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PREDICTION OF COMPRESSIVE CAPACITY OF GUSSET PLATES USING A COMPUTATIONAL INTELLIGENCE TECHNIQUE

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Abstract: Gusset plate buckling is a major failure mode in bracing systems and truss bridges. The stress distribution in the connection area under compression is complex, and as such, it is difficult to accurately evaluate the compressive capacity of gusset plates. Current procedures for the compressive design of gusset plates involve highly simplified approaches, which typically result in poor and inconsistent prediction of compressive strength. In this research, a computational intelligence technique, called gene expression programming (GEP), which is a powerful variant of genetic programming, is employed to develop predictive models for compressive capacity of corner gusset plates. The empirical models are developed based on a comprehensive database, consisting of test results and test-validated finite element models, collected from the literature. The database under consideration covers several key parameters influencing the buckling capacity of gusset plates, and encompasses reasonably wide ranges for the mechanical and geometrical properties of typical gusset plates used in the industry. The predictive models correlate the ultimate compressive strength of gusset plates with their mechanical and geometrical properties. A comparative study is conducted to evaluate the prediction performance of these models, and compare the results of the best performing model with those of the well-known column analogy method. The results indicate that the developed expressions are capable of estimating the compressive capacity of gusset plates with high accuracy, and their prediction performances are significantly better than that of the existing analysis model.

1 INTRODUCTION

Gusset plates are commonly used in a variety of bracing systems and truss bridges as a means of transferring the axial forces of the brace elements to the supporting structural members. The compressive behaviour of gusset plates is quite complex, and has been the subject of numerous research projects in the past few decades (Safari Gorji and Cheng 2019). The catastrophic collapse of I-35W bridge (Minnesota, USA) in 2007, which was due primarily to the failure of gusset plates, once again has highlighted the critical importance of design of gusset plate in compression. The ultimate capacity and failure mechanism of gusset plates depends largely on the type of connections, boundary conditions and plate geometry. As such, it is difficult to evaluate the internal stress distribution in the connection area and determine the compressive strength of the gusset. For this reason, engineers have conventionally adopted highly simplified approaches to tackle the complex task of designing gusset plates in compression. These simplified approaches, which often result in inconsistent reliability for various gusset configurations and boundary conditions, can become overly conservative or potentially unsafe on the other extreme. Current design procedures to predict the buckling capacity of gusset plates are generally based on the effective length factor method, also called column analogy method, originally proposed by Thornton (1984).

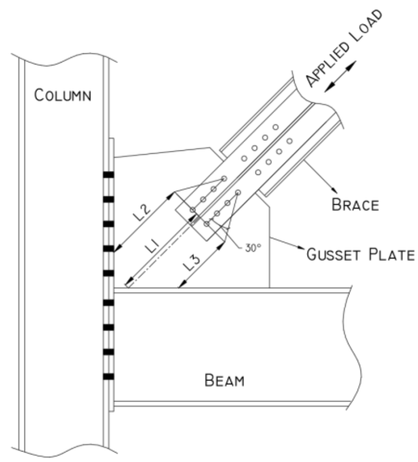

Figure 1: A typical corner gusset plate connection and the Whitmore stress dispersion angle

According to the Thornton method, the compressive capacity of the gusset plate is assumed to be equal to the compressive strength of an imaginary rectangular column below the effective width drawn by two lines with 30° stress dispersion angle, originally suggested by Whitmore (1952), and an average buckling length, as shown in Figure 1. The compressive strength of this assumed column can be calculated using the column curves available in the codes. For this calculation, Thornton (1984) suggested an effective length factor of K=0.65, which corresponds to the fixed-fixed boundary condition. While this method provides a relatively good estimation of buckling capacity of gusset plates, several previous experimental and analytical studies have shown that this method can be conservative and inaccurate in many cases. Several researchers have attempted to improve the prediction performance of the Thornton method by modifying some of the basic assumptions of the model such as the values of effective length factor and stress dispersion angle (Yam and Cheng 2002; Dowswell 2006). However, accurate estimation of the ultimate capacity of gusset plates is still a challenge for efficient design of these elements.

