Building Tomorrow's Society
Bâtir la Société de Demain

Fredericton, Canada
June 13 – June 16, 2018/ *Juin 13 – Juin 16, 2018*

VEHICLE ROUTING PROBLEMS: TRUE OPTIMAL SOLUTIONS

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Abstract: Optimizing vehicular use of the transportation infrastructure results in benefits for private transport companies, public departments of transportation infrastructure, and the general traveling public. Benefits include reduced operating costs, reduced energy use, decreased pollution, reduced demand for resources in terms of vehicles and infrastructure, improved safety in terms of saved lives and fewer injuries, improved level of service with extended economic life and sustainability for the existing infrastructure. Transportation engineers have been challenged for decades with finding a true optimal solution. Developing a methodology to find the optimal solution to the vehicle routing problem is the first step. This has been the primary goal of this research. The focus has been on an exact solution to the point-to-point vehicle routing problem. In the simplest situation this means that a single vehicle is used to deliver (or pick-up) goods to (or from) a number of customers that are located throughout a service area; the objective is to leave the depot and service each customer using the most efficient route from customer to customer and finally returning back to the depot – thus defining the traveling salesperson problem. If the capacity of one vehicle is exceeded then more vehicles are dispatched, and the general vehicle routing problem has to be solved such that the customer base is serviced most efficiently with the least number of vehicles. A novel way to merge integer linear programming and constraint programming has been developed that is efficient and effective in solving the vehicle routing problem.

1 INTRODUCTION

The benefits of the application of vehicle logistics optimization has been discussed by the current author and coauthors of supervised research projects in previous conference publications based on the related PhD Dissertation (Satir 2006) and MScE Thesis (Satir 2003). For example, Christie and Satir (2004 and 2006) identified a number of potential spin-off benefits to the transportation industry, both for those operating the vehicles and those operating the transportation infrastructure and for the general public. One paper focused on the benefits in terms of reducing energy use and reducing emissions with between 20 and 40 percent reduction in some cases with the application of computerized vehicle routing and scheduling optimization (Christie and Satir 2006). In another paper other traffic spin-off benefits were identified with those relating to pickup and delivery operations whereby up to 50 percent reduction in the related truck traffic could potentially be realized (Christie and Satir 2004). This leads to improved level of service of the infrastructure, extended economic life, reduction in truck related accidents and reduced load repetition on pavements leading to improved pavement life.

This identifies the need for continued improvement in the optimization methods available to transportation engineers and others involved in both the vehicle and transportation infrastructure systems. This is the primary motive for the current research with the goal to develop new and improved methods to solve the traveling salesperson problem (TSP) and the more general multiple vehicle routing problem (VRP). In terms of a truly optimal solution to the TSP, little further progress has been made since the formulations of the problem in 1954 (Dantzig, Fulkerson and Johnson 1954), denoted as DFJ, and the formulation presented in 1960 (Miller, Tucker and Zemilin 1960), denoted as MTZ. No exact solution methods to solve the VRP have been presented to date. This research presents a new formulation to provide an exact solution to both problems.

2 CURRENT EXACT ASIGNMENT FORMULATIONS TO SOLVE TSPs

For decades transportation researchers and practicing professionals in engineering and other transportation related fields have been grappling with finding exact solutions to the famous TSP and the more general VRP. Two assignment formulations were presented for the TSP. The conventional formulation for N cities identified by nodes 1,2,3 …, n (Dantzig, Fulkerson and Johnson, 1954; Miller, Tucker and Zemlin, 1960) is:

Xij is a binary (0, not selected, or 1, selected) decision variable that represents a link connecting city i to city j with an associated cost of Cij, and a complete tour, starting arbitrarily from say node 1, continues sequentially to each of the other cities until the completed tour ends back at node 1 with all N cities included without any sub-tours. The objective is to -

[1] Minimize: $\sum_{i,j} CijXij$ for i = 1, 2 … n; j = 1, 2 …, n

[2] Subject to: $\sum_{i} Xij = 1$ for all j

[3] $\sum_{j} Xij = 1$ for all i

Both the DFJ and MTZ methods incorporate this assignment formulation initially, but sub-tours may result with this integer linear programming formulation.

