



ANALYTICAL MODELLING OF HEAVY TIMBER ASSEMBLIES WITH REALISTIC BOUNDARY CONDITIONS SUBJECTED TO BLAST LOADING

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Abstract: Several existing studies, investigating the performance of heavy timber assemblies with realistic boundary conditions, have concluded that using simplified modelling tools such as single-degree-of-freedom (SDOF) modelling may not be sufficient to adequately describe their behaviour and predict the level of damage observed during blast events. A two-degree-of-freedom (TDOF) model, dubbed *BlasTDOF*, that captures the effects of boundary conditions in the overall system response and includes considerations for high strain-rate effects and semi-rigid boundary conditions is presented and discussed in this paper. Sensitivity analyses were conducted for various cases using single- and two-degrees-of-freedom modelling in order to make recommendations on the needs and appropriateness of using more advanced modelling. It was determined that the use of SDOF modelling is adequate when the connection resistance and stiffness exceed those of the timber element by ratios of one and ten, respectively. For the cases where these conditions could not be met, the use of TDOF modelling was determined to be required in order to accurately model the timber assembly.

1 INTRODUCTION

Threats of blast explosions on buildings and infrastructure are typically addressed through blast design provisions (e.g. Unified Facilities Criteria Program 2008, ASCE 2011, CSA 2012). These standards provide designers with guidance to perform blast analysis and design, and include items such as high strain-rate effects, response limits, pressure-impulse diagrams, etc. The majority of these provisions deal with the response of the load-bearing elements (e.g. columns, walls) under idealized boundary conditions and requires that connections be overdesigned relative to the loaded structural elements. This approach may not adequately reflect the performance of structural systems with relatively flexible end conditions. This is generally the case for structural steel and timber assemblies, where the joints are generally assumed to be pinned or (for properly detailed steel connections) fully fix. Having some deformation in the connections of timber assemblies may even be desired since the wood structural elements are likely to experience brittle failure (mainly in flexure and/or shear) when subjected to a blast load. The connections could help in absorbing some of the imparted energy on the structure, however, it is imperative that the connections themselves do not fail prior to achieving full capacity in the main structural element. This balancing act between providing energy dissipation in the connections while still maintaining the deformation capacity such that premature failure in the system is not experienced requires careful investigation of the behaviour of the connections in isolation as well as systems containing such connections.

It is common for designers to use single-degree-of-freedom (SDOF) analysis, based on idealized boundary conditions, which does not explicitly consider the behaviour of the connections. As stated in CSA (2012), more refined methods should be used if the dynamic response of the structural system cannot be represented by SDOF methods. While this statement provides some guidance for designers, it does not explicitly specify when it is appropriate or even necessary to resort to more robust modelling techniques.

The majority of studies investigating the applicability of simplified modelling methodologies (such as SDOF) in blast research have dealt primarily with idealized boundary conditions (Jacques et al. 2012, Lacroix and Doudak 2015, Viau and Doudak 2016a, Poulin et al. 2017, Lacroix and Doudak 2018). Studies investigating structural elements with realistic boundary conditions subjected to blast loads have generally concluded that limiting the modelling to SDOF will often lead to inaccurate predictions (Viau and Doudak 2016b, El-Hashimy et al. 2017, Côté and Doudak 2019). Without resorting to a more resource-intensive finite element analysis (FEA), other methods have been used effectively to model these assemblies, including energy methods (Lavarney and Pollino 2015), SDOF analysis with modified resistance curve (Whitney 1996, Gagnet et al. 2017), and two-degree-of-freedom (TDOF) modelling (Park and Krauthammer 2009, Jacques and Saatcioglu 2018).

Numerical solutions available through the use of FEA are generally not justified when considering the computational efforts involved in the development and validation of these models. This is particularly the case when dealing with non-homogenous materials (e.g. cross-laminated timber) and nonlinear connections. A good balance between simplicity and accuracy can be obtained with TDOF modelling, which consists of lumping the behaviour of each subcomponent (i.e. connections and load-bearing elements) into equivalent subsystems. This inherently allows for two failure modes, as well as the effects of realistic boundary conditions (i.e. translational and rotational flexibility), to be captured by the model. This paper summarizes the findings of an investigation on the applicability of SDOF and TDOF modelling for timber assemblies with realistic boundary connections subjected to blast loads. This investigation was conducted through sensitivity analyses of various parameters such as capacities and stiffness of both the connections and the load-bearing timber element.

2 TWO-DEGREE-OF-FREEDOM (TDOF) ANALYSIS

The following section describes the development and process of the proposed TDOF model. While the model is developed for cross-laminated timber (CLT) and glued-laminated timber (glulam) assemblies with various end connections, the methodologies can be extended beyond this application provided that proper material and connection characteristics are obtained.