Empirical models are an alternative approach to solve complex engineering problems, especially when the mechanics based models become erroneous due to highly simplified assumptions or considerable uncertainty. Recent advancements in computational intelligence (CI) and soft computing techniques offer the potential to improve the prediction performance of empirical models for highly complex and nonlinear engineering problems. In the last decade, powerful evolutionary computing approaches such as genetic programming (GP) tools have gained popularity among researchers because of their efficiency and superiority over more traditional statistical approaches (Cevik 2007, Gandomi et al. 2011, Safari Gorji and Cheng 2019). This paper presents the development of empirical models for predicting the compressive capacity of gusset plate connections using a computational intelligence technique called gene expression programming (GEP), which is a powerful variant of GP. For this purpose, a comprehensive database covering several key variables influencing the ultimate capacity and behaviour of gusset plates in compression, were collected from the literature. The database used for developing the models consisted of experimental results and test-validated finite element (FE) models, and covered mechanical and geometrical properties of gusset plate connections. Three predictive models are developed using the GEP algorithm and their performance are evaluated and compared. The best performing model is selected for further examination and validation using several statistical performance criteria. Furthermore, the prediction performance of the best GEP based model is compared with the results of column analogy method and the results are discussed.

2 MODELLING USING GENETIC PROGRAMMING

Genetic programming (GP) is one of the most powerful computational intelligence techniques, which was originally developed by Koza (1992). The GP algorithm, as defined by Koza, encodes and evolves computer programs to solve problems, in a way similar to the Darwinian process of biological evolution. GP tools, which are widely used in optimization problems, enables one to discover relationship between variable in a dataset, especially when the solutions using traditional methods become erroneous due to complexity and presence of uncertainty in variables. In the last two decades, the GP algorithm has been used by engineers and researchers to develop predictive models that are difficult to solve using engineering principles.

In this research, a powerful variant of GP, called gene expression programming (GEP) developed by Ferreira (2006), is employed to derive empirical predictive models for ultimate capacity of gusset plates in compression. The GEP algorithm, which can be implemented in various programming languages such as MATLAB and Python, creates computer models or programs. The computer programs generated by the GEP algorithm are complex tree structures with the ability to learn and adapt. These computer programs are encoded in simple chromosomes of fixed length, where each chromosome consists of genes that are represented by an expression tree to solve or approximately solve a given problem. Figure 2 shows an example of a simple expression tree, which can be represented in mathematical format as follows:

[1] $A \times B + C / (D - E)$

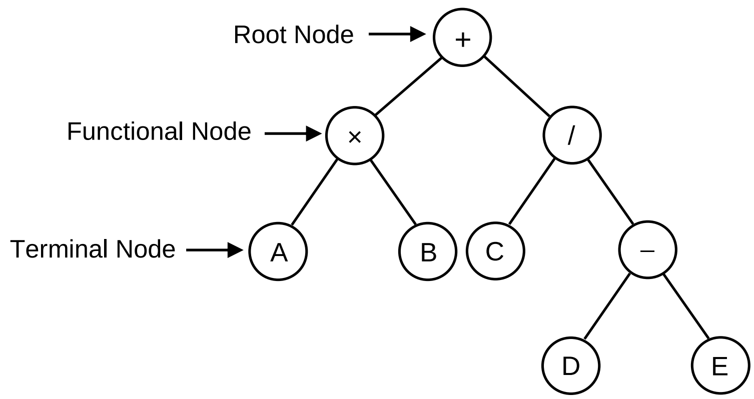

Figure 2: A simple example of an expression tree

This section describes the development of compressive strength models for gusset plates using the GEP algorithm.

2.1 Data Collection

In order to develop a reliable empirical model, a comprehensive database was collected from past research on compressive behaviour of corner gusset plate connections. The research database, which includes several key variables influencing the compressive behaviour of gusset plates, consists of test results of 41 gusset plate specimens and 164 test-validated finite element models. These variables include wide ranges of mechanical and geometrical properties of gusset plates typically used in the industry. Many of the experimental specimens used in this research were tested by the senior author and his associates through a comprehensive research program at the University of Alberta (Cheng and Hu 1987; Yam and Cheng 1993; Rabinovitch and Cheng 1993; Cheng et al. 1994; Nast et al. 1999), and the rest of experimental data were from experimental studies by other researchers (Brown 1988, Gross and Cheok 1988, Chen and Chang 2012, Naghipour et al. 2013). The finite element data were compiled from several previously published numerical research on gusset plates in compression (Chakrabarti 1987, Walbridge et al. 1998, Sheng et al. 2002, Naghipour et al. 2013, and Fang et al. 2015), many of which were collected by Dowswell 2006. It is important to note that the primary focus herein is to investigate the ultimate strength of gusset plates in which the plate buckling is the governing failure mode. Details of the experimental setup and finite element models employed to generate database used in this research are out of the scope of this paper. A more detailed survey of the past research on gusset plates in compression along with the details of the database used to develop the empirical models can be found elsewhere (Safari Gorji and Cheng 2019) and are not presented here for brevity.