The DFJ method eliminates sub-tours using an iterative procedure whereby the linear programming assignment procedure is run followed by a non-linear step where sub-tours are identified and sub-tour elimination constraints are added, and the assignment procedure is rerun with the additional constraints. This iterative process continues until no further sub-tours can be identified. The sub-tour elimination constraint is presented in the following equation [4]:

[4] $\sum_{i \in S, j \in S} Xij \leq |S| - 1$ for all $S \subset \{2, \ldots, n\}$

With the MTZ formulation, constraint variables are included to eliminate sub-tours. An integer variable, u_i represents the relative position of each node (city) in the complete tour, with node 1 arbitrarily set as the first position. The resulting constraints are:

[5] $u_1 = 1$

[6] $2 \leq u_i \leq n$

[7] $u_i - u_j + nx_{ij} \leq n - 1$ for i, j = 2, 3, …, n

Both the DFJ and MTZ formulations have inefficiencies that grow with increasing problem size, and although the MTZ constraint formulation avoids an iterative application of integer linear programming the constraint formulation itself is weak (Pataki, 2000; Gutin and Punnen, 2007). Some improvements in the MTZ sub-tour elimination constraints have been made, for example, by M. Desroches and G. Laporte (Desroches and Laporte, 1991). The other shortcoming of the DFJ and MTZ formulations is that they are not designed to effectively solve the more general VRP. Heuristic methods cannot necessarily be proven to be exact optimal solutions to the TSP and are not generally considered in this paper. However, because the Concorde TSP Solver is fairly well known and has been used to solve some fairly large symmetric TSPs it is acknowledged here (Applegate, Bixby, Chvatal and Cook, 2006). Again Concorde is not designed to solve asymmetric TSPs nor the more general VRPs. Most recently Zamorano and Stolletz (2017) used branch-and-price methods to solve related multi-period technician routing and scheduling problems (MPTRSP). Mathlouthi, Gendreau and Potvin (2018) used mixed integer linear programming to solve a similar MPTRSP. Woods, Punnen and Stephen (2017) developed a linear time algorithm to solve the 3-neighbor TSP. A general linear programming formulation is still needed to solve both TSPs and VRPs.

3 PROPOSED METHOD TO SOLVE TSPs AND VRPs

The research presented here is designed to solve symmetric and asymmetric TSPs and VRPs using the mixed integer linear programming assignment formulation, as does the DFJ and MTZ methods. However, the assignment formulation has been revised to also accommodate VRPs. New sub-tour elimination constraints are considered that help alleviate the shortcomings of both the DFJ and MTZ sub-tour constraints by merging the standard integer linear programming methods with the approaches used in constraint programming. Although not noted in the literature, actually this is, in effect, the approach used with the MTZ method, but their sub-tour elimination constraints are weak. The current method presented here uses strong constraints. This requires the inclusion of constraint variables in addition to the usual decision variables found in standard linear programming solutions. Details are presented in the following.

3.1 Assignment-Transshipment Formulation to solve symmetric and asymmetric TSPs

As with the DFJ and MTZ methods, the integer programming assignment formulation is used with equations [1], [2] and [3]. The potential sub-tours are eliminated by using a transshipment formulation of the shortest route problem. The rationale is that, in the complete TSP tour, all N nodes or cities are connected from one to the other sequentially, in which case any individual node can be reached from say node 1. If there were sub-tours some of the nodes could not be reached from node 1, assuming some portion of the TSP tour is to be followed. Therefore a transshipment formulation of the shortest route problem is introduced for each node (> 1) to ensure a possible connection from node 1 to each of the other nodes. With this additional constraint introduced the TSP assignment formulation is solved to optimality resulting in a complete tour. Tables 1a and 1b depict this formulation of the TSP in an Excel spreadsheet format.