2.1 Model Definition

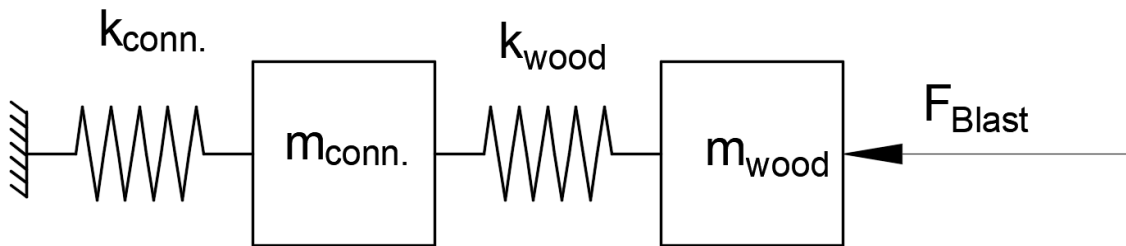
The wood assembly can be represented as a continuous frame element connected at its ends with translational and rotational springs. In order to discretize the continuous wood beam element, the deflected shape function must be determined in order to obtain the appropriate load-mass factor (k_{LM}). The factor is obtained by equating the kinetic energy and strain energy of the real structural system, based on the assumed static deflected shape, to that of the equivalent system. The end springs account for the translational and rotational stiffness of the connections by associating their behaviour to a load-displacement relationship. As the end translational connections are acting in parallel, they can be lumped together into a single equivalent translational spring. The equivalent mass of the wood member and connections are represented as m_{wood} and $m_{conn.}$, respectively, while their respective stiffnesses are represented by k_{wood} and $k_{conn.}$. In the full-scale test, the force enacted onto the system consists of a pressure collected by a load-transfer-device (LTD) and applied to the specimens via two concentrated point loads (see Figure 1). The stiffness of the assembly can be modelled as two springs in series, each represented with a SDOF, as shown in Figure 1c.



(a) Actual CLT Test Assembly



(b) Actual Glulam Test Assembly



(c) Idealized TDOF System

Figure 1: Two-Degree-of-Freedom Idealization

For the undamped TDOF system shown in Figure 1c, the following two equations of motions must be solved simultaneously:

$$[1] \quad k_{LM}m_{wood}\ddot{x}_{wood} + R_{wood} = F_{Blast}$$

$$[2] \quad m_{conn}\ddot{x}_{conn} + R_{conn} - R_{wood} = 0$$

where k_{LM} is the load-mass transformation factor, used to transform the continuous wood member into an equivalent SDOF, m and R are the component masses and resistances, respectively, F_{Blast} is the applied concentrated blast force, and \ddot{x} are the component accelerations.

The system described through Equations 1 and 2 can be solved numerically using the constant average acceleration method (Newmark 1959). The absence of the stiffness terms can be observed in Equations 1 and 2, as they have been replaced with the respective resistance terms. The nonlinear response expected though yielding in the connections as well as the post-peak response of the CLT panels make it desirable to introduce the resistance term since this makes for a more stable numerical solution. Numerical instabilities may be encountered in cases where the stiffness term approaches zero or becomes negative.

2.2 Model Inputs

The linear-elastic portion of the resistance curve can be defined by the maximum resistance (R_{max}) which occurs at the elastic limit (x_e). For a beam with two equal point loads at third spans, these parameters can be obtained from Equations 3 and 4:

$$[3] \quad R_{max} = \frac{6M_{dyn}}{L}$$

$$[4] \quad x_e = \frac{R_{max}}{K}$$

where M_{dyn} is the maximum dynamic moment, which can be obtained experimentally or from published static data, modified for high strain-rate effects (CSA 2012), and L is the clear-span of the flexural wood member.

The initial stiffness of the wood member for two concentrated point loads (K) can be modified to consider the rotational stiffness of the connections at the beam ends. This is done through the derivation of an analytical solution of an Euler-Bernoulli beam with semi-rigid springs at its ends. The solution considers a nondimensional constant (B) defined as the ratio of the rotational stiffness ($K_r l$) to that of the beam stiffness, (EI) (Equation 5). The solution for the modified stiffness is presented in Equation 6.

$$[5] \quad B = \frac{K_r l}{EI}$$

$$[6] \quad K = \frac{1296EI}{L^3} \left(\frac{B + 2}{5B + 46} \right)$$

By setting $B = 0$, the solution in Equation 6 corresponds to the case of simply-supported beam, and by setting $B = \infty$, the solution corresponds to a beam with fully-fixed ends. The input values of the rotational stiffness can also be obtained via experimental testing of the joints in question. It should be noted that in the case of timber joints, the effects of rotational stiffness are generally low and tend not to affect the response significantly.