2.2 Empirical Model Development

In order to develop an accurate predictive model geometrical variables of the connections and mechanical properties of gusset plates were considered. A careful review of the past experimental and analytical studies as well as a sensitivity analysis of the database revealed that the most important parameters influencing the compressive capacity of gusset plates are as follows: plate yield strength (F_y), gusset plate plate thickness (t), plate buckling length (L), plate cantilever length (C), connection length (L_c), and fastener distance perpendicular to the brace axis (S). As such, in the modelling process, the buckling capacity of gusset plates was considered to be a function of these six variables as follows:

[2] $P_u = f(F_y, t, L, C, L_c, S)$

where the values of L, C, L_c and S are shown in Figure 3 for bolted and welded gusset plate connections. It should be recognized that the compressive capacity of gusset plates is also affected by other factors such as initial imperfection and brace inclination angle. It is worth nothing that the buckling capacity of gusset plates is also affected by some other factors such as brace inclination angle and initial imperfections. Yam and Cheng (2002) reported that the brace inclination angle did not have a considerable effect on the compressive capacity of their gusset plate specimens. Also, the initial imperfection of many of the gusset plates considered were not reported, as such, this parameter was not considered in the model explicitly.

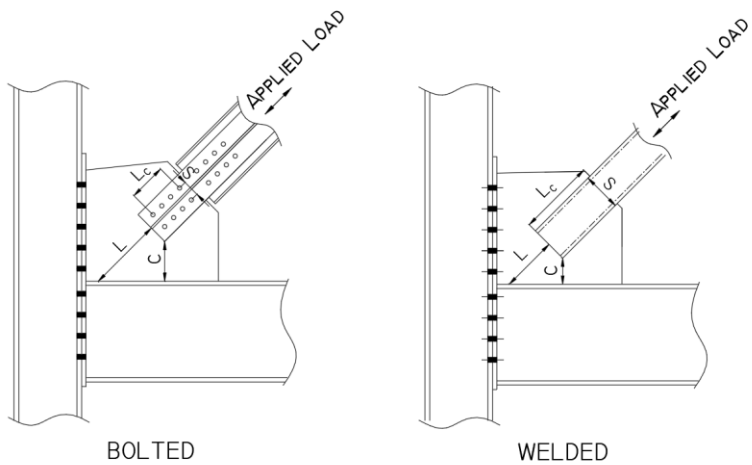

Figure 3: Geometrical variables used in the GEP algorithm

To develop the models using the GEP algorithm, the database was randomly divided into three categories, namely: (1) learning, (2) validation and (3) testing subsets, with the number of data points being 143, 31 and 31, respectively. The learning and validation data, referred to as training data, were used to develop the model, and the testing data, referred to as unseen data, was used to evaluate the performance of the model with a set of data that was not used in the modelling process. The accuracy of an empirical model developed using the GEP algorithm depends largely on the parameters and analysis criteria used when running the program. As such, numerous runs were conducted to achieve a model with minimal errors. Moreover, to minimize the influence of random selection three different combinations of learning, validation and testing data sets were generated. The main factors that affect the size, complexity and precision of the GEP models are the number of genes and chromosomes, head size of the genes as well as the linking function between sub-expression trees (sub-ETs). The above mentioned parameters were selected based on a sensitivity analysis and recommendations from past research (Gandomi et al. 2011, Cevik 2007). Generally, higher number of genes results in more complex structure for the GEP models. To select the best performing model Gandomi et al. (2011) suggested an objective function, which evaluates the performance of the models considering both the error values and coefficient of determinations for learning and validating datasets simultaneously. The best model should minimize the value of this objective function presented below:

[3] $\mathrm{OBJ} = \dfrac{n_L - n_V}{n_T}\left(\dfrac{\mathrm{MAE}_L}{R_L^2}\right) + \dfrac{2n_V}{n_T}\left(\dfrac{\mathrm{MAE}_V}{R_V^2}\right)$

where n_L, n_T and n_V are the number of learning, training and validation data sets, respectively. MAE_L and MAE_V are respectively the mean absolute error for learning and validation data sets, and R is the correlation coefficient.