Table 1a: Assignment(Transportation)-Transshipment Formulation of the TSP(VRP): Part A								
COST Matrix, C								
From\To	j = 1	2	---	---	---	n		
i = 1	C_{11}	C_{12}	---	---	---	C_{1n}		
2	C_{21}	C_{22}	---	---	---	C_{2n}		
|	---	---	---	---	---	---		
|	---	---	---	---	---	---		
|	---	---	---	---	---	---		
n	---	---	---	---	---	C_{nn}		
SOLUTION Matrix, O		**Assignment(Transportation) Formulation of TSP(VRP)**						
From\To	j = 1	2	---	---	---	n	i-SUM =	Constraint
i = 1	X_{11}	X_{12}	---	---	---	X_{1n}	0	#Vehicles
2	X_{21}	X_{22}	---	---	---	X_{2n}	0	1
|	---	---	---	---	---	---	0	1
|	---	---	---	---	---	---	0	1
|	---	---	---	---	---	---	0	1
n	---	---	---	---	---	X_{nn}	0	1
j-SUM =	0	0	0	0	0	0	**TOTAL COST =**	$\sum_{i,j} CijXij$
Constraint	#Vehicles	1	1	1	1	1		
							for all i, j = 1 to n	
CONSTRAINT MATRICES, S_k		**Transshipment Formulation of Shortest Route Problem**						
	Shortest route from node 1 along TSP tour to node k = 2							
Fr1\To2	j = 1	2	---	---	---	n	i-SUM =	Constraint
i = 1	X_{112}	X_{122}	---	---	---	X_{1n2}	0	2
2	X_{212}	X_{222}	---	---	---	X_{2n2}	0	1
|	---	---	---	---	---	---	0	1
|	---	---	---	---	---	---	0	1
|	---	---	---	---	---	---	0	1
n	---	---	---	---	---	X_{nn2}	0	1
j-SUM =	0	0	0	0	0	0		
Constraint	1	2	1	1	1	1		
	Shortest route from node 1 along TSP tour to node k = 2, 3, ---, n							
Fr1\To n	j = 1	2	---	---	---	n	i-SUM =	Constraint
i = 1	X_{11k}	X_{12k}	---	---	---	X_{1nk}	0	2
2	X_{21k}	X_{22k}	---	---	---	X_{2nk}	0	1
|	---	---	---	---	---	---	0	1
|	---	---	---	---	---	---	0	1
|	---	---	---	---	---	---	0	1
n	---	---	---	---	---	X_{nnk}	0	1
j-SUM =	0	0	0	0	0	0		
Constraint	1	1	1	2	1	1		

Table 1b: Assignment(Transportation)-Transshipment Formulation of the TSP(VRP): Part B

Multiplication Matrix, M, to eliminate diagonal values in Constraint Matrices

From\To	j = 1	2	---	---	---	n
i = 1	0	1	---	---	---	1
2	1	0	---	---	---	1
|	---	---	---	---	---	1
|	---	---	---	---	---	1
|	---	---	---	---	---	1
n	1	1	---	---	---	Mnn = 0

Product Matrix, P, to prevent diagonal assignments in Solution Matrix

From\To	j = 1	2	---	---	---	n
i = 1	1	0	---	---	---	0
2	0	1	---	---	---	0
|	---	---	---	---	---	0
|	---	---	---	---	---	0
|	---	---	---	---	---	0
n	0	0	---	---	---	Pnn = 1

Constraint = SUMPRODUCT (Product Matrix, SOLUTION Matrix) = $\boxed{\sum_{i,j} PijXij = 0}$

THIS CONSTRAINT ONLY NEEDED FOR VRP

for all i, j = 1 to n

CONSTRAINT MATRICES * Multiplication Matrix

Fr1\To2	j = 1	2	---	---	---	n
i = 1	$X_{112} * M_{11}$	$X_{122} * M_{12}$	---	---	---	$X_{1n2} * M_{1n}$
2	$X_{212} * M_{21}$	$X_{222} * M_{22}$	---	---	---	$X_{2n2} * M_{2n}$
|	---	---	---	---	---	---
|	---	---	---	---	---	---
|	---	---	---	---	---	---
n	---	---	---	---	---	$X_{nn2} * M_{nn}$