Research done on glulam beams subjected to blast loads has shown that the dynamic behaviour can be modelled using a linear-elastic resistance curve since little-to-no post-peak behaviour was observed (Lacroix and Doudak 2018). For CLT, the cross-laminations allow for some post-peak resistance, which can be described as ratios of the maximum resistance. Research on CLT under blast loads shows that the post-peak behaviour tends to be consistent, in that failure of the outer tension laminates causes a drop-in load to an intermediate region based on the remaining transverse and longitudinal layer (Poulin et al. 2017).

The mass of the wood assembly is assumed to be that of the wood member as well as the weight of the load transfer device (Figures 1a and 1b). While the mass of the connections is negligible, a non-zero mass must be entered in the TDOF model, otherwise the dynamic analysis will not converge to a solution.

For the purpose of TDOF modelling, the translational (i.e. out-of-plane) stiffness of the end connections can be idealized as a separate axial spring with associated load-displacement relationship. Considerations of high strain-rate effects in timber connections are not well developed yet, however, ongoing research is underway at the University of Ottawa to address this issue (McGrath et al. 2019, Viau and Doudak 2019).

2.3 BlasTDOF Algorithm

In order to conduct TDOF analysis for a wide range of structural components, a numerical algorithm (*BlasTDOF*) was developed. *BlasTDOF* is capable of analyzing two-component systems subjected to blast loads, and permits the user to input custom resistance curves and masses, as well as the pressure-time histories. *BlasTDOF* is comprised of three modules; an input module, a dynamic analysis module, and an export module. A flowchart of the program's algorithm is presented in Figure 2.