3 RESULTS AND DISCUSSION

3.1 GEP-Based Modes

Table 1 shows the best performing models from the three different datasets generated. Table 2 summarizes the corresponding parameters used in the algorithms for each of these models. While initially six input variables were considered for all runs, the first model (GEP 1) used only five variables, for which the algorithm placed higher importance when developing this model. Attempts were made in the modeling process to avoid highly complex models for simplicity.

Table 1: Example table caption

Model	Number of variables used
GEP 1	$P_u = \frac{1}{6}\left(\sqrt[3]{F_y} + \sqrt[3]{2L_c} - 7.22 - 12.82t^{-1}\right)(15 - 0.27C + 7.9t + 0.31S)\sqrt[3]{tF_y}$
GEP 2	$P_u = \frac{(2t-0.4)\sqrt{F_yL_c\left(S^{\frac{1}{3}}+L^{\frac{1}{2}}+F_y^{\frac{1}{2}}-14.43\right)}}{\left(8.98+\frac{C}{t}+\frac{l_c}{L}\right)}$
GEP 3	$P_u = \left(\frac{1.5tF_y}{C+114.4}+0.25\left(F_y+L_c\right)\right)\left(t-\frac{2605.5L^{-1}+2C}{L_ct^{-1}-1.7+t+S}\right)$

Table 2: Example table caption

Model	Number of Chromosomes	Number of Genes	Head Size	Linking Function	Number of Variables Used
GEP 1	100	2	9	Multiplication	5
GEP 2	30	2	8	Multiplication	6
GEP 3	50	2	8	Multiplication	6

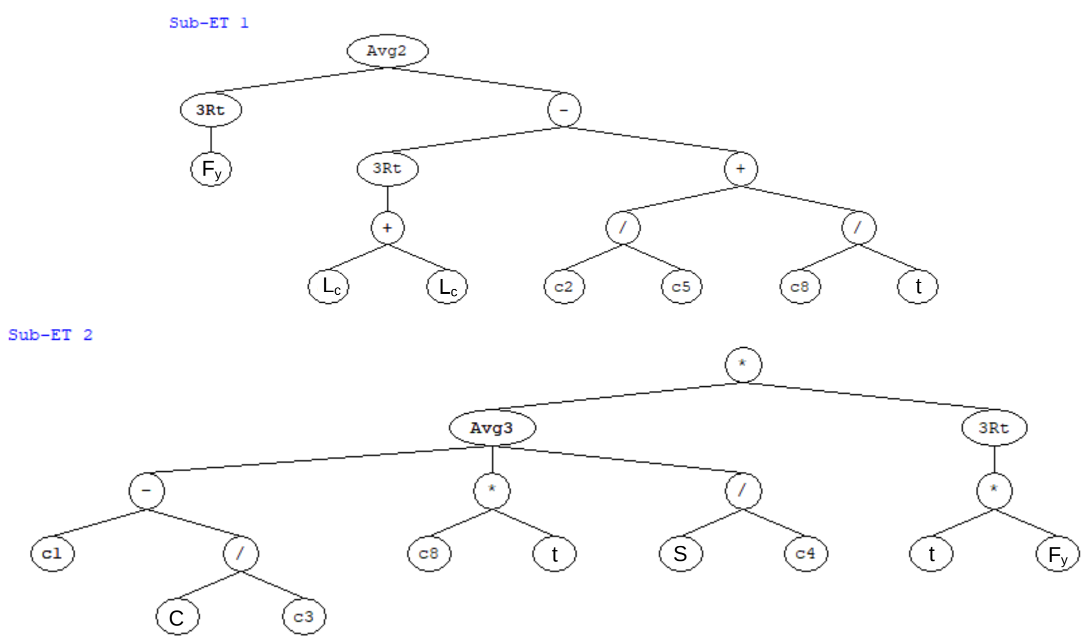

Figure 4: Expression tree of the GEP 1 model

The outcome of the GEP algorithm can be expressed in several different formats. The schematic version of the model, referred to as expression tree (ET), consists of sub-ETs that are linked together with a linking function defined prior to running the program. Figure 4 shows the ET for the first GEP model presented in Table 1. The two sub-ETs shown in the figure are linked together by the multiplication function. For the first sub-ET, the values of constants C2, C5 and C8 are respectively 6.78, 0.94 and 12.82, and for the second sub-ET C1, C3, C4 and C8 are 15.04, 3.76, 3.25 and 7.89, respectively.