Constraints for k = 2 : $X_{ij} \geq (X_{ij2} * M_{ij})$

Fr1\To n	j = 1	2	---	---	---	n
i = 1	$X_{11k} * M_{11}$	$X_{12k} * M_{12}$	---	---	---	$X_{1nk} * M_{1n}$
2	$X_{21k} * M_{21}$	$X_{22k} * M_{22}$	---	---	---	$X_{2nk} * M_{2n}$
|	---	---	---	---	---	---
|	---	---	---	---	---	---
|	---	---	---	---	---	---
n	---	---	---	---	---	$X_{nnk} * M_{nn}$

Constraints for all k ≥ 2 : $X_{ij} \geq (X_{ijk} * M_{ij})$

In Table 1a, the Cost Matrix and the Assignment formulation of the Solution Matrix represent the standard formulation of the basic TSP used with both the DFJ and MTZ methods. The cell entries in the Solution Matrix noted as #Vehicles are replaced with the numeric value of 1 (for 1 person or vehicle), thus completing the Assignment formulation. The Constraint Matrices shown in Table 1a are introduced as sub-tour elimination constraints and are effective whether the cost matrix is symmetric or asymmetric. The inspiration for this approach arose from a review of the Handbook of Constraint Programming (Rossi, van Beek and Walsh 2006), particularly chapters 11, 12 and 15. The ideas of constraint formulations, constraint satisfaction problems, constraint logic programming and using operations research methods in constraint programming led to the idea of reversing the process and finding a way to use constraint programming methods and approaches in operations research methods to solve the TSPs and VRPs. This is the first time this approach has ever been presented.

The fact that a valid solution to the TSP allows for a viable routing from any given node to each and every other node in the complete tour then provided the fundamental system of constraints to eliminate any possible sub-tours. The analyst can choose any given node as the basic starting node, so node 1, representing the depot, has been arbitrarily chosen with no loss of generality. Then an integer linear programming formulation of the shortest route problem is required and is readily available in the transshipment problem formulation (see, for example, Ragsdale 2008; Hillier and Lieberman 2005). Now binary constraint variables, X_{ijk}, are introduced with a value 0 if not selected as a link (node i to node j) or 1 if selected as a link (i to j) forming a part of the shortest route open tour from node 1 to node k (see the Constraint Matrices in Table 1a where k represents the target node destination for the relevant shortest route problem). The diagonal values, where i = j, are eliminated to provide the final transshipment shortest route solution from node 1 to node k. To accomplish this, the individual cell values, X_{ijk}, of each of the Constraint Matrices, C_k, shown in Table 1a is multiplied by the corresponding cell values, M_{ij}, in the Multiplication Matrix, M, shown in Table 1b. The final sub-tour elimination constraints are then shown in Table 1b as the CONSTRAINT MATRICES*Multiplication Matrix for each k≥2 where each $X_{ij} \geq (X_{ijk} * M_{ij})$ represent the individual constraints. Unlike the typical integer linear program these constraint values are not constant; they are a function of the constraint variables that are solved for simultaneously in conjunction with the standard decision variables which determine the final TSP tour and the final objective value shown in equation [1] and in Table 1a. The constraint variables do not directly determine the objective value; however, the constraint variables, X_{ijk}, constrain the values in the Solution Matrix in Table 1a such that all $X_{ij} \geq (X_{ijk} * M_{ij})$ for all k ≥ 2 to n.

As one final, but important, note, no integer (binary) constraint is required for the constraint variables because the decision variables are already restricted to be integer (binary) values. By default then the constraint variables will have integer values, and there is no need to actually constrain them to be integer variables. Therefore they can be solved as continuous variables with standard linear programming procedures, which reduces the required computational time considerably.