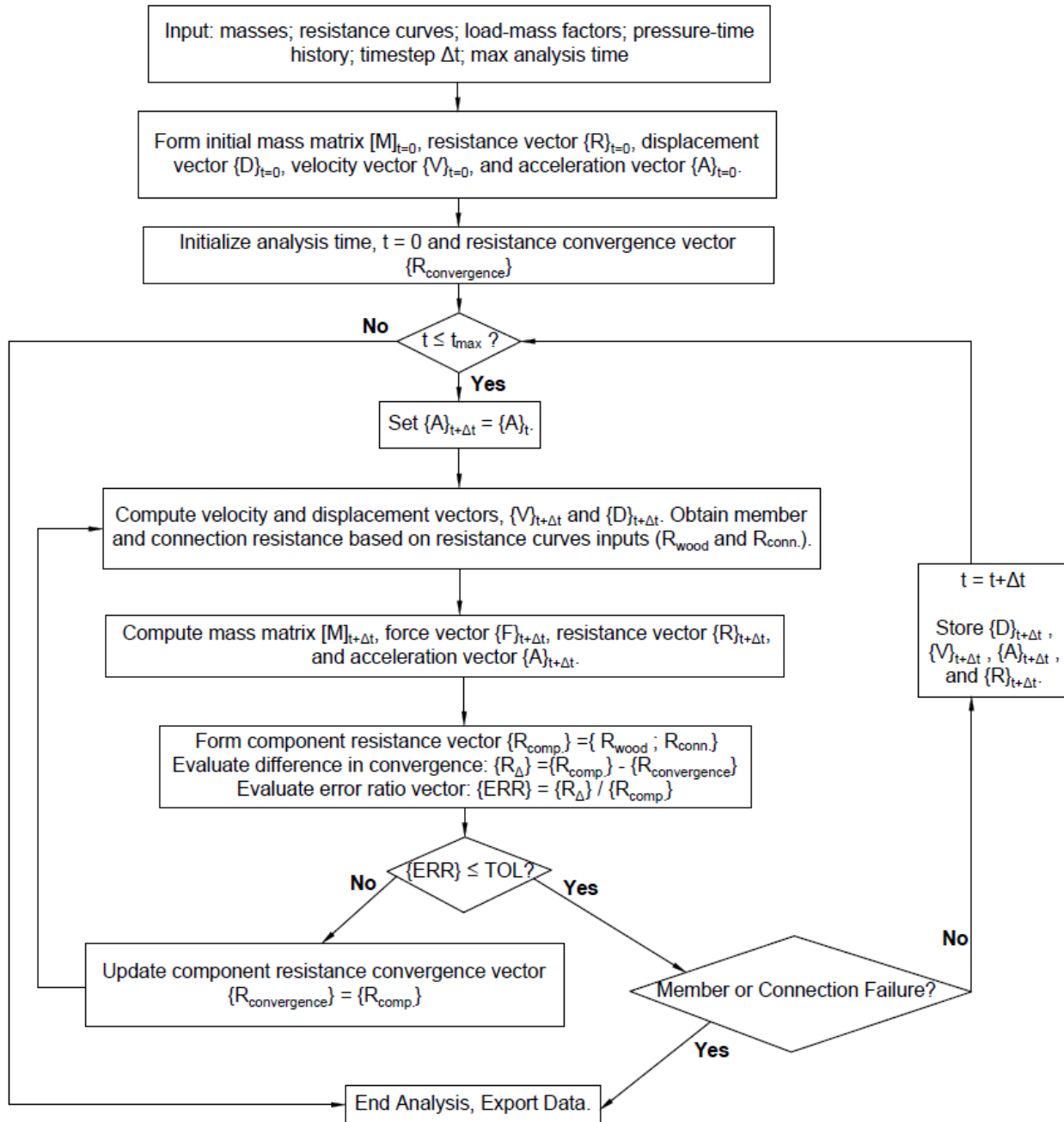


Figure 2: BlasTDOF Algorithm

3 SENSITIVITY ANALYSES

The main objective of the sensitivity analyses was to establish cases where the use of TDOF modelling would be required and where SDOF modelling could be justified and not lead to significant erroneous results. Two parameters, namely the ratio of the connection stiffness and maximum resistance to the corresponding values for the wood member, were evaluated. For all analyses, a bi-linear resistance curve, with a ductility limit of 2.0, was used to represent the behaviour of the connections. This was consistent with observed behaviour from experimental studies. The reference resistance curves of the CLT and glulam members are shown in Figures 3a and 3b, respectively. These are based on proposed models from recent studies on CLT panels (Poulin et al. 2017) and glulam members (Lacroix and Doudak 2018) subjected to blast loads. A mass of 385 kg for the CLT panel and load-transfer-device was used for the CLT groups, while a mass of 321 kg was used for the glulam groups. Forcing functions described by idealized triangular pressure-time histories were used and are presented in Figures 3c and 3d for the CLT and glulam cases, respectively. The forcing functions represent a reflected pressure and impulse (i.e. area under the pressure-time curve) combination which will allow the CLT and glulam member to reach their respective ultimate failure displacement. The sensitivity analyses are summarized in Table 1.

