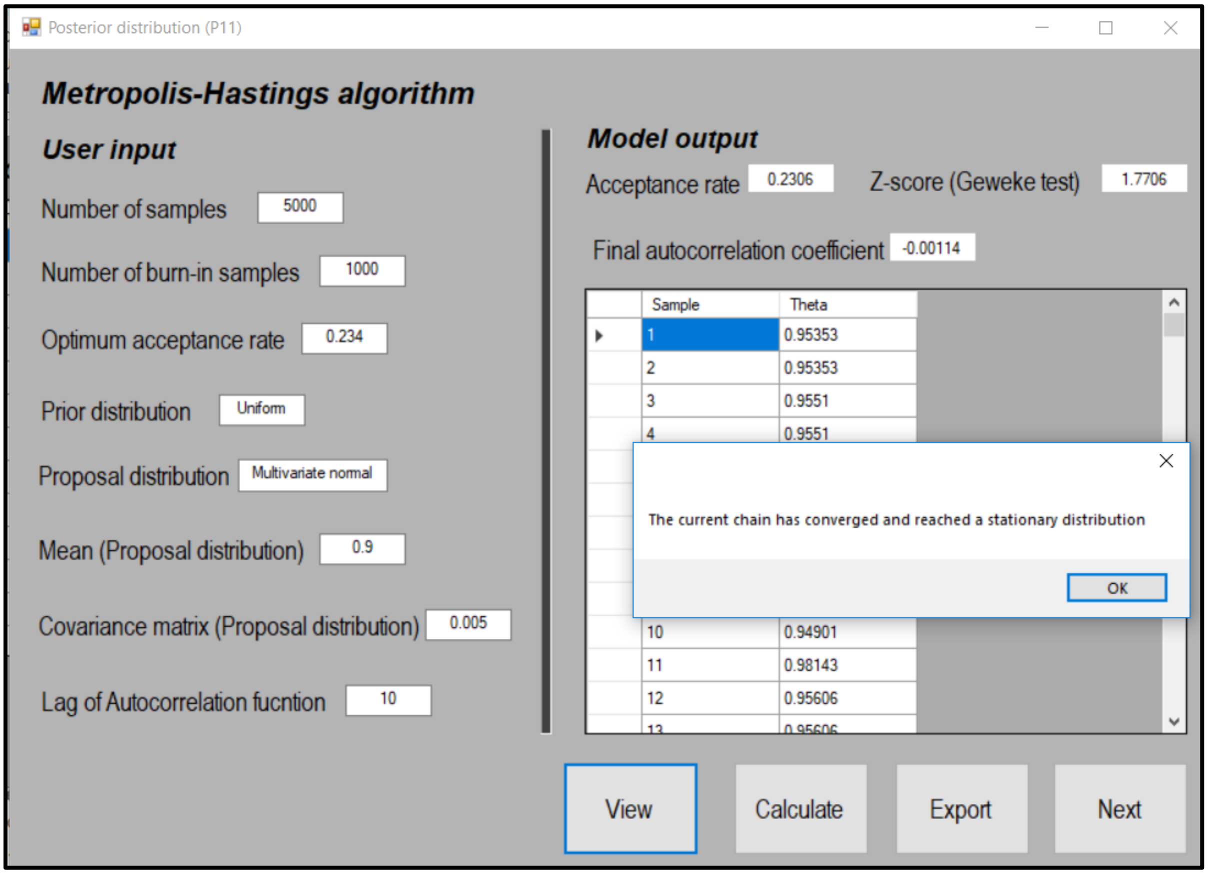

Figure 2: Interface of the Metropolis-Hastings algorithm model

As shown in Figure 2, the acceptance rate is 0.2306, Z-score is 1.7706, and final autocorrelation coefficient is -0.00114 for the in-state probabilityP_{11}. If the calculated probabilities satisfy the three tests, a message box will appear indicating that the current chain has converged. Otherwise, a message box will appear indicating that the current chain did not converge. As shown in Figure 2, the constructed chain of P_{11} satisfies the three convergence diagnostics, which means that the current chain has converged. Since, the Markov chains fulfill the convergence diagnostics. Thus, the type and parameters of the prior and proposal distributions are correctly defined.

A comparison between different deterioration models is shown in Table 1. The proposed method is denoted as $H-B$. The chi-squared critical values at 180 degrees of freedom and a significance level of 5% equals to 212.304. In terms of $RMSE$, $H-B$ achieved the lowest $RMSE$ ($RMSE = 0.7716$). On the other hand, gamma distribution achieved the highest $RMSE$ ($RMSE = 1.4584$). Thus, $H-B$ achieved the best performance based on $RMSE$. For MAE, $H-B$ provided the lowest MAE ($MAE = 0.5401$). On the other hand, gamma distribution achieved the highest MAE ($MAE = 0.9899$). Thus, $H-B$ provided the best performance according to MAE. The gamma and weibull distributions fail to pass the chi-squared test because the chi-squared critical value is larger than the chi-squared statistic. $H-B$ provided the best performance according to x^2 ($x^2 = 46.0583$). Based on the previous statistics, $H-B$ outperformed other models in terms of $RMSE$, MAE, and x^2.