3.2 Transportation-Transshipment Formulation to solve symmetric and asymmetric VRPs

There are only a few changes in the formulation when solving the VRP. The starting node has to be the depot node, which is node 1 in the current representation. Because that is the node from which all vehicles leave, each and every other node must be accessible in a valid solution. Again the # Vehicles denoted in the SOLUTION MATRIX now has to be entered in the two cells in Table 1a as a numeric value of 2 or greater (which represents the actual number of vehicles being used to service the nodes/customers/cities, which completes the Transportation formulation. Although not required for solving TSPs, the Product Matrix, P, in Table 1b is required for solving the VRP SOLUTION Matrix for the Transportation formulation in order to prevent diagonal assigned values of 1 in the SOLUTION Matrix. The additional constraint required is:

[8] $\sum_{i,j} PijXij = 0$, for all i, j = 1 to n

With this one additional constraint the Transportation-Transshipment formulation then provides an integer programming solution to the VRP for the number of vehicles provided to service all the nodes from the depot node. Again it is important to apply the binary constraint to the SOLUTION Matrix variables (decision variables) only and not to the CONSTRAINT Matrices variables (constraint variables). As with the TSP solution, the constraint variables are treated as continuous variables which will inherently solve to have binary integer values. This saves considerable computing time which is very important with larger problems.

4 APPLICATION OF THE PROCEDURE

The Transportation-Transshipment formulation of the VRP is demonstrated with a small example. Figure 1 shows a screenshot of the Excel spreadsheet with the Cost Matrix for an asymmetric problem with only 6 nodes. Node 1 represents the depot and the nodes are serviced by 2 vehicles. Table 1 can be referred to for descriptive details.

The SOLUTION MATRIX is the Transportation formulation of the solution. To prevent sub-tours in the SOLUTION MATRIX the CONSTRAINT MATRICES are shown in a Transshipment formulation. Note that the '2' shown in each of the CONSTRAINT MATRICES (for k= 2 to n) is to allow for transshipment movements, and it does not represent the number of vehicles dispatched from the depot node. For details describing the shortest route solution using the Transshipment formulation consult an operations research textbook (for example, Hillier and Lieberman 2005). The problem was solved using an Add-in to Excel called OpenSolver version 2.9.0 (Mason 2012) and the GUROBI mixed integer linear programming solver engine version 7.5 (GUROBI Optimization 2017). Figure 2 shows the OpenSolver user input screen used to identify the objective cell to be minimized, the decision and constraint variables, the various linear constraints and the solver engine (Gurobi) to be used. Figure 3 shows the OpenSolver model runtime statistics: number of continuous and integer variables, number of simplex iterations and processing time, and the final objective value along with the integer gap, if any.

COST Matrix						
From\To	1	2	3	4	5	6
1	0	27	43	16	30	26
2	7	0	16	1	30	25
3	20	13	0	35	5	0
4	21	16	25	0	18	18
5	12	46	27	48	0	5
6	23	5	5	9	5	0

Multiplication Matrix, M						
From\To	1	2	3	4	5	6
1	0	1	1	1	1	1
2	1	0	1	1	1	1
3	1	1	0	1	1	1
4	1	1	1	0	1	1
5	1	1	1	1	0	1
6	1	1	1	1	1	0

SOLUTION Matrix	Depot = 1	TSP=1 Vehicle		VRP=X Vehicles					
From\To	1	2	3	4	5	6	SUM =	Constraint	
1	binary 0	0	0	1	0	1	2	2	=# of vehicles
2	1	0	0	0	0	0	1	1	
3	0	0	0	0	1	0	1	= 1	
4	0	1	0	0	0	0	1	1	
5	1	0	0	0	0	0	1	1	
6	0	0	1	0	0	0	1	1	
SUM =	2	1	1	1	1	1	TOTAL	min	
Constrain	= 2	1	1	1	1	1	COST =	87	
	=# of vehicles								