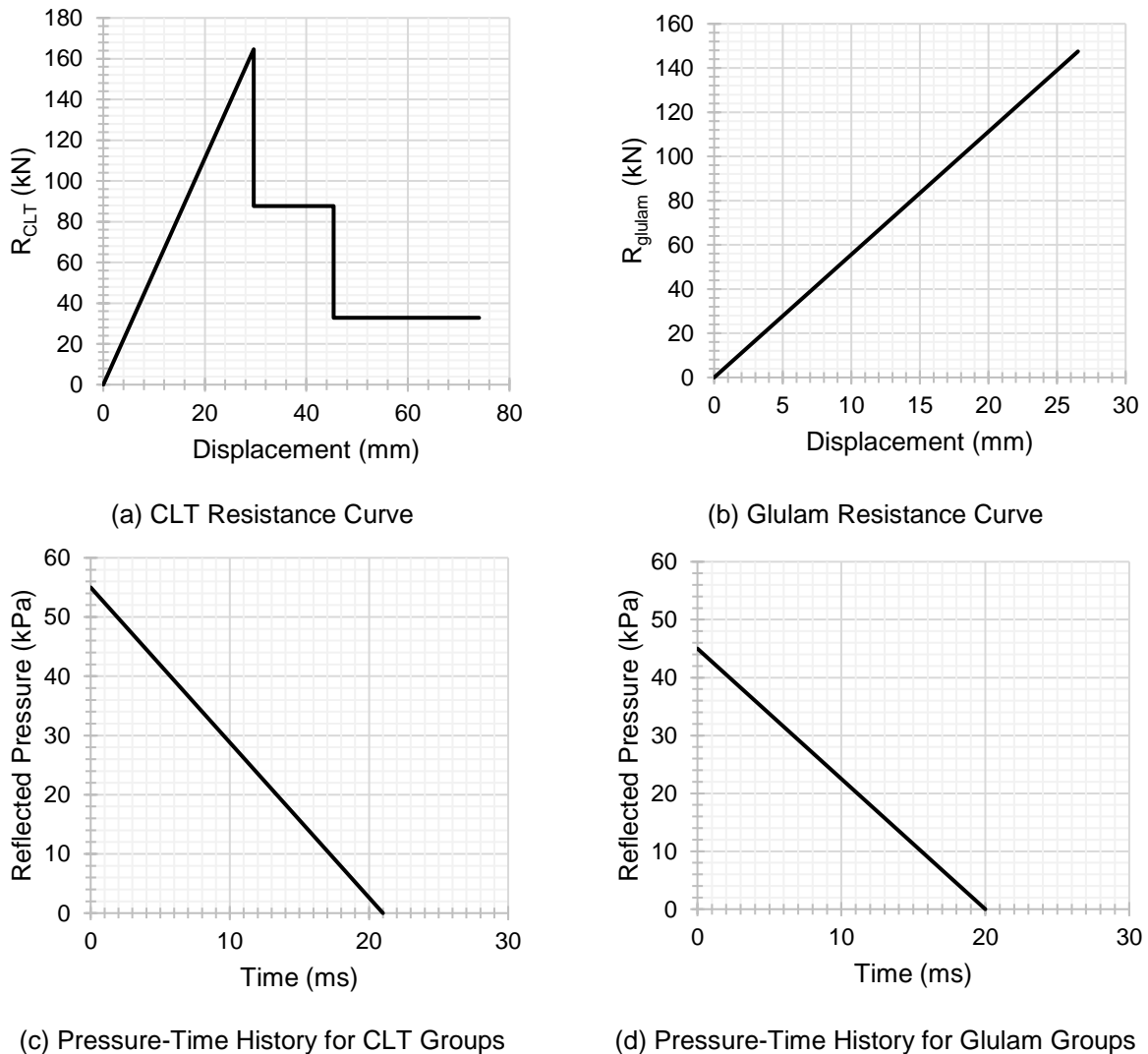


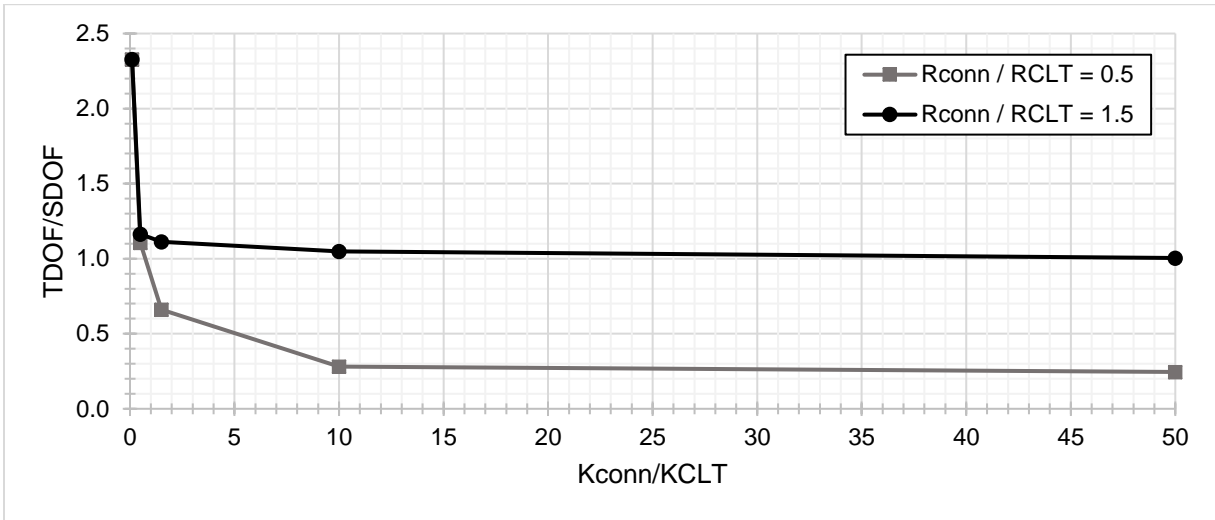
Figure 3: Reference Resistance Curves and Pressure-Time Histories for Sensitivity Analyses