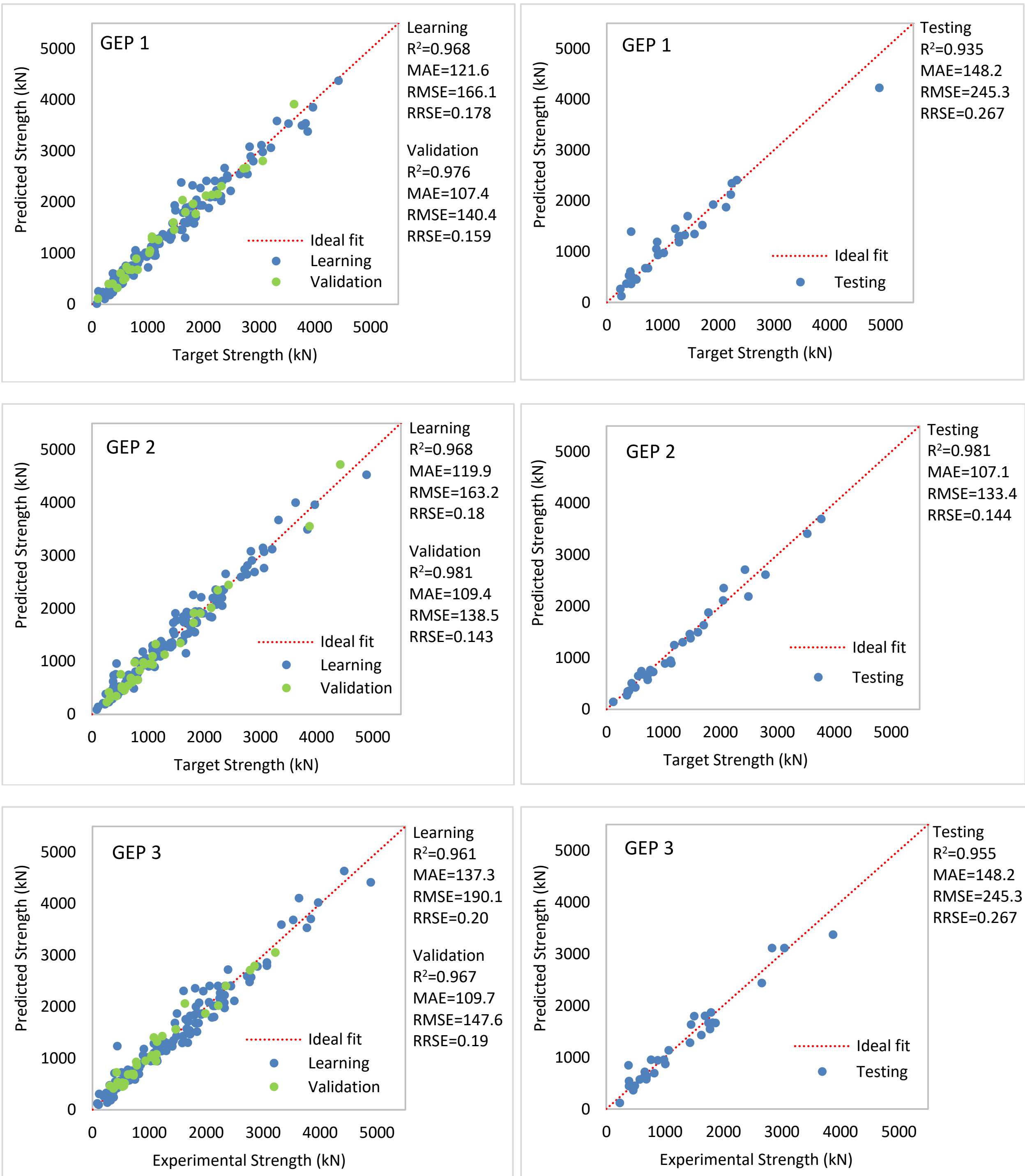

Figure 5: Predicted values versus target values of compressive strength of gusset plates

Figure 5 shows comparisons of the predicted values of compressive capacity of gusset plates with those of the target values. As shown, for all three models, the error values are generally low and the coefficients of determination are greater than 0.95 (R^2 > 0.95) for all learning, validation and testing data sets, except for the testing data set of the GEP 1 model, where this value is 0.935. This indicates that all of the selected models are capable of estimating the shear capacity of the walls with high accuracy. However, a comparative study of the predictive models revealed that the GEP 2 model performs the best among all models, especially for the testing data (i.e., the set of data that was not used in training the model). As such, model GEP 2 is selected herein for further investigation and validation in the next sections.