Product Matrix, P						
From\To	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1
				SumProduct =		0= 0
		THIS CONSTRAINT ONLY NEEDED FOR VRP				

CONSTRAINT MATRICES

Shortest route from node 1 along TSP/VRP tour to node k=2

Fr1\To2	1	2	3	4	5	6	SUM =	Constraint
1	1	0	0	1	0	0	2	2
2	0	1	0	0	0	0	1	1
3	0	0	1	0	0	0	1	= 1
4	0	1	0	0	0	0	1	1
5	0	0	0	0	1	0	1	1
6	0	0	0	0	0	1	1	1
SUM =	1	2	1	1	1	1		
Constrain	= 1	2	1	1	1	1		

CONSTRAINT Matrices * Multipication Matrix

Fr1\To2	1	2	3	4	5	6
1	0	0	0	1	0	0
2	0	0	0	0	0	0
3 ≥	0	0	0	0	0	0
4	0	1	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Shortest route from node 1 along TSP/VRP tour to node k=3

Fr1\To3	1	2	3	4	5	6	SUM =	Constraint
1	1	0	0	0	0	1	2	2
2	0	1	0	0	0	0	1	1
3	0	0	1	0	0	0	1	= 1
4	0	0	0	1	0	0	1	1
5	0	0	0	0	1	0	1	1
6	0	0	1	0	0	0	1	1
SUM =	1	1	2	1	1	1		
Constrain	= 1	1	2	1	1	1		

Fr1\To3	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	0	0
3 ≥	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	1	0	0	0

Shortest route from node 1 along TSP/VRP tour to node k=4

Fr1\To4	1	2	3	4	5	6	SUM =	Constraint
1	1	0	0	1	0	0	2	2
2	0	1	0	0	0	0	1	1
3	0	0	1	0	0	0	1	= 1
4	0	0	0	1	0	0	1	1
5	0	0	0	0	1	0	1	1
6	0	0	0	0	0	1	1	1
SUM =	1	1	1	2	1	1		
Constrain	= 1	1	1	2	1	1		

Fr1\To4	1	2	3	4	5	6
1	0	0	0	1	0	0
2	0	0	0	0	0	0
3 ≥	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Shortest route from node 1 along TSP/VRP tour to node k=5

Fr1\To5	1	2	3	4	5	6	SUM =	Constraint
1	1	0	0	0	0	1	2	2
2	0	1	0	0	0	0	1	1
3	0	0	0	0	1	0	1	= 1
4	0	0	0	1	0	0	1	1
5	0	0	0	0	1	0	1	1
6	0	0	1	0	0	0	1	1
SUM =	1	1	1	1	2	1		
Constrain	= 1	1	1	1	2	1		

Fr1\To5	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	0	0
3 ≥	0	0	0	0	1	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	1	0	0	0

Shortest route from node 1 along TSP/VRP tour to node k=6

Fr1\To6	1	2	3	4	5	6	SUM =	Constraint
1	1	0	0	0	0	1	2	2
2	0	1	0	0	0	0	1	1
3	0	0	1	0	0	0	1	= 1
4	0	0	0	1	0	0	1	1
5	0	0	0	0	1	0	1	1
6	0	0	0	0	0	1	1	1
SUM =	1	1	1	1	1	2		
Constrain	= 1	1	1	1	1	2		

Fr1\To6	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	0	0
3 ≥	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Figure 1: Screenshot of Excel Spreadsheet Demonstrating VRP Solution for 2 Service Vehicles