Table 1: Sensitivity Analyses

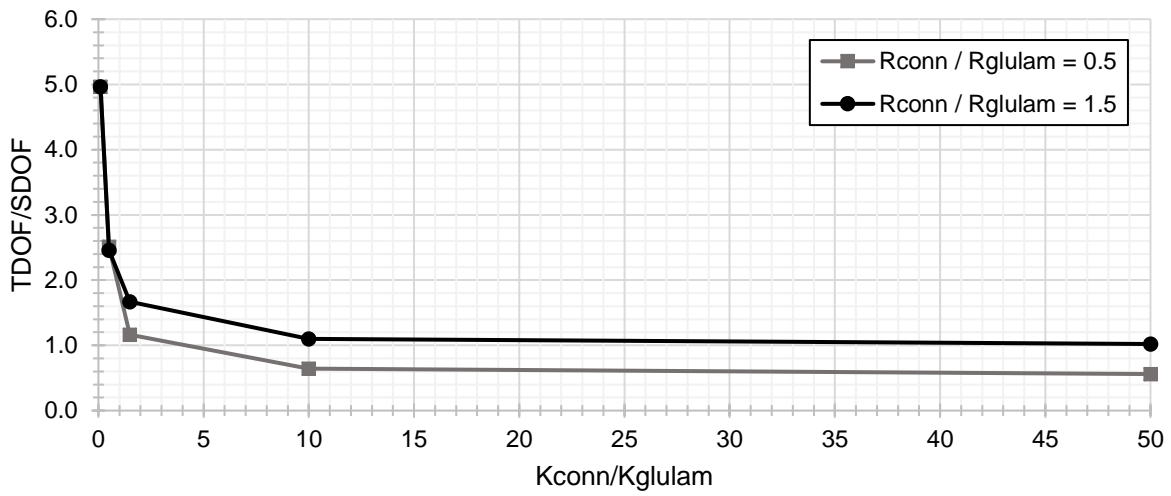
Groups	Variable	Variable Ratios	Constants
CLT-K	$K_{\text{conn}} / K_{\text{CLT}}$	0.1, 0.5, 1.5, 10, 50	Case 1: $R_{\text{conn}} / R_{\text{CLT}} = 0.5$ Case 2: $R_{\text{conn}} / R_{\text{CLT}} = 1.5$
CLT-R	$R_{\text{conn}} / R_{\text{CLT}}$	0.5, 0.8, 1, 1.05, 1.125, 1.25	Case 1: $K_{\text{conn}} / K_{\text{CLT}} = 0.5$ Case 2: $K_{\text{conn}} / K_{\text{CLT}} = 1.5$ Case 3: $K_{\text{conn}} / K_{\text{CLT}} = 10$
LAM-K	$K_{\text{conn}} / K_{\text{glulam}}$	0.1, 0.5, 1.5, 10, 50	Case 1: $R_{\text{conn}} / R_{\text{glulam}} = 0.5$ Case 2: $R_{\text{conn}} / R_{\text{glulam}} = 1.5$
LAM-R	$R_{\text{conn}} / R_{\text{glulam}}$	0.5, 0.8, 1, 1.05, 1.125, 1.25	Case 1: $K_{\text{conn}} / K_{\text{glulam}} = 0.5$ Case 2: $K_{\text{conn}} / K_{\text{glulam}} = 1.5$ Case 3: $K_{\text{conn}} / K_{\text{glulam}} = 10$

Groups CLT-K and LAM-K consisted of varying the stiffness ratios for two cases of maximum connection resistance, one being smaller and one larger than the maximum resistance of the wood member. This is meant to represent two different design philosophies, where the connection is oversized to reach the ultimate capacity of the structural member, or where the connection is intentionally under-designed in order to dissipate energy in the ductile connections rather than the brittle wood member. Figures 4a and 4b show that for the case where the connection is stronger than the wood member, SDOF analysis can accurately predict the response only if the stiffness of the connection is at least ten times that of the wood member. Using SDOF analysis to analyze a case containing connections with lower stiffness may lead to significant error in results, and in those cases, the use of TDOF may be required. For the case of oversized wood member, the results clearly show that using SDOF analysis can no longer adequately predict the correct displacement or failure mode, regardless of the stiffness ratio. Additionally, it can be observed that convergence is faster for the CLT panel, which may be attributed to its significant post-peak region.