3.2 Model Validation and Performance Evaluation

To further evaluate the validity of the predictive model developed using the GEP algorithm, several statistical model acceptance criteria suggested by past researchers were investigated. For acceptability of the empirical model developed based on a database, Frank and Todeschini (1994) and Gandomi et al. (2011) recommended that the number of input variables should be at least three times greater than the total data points, with five times being a safer value. In this research, this number is about 34 (205/6), which satisfies this criterion. For model verification purpose, Golbraikh and Tropsha (2002) suggested that the slope of the regression line (k or k') between the target data (h_i) and predicted data (t_i) should be close to 1.0. Also, Gandomi et al. (2011) recommended that the absolute values of two performance indices m and n, as defined in Table 3, should be less than 0.1. To assesses the predictability of the model, another statistical validation criterion was introduced by Roy and Roy (2008), in which the factor R_m, defined in Table 3, should be less than 0.5. The calculated values of the above mentioned model validation criteria for the selected model are summarized in Table 3. As shown, the derived empirical model satisfied all of these criteria, confirming that this GEP based model is statistically reliable based on these performance measures.

Table 3: Statistical performance measures used to assess the validity of the developed model

Model	Formula	Condition	Values
1	$k = \frac{\sum_{i=1}^{n}(h_i \times t_i)}{h_i^2}$	0.85 < k <1.15	0.989
2	$k' = \frac{\sum_{i=1}^{n}(h_i \times t_i)}{t_i^2}$	0.85 < k <1.15	1.002
3	$R_m = R^2 \times \left(1 - \sqrt{\lvert R^2 - R_0^2 \rvert}\right)$	$R_m > 0.5$	0.945
4	$m = \frac{R^2 - R_0^2}{R^2}$	$\lvert m \rvert < 0.1$	-0.0283
5	$n = \frac{R^2 - R'^2_0}{R^2}$	$\lvert n \rvert < 0.1$	-0.0287
	where		
	$R_0^2 = 1 - \frac{\sum_{i=1}^{n}\left(t_i - h_i^0\right)^2}{\sum_{i=1}^{n}(t_i - \bar{t}_i)^2}$ $and\ \ h_i^0 = k \times t_i$		
	$R'^2_0 = 1 - \frac{\sum_{i=1}^{n}\left(h_i - t_i^0\right)^2}{\sum_{i=1}^{n}\left(h_i - \bar{h}_i\right)^2}$ $and\ \ t_i^0 = k' \times h_i$		

Figure 6 shows a comparison of the prediction performance of the selected GEP based expression with that of the column analogy method for the testing dataset as well as the whole set of data. As shown, the proposed empirical model for the compressive capacity of gusset plates performs significantly better than the column analogy method. For the GEP based model, the average test-to-predicted ratio, standard deviation (STD), coefficient of determination (R^2), and mean absolute percentage error (MAPE) are respectively 1.02, 0.15, 0.97, and 11.9%, where these values for the Thornton method are 1.35, 0.57, 0.82 and 30.5%, respectively. It should be noted that the error values of the GEP model is considerably lower than the existing method, and its correlation coefficient, which is quite close to unity, is significantly higher than the later model. The superior prediction performance of the derived expression relative to the existing method further confirms the validity of the model.