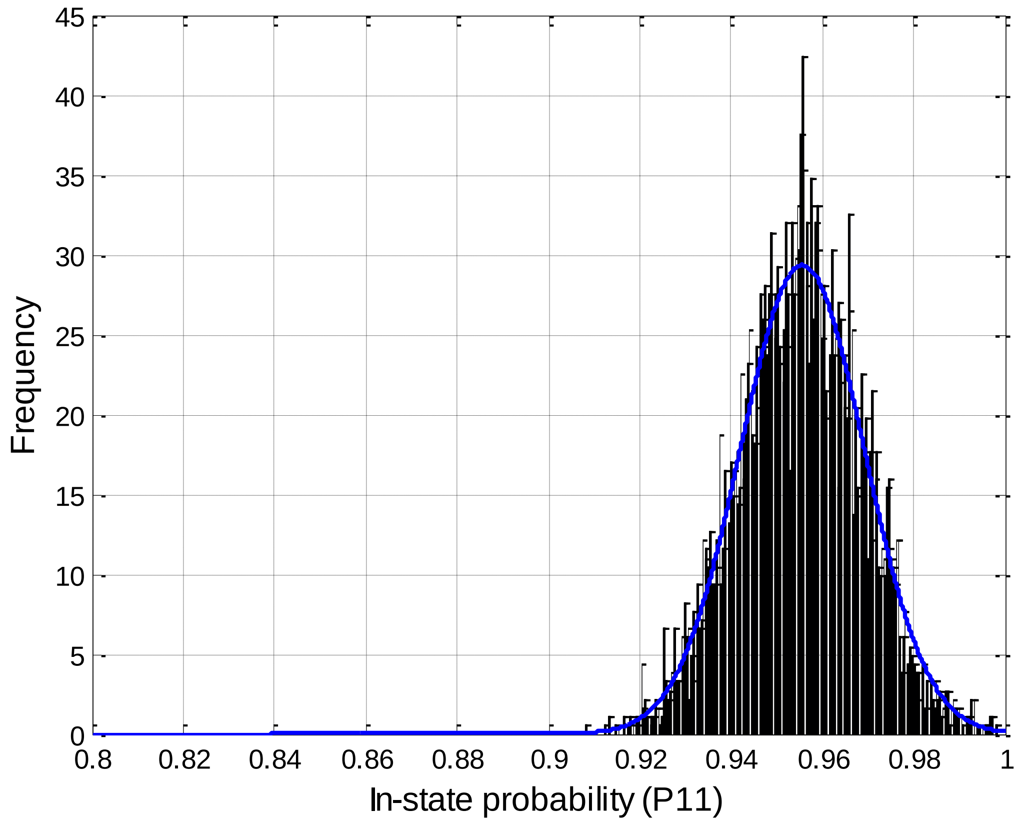

Figure 3: Posterior distribution of the in-state probability P_{11}

Table 1: Comparison between deterioration models

Deterioration model	RMSE	MAE	x^2
Hybrid Bayesian model	0.7716	0.5401	46.0583
Weibull distribution	1.4527	0.9834	356
Gamma distribution	1.4584	0.9889	356.6667

8 SUMMARY and CONCLUSION

This paper presented a hybrid Bayesian-based model capable of predicting future performance of concrete bridge decks. The proposed method utilizes regular inspection records provided by the Ministry of Transportation in Quebec (MTQ). The probability distributions of the developed likelihood functions needed in the Markov Chain combine both Latin hypercube sampling and Bayesian belief network to address the stochastic nature of the transition probabilities used in the developed method. The results demonstrate that Bayesian belief networks enable the impact assessment of the identified defects severities on the overall bridge deck condition. Five bridge defects are considered: corrosion, delamination, cracking, spalling and pop-out. Metropolis-Hastings algorithm is employed to calculate the posterior distributions of the in-state probabilities of P_{11}, P_{22}, and P_{33}. A computerized tool is developed to facilitate the use of the proposed method by the user. The proposed method outperformed the commonly-utilized weibull distribution, whereas the proposed model achieved an improvement in the $RMSE, MAE$, and x^2 equal to 46.885%, 45.078% and 87.062%, respectively. Based on the conducted comparison, the proposed method provides promising results in terms of the prediction accuracy. Thus, the developed model helps in providing a reliable depiction of the future performance of the bridge decks which consequently helps in determining the optimal maintenance, repair and rehabilitation activities in a proper manner as well as decision making for both project level and element level.

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