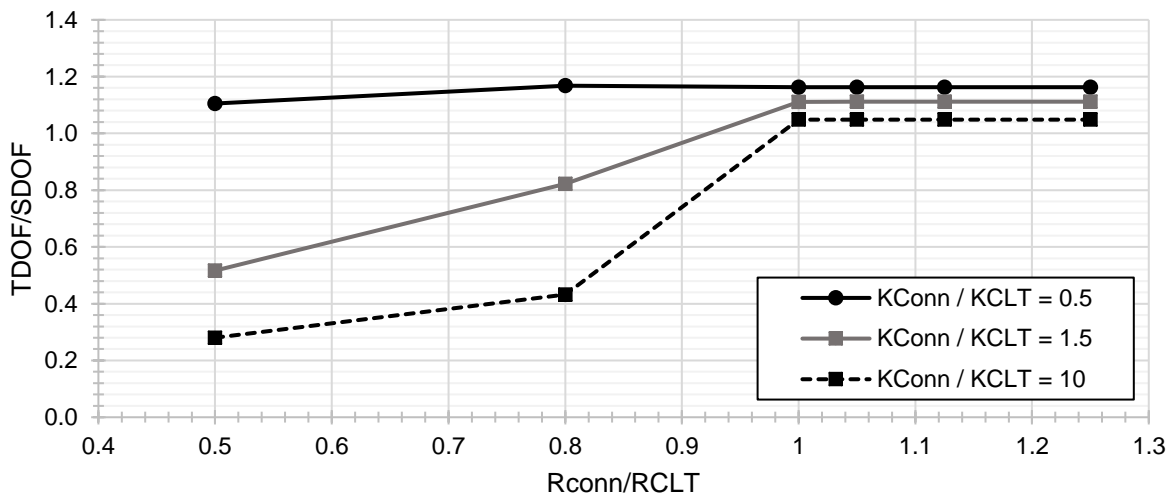
Groups CLT-R and LAM-R consisted of varying the maximum resistance ratio for three values of connection stiffness. As shown in Figures 4c and 4d, for both groups, a plateau seems to form when the ratio of connection to wood member resistance becomes greater than one. This can be explained by the fact that beyond a ratio of one, the behaviour of the wood member will tend to govern the overall displacement, and additional connection resistance will no longer play a role. It is also observed that the more flexible the connections are, the higher the TDOF/SDOF ratio is for the plateau values. As seen in the previous analysis, once the resistance of the connection becomes less than that of the wood member, the SDOF predictions will diverge from the actual displacement and failure mode. It is interesting to note that when CLT is used, the use of SDOF analysis seems adequate for all stiffness ratios as long as the connection capacity is greater than the panel capacity. A better fit is obtained when the stiffness of the connection is relatively higher than the panel stiffness, however in general all cases produce a reasonable agreement between the two analysis methods. Interestingly, the outcome looks significantly different for glulam, where only the case with very high relative connection stiffness ratio (i.e. > 10) yields adequate use of the SDOF modelling methodology. As expected, the scenario where the connection stiffness is ten times that of the wood member and with a resistance that is half that of the CLT panel yields the least accurate prediction when using SDOF modelling. This is attributed to the fact that the connection will fail at a significantly lower displacement than that of a more flexible connection, and the wood member will play a significantly lesser role in the response of the system as a whole.



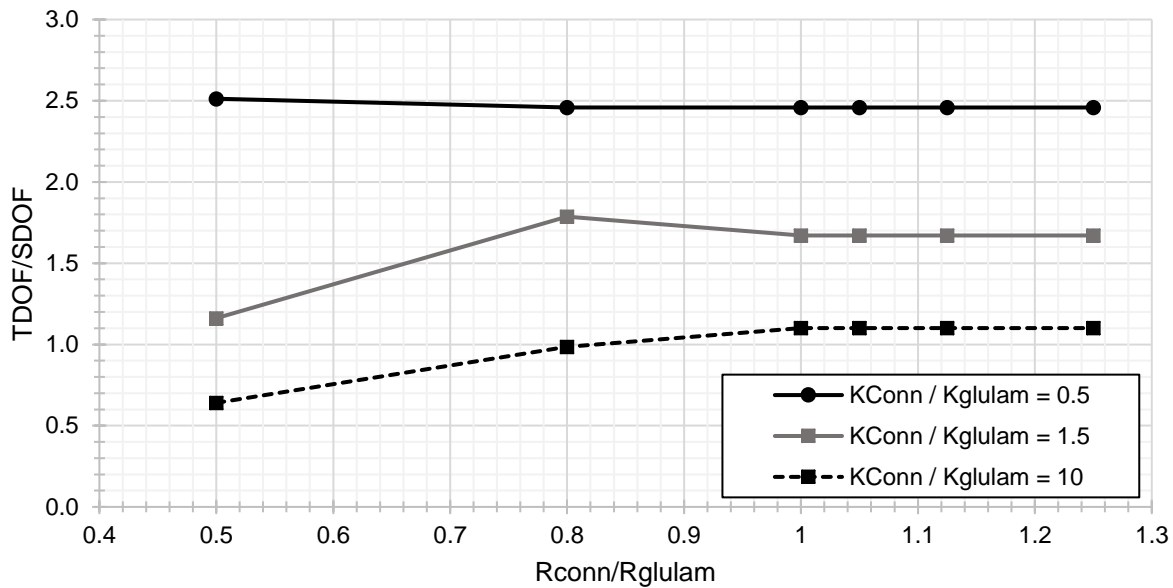
(a) CLT-K



(b) LAM-K



(c) CLT-R



(d) LAM-R

Figure 4: Results of Sensitivity Analyses for Predictions of Maximum Deflection

4 CONCLUSIONS AND RECOMMENDATIONS

This paper discusses the applicability of SDOF and TDOF modelling for timber assemblies with realistic boundary connections subjected to blast loads. A TDOF analysis program, *BlasTDOF*, was presented, and the developed numerical algorithm was described. The sensitivity of varying stiffness and capacity of connections relative to the timber structural elements was investigated. The results from the sensitivity analyses show that:

- In the case where the connection is stronger than the wood member, SDOF analysis can accurately predict the response only if the stiffness of the connection is at least 10 times that of the wood member. For the case of oversized wood members, the results clearly show that using SDOF analysis can no longer adequately predict the correct displacement or failure mode, regardless of the stiffness ratio.
- In general, a consistent ratio of SDOF to TDOF results is obtained when the ratio of connection to wood member resistance becomes greater than one. Once the resistance of the connection becomes less than that of the CLT panel, the SDOF predictions will diverge from the actual displacement and failure mode.
- When CLT is considered, the use of SDOF analysis seems adequate for all stiffness ratios as long as the connection capacity is greater than the panel capacity. For glulam, only the case of oversized connections with a very high relative stiffness ratio yields adequate use of the SDOF modelling technique.