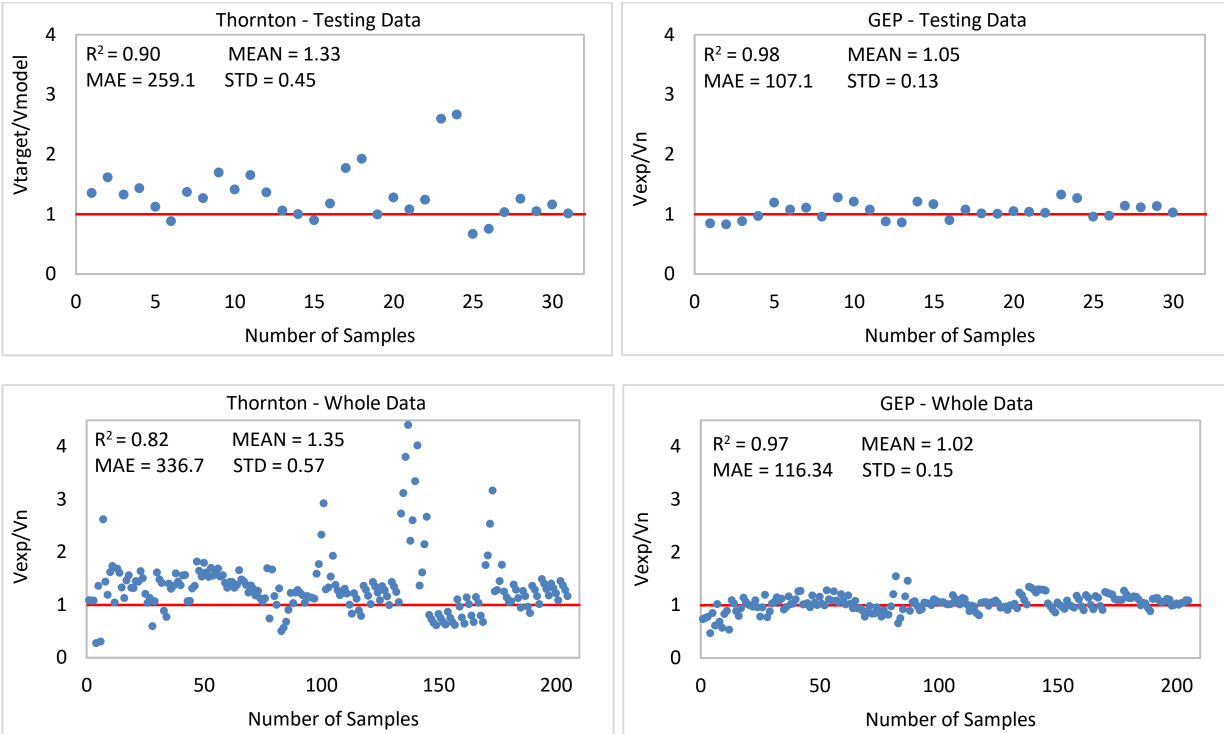

Figure 6: Comparisons of target-to-predicted ratios between the GEP model and the column analogy model

4 SUMMARY AND CONCLUSION

In this study, a computational intelligence technique, namely gene expression programming (GEP), was used to derive empirical expressions for predicting the compressive capacity of corner gusset plate connections. The predictive models, which were derived based on a comprehensive database from past research, used several key parameters influencing the gussets' compressive behaviour as input variables. To achieve an accurate model and minimize the influence of random selection, three combinations of training, validation and testing data sets were considered for the analysis. Three of the best performing predictive expressions were compared and their robustness and prediction accuracy were evaluated in terms of coefficient of determination and error values. While all three empirical models accurately estimated the compressive capacity of the gusset plates considered, the most accurate model, referred to as GEP 2, was considered for further investigation and validation.

In addition to verifying the validity of the selected model using an "unseen data", the reliability of the model was examined based on several statistical performance criteria. A comparative study was conducted to compare the predictions of the derived empirical expression with those of the column analogy method originally proposed by Thornton. The results indicated that the estimations of the compressive strength using the proposed expression agree well with the target data, and the prediction performance of the model is significantly better than that of the Thornton method. In comparison with the column analogy method commonly used in the industry, the proposed empirical model resulted in considerably lower error values and higher correlation coefficient, with the values of MAPE and R^2 being 11.9% and 0.97, respectively. For the dataset considered, these values for the later method were 30.5% and 0.82, respectively.

It is important to note that, since the expression derived using the GEP algorithm is based purely on data, as in other empirical predictive models, it should be used only in the variable ranges used in deriving the model, which covers a wide range of geometrical variables and mechanical properties of gusset plates. As such, the model can be further refined to include additional experimental database in the future. The proposed model is expected to be very useful as a reliable tool for assessing the buckling capacity of gusset plates, especially in pre-design phase and for pre-planning purposes.

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