References

- ASCE. 2011. Blast Protection of Buildings. *ASCE/SEI 59-11*. Reston, VA: American Society of Civil Engineers.
- Côté, D., and Doudak, G. 2019. Experimental investigation of cross-laminated timber panels with realistic boundary conditions subjected to simulated blast loads. *Engineering Structures*, **187**: 444-456.

- CSA. 2012. Design and assessment of buildings subjected to blast loads. CSA S850. Mississauga, ON: CSA Group.
- El-Hashimy, T., Campidelli, M., Tait, M., and El-Dakhakhni, W. 2017. Modeling of Reinforced Masonry Walls with Boundary Elements Under Blast Loading. *13th Canadian Masonry Symposium*, Halifax, NS, 4-7 June 2017.
- Gagnet, E. M., Hoemann, J. M., and Davidson, J. S. 2017. Blast resistance of membrane retrofit unreinforced masonry walls with flexible connections. *International Journal of Protective Structures*, **8**(4): 539-559.
- Jacques, E., Lloyd, A., and Saatcioglu, M. 2012. Predicting reinforced concrete response to blast loads. *Canadian Journal of Civil Engineering*, **40**(5): 427-444.
- Jacques, E., and Saatcioglu, M. 2018. Computer Software for the Design of Blast Resistant Window Retention Anchors. *CSCE 2018 Annual Conference*, Canadian Society for Civil Engineering, Fredericton, NB, June 13 - 16.
- Lacroix, D. N., and Doudak, G. 2015. Investigation of Dynamic Increase Factors in Light-Frame Wood Stud Walls Subjected to Out-of-Plane Blast Loading. *Journal of Structural Engineering*, **141**(6): 04014159.
- Lacroix, D. N., and Doudak, G. 2018. Determining the Dynamic Increase Factor for Glued-Laminated Timber Beams. *Journal of Structural Engineering*, **144**(9): 04018160.
- Lavarney, D., and Pollino, M. 2015. Mitigation of Air-Blast Pressure Impulses on Building Envelopes through Blast Resistant Ductile Connectors. *Journal of Engineering and Architecture*, **3**(2): 9-24.
- McGrath, A., Viau, C., and Doudak, G. 2019. Investigating the Response of Bolted Wood Connections to the Effects of Blast Loading. *CSCE 2019 Annual Conference*, Canadian Society for Civil Engineering, Laval, QC, June 12 - 15.
- Newmark, N. M. 1959. A Method of Computation for Structural Dynamics. *Journal of the Engineering Mechanics Division*, **85**(EM 3): 67-94.
- Park, J. Y., and Krauthammer, T. 2009. Inelastic two-degree-of-freedom model for roof frame under airblast loading. *Structural Engineering and Mechanics*, **32**(2): 321-335.
- Poulin, M., Viau, C., Lacroix, D. N., and Doudak, G. 2017. Experimental and Analytical Investigation of Cross-Laminated Timber Panels Subjected to Out-of-Plane Blast Loads. *Journal of Structural Engineering*, **144**(2): 04017197.
- Unified Facilities Criteria Program. 2008. Structures to resist the effects of accidental explosions (UFC 03-340-02). Washington, D.C.: United States of America Department of Defense.
- Viau, C., and Doudak, G. 2016a. Investigating the Behavior of Light-Frame Wood Stud Walls Subjected to Severe Blast Loading. *Journal of Structural Engineering*, **142**(12): 04016138.
- Viau, C., and Doudak, G. 2016b. Investigating the behaviour of typical and designed wall-to-floor connections in light-frame wood stud wall structures subjected to blast loading. *Canadian Journal of Civil Engineering*, **43**(6): 562-572.
- Viau, C., and Doudak, G. 2019. Effect of High Strain-Rates on Heavy Timber Connections. *CSCE 2019 Annual Conference*, Canadian Society for Civil Engineering, Laval, QC, June 12 - 15.
- Whitney, M. G. 1996. Blast Damage Mitigation Using Reinforced Concrete Panels and Energy Absorbing Connectors. San Antonio, Texas: Wilfred Baker Engineering, Inc.