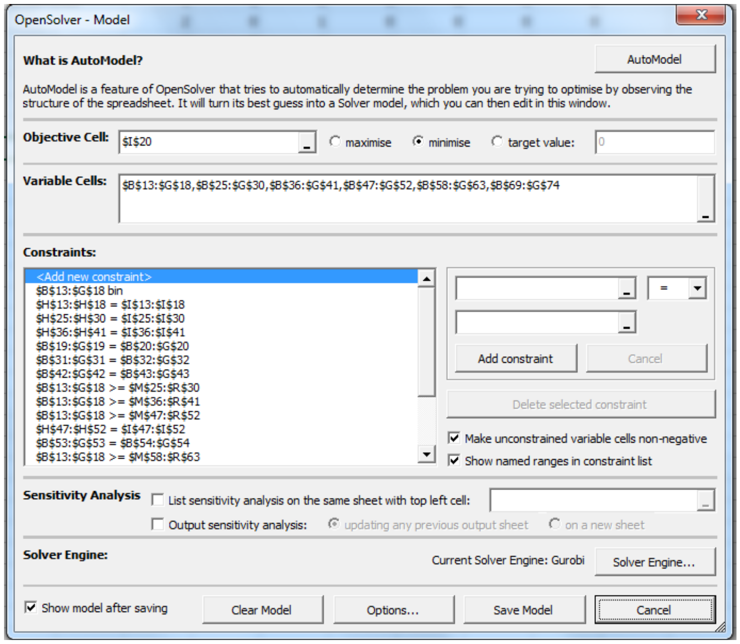

Figure 2: User input screen for the OpenSolver Linear Programming Model

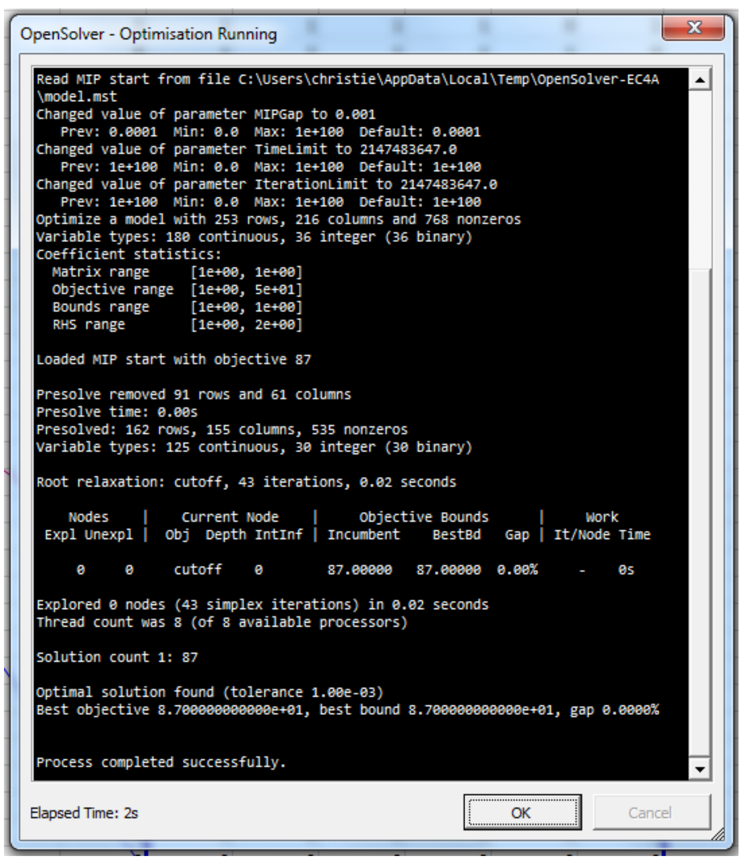

Figure 3: OpenSolver model runtime statistics

Figure 1 depicts the objective value to be 87 (which represents the total cost, time or distance, depending on the user's chosen unit measure) when serviced by 2 vehicles. The same problem was run with 1 vehicle with a resulting objective of 63 and again with 3 vehicles with a resulting objective value of 119. Although this is a very small example it does demonstrate the relative efficiencies and importance of choice of number of service vehicles. The least cost choice is to provide the service with 1 vehicle, and the cost is almost doubled by using 3 vehicles.

For the same 6 node example, one may question which of the nodes should be the depot node to optimize the service cost. To determine this, two vehicles were again used to service the nodes. However, each node in turn was considered to be the depot node. The resulting total cost for each was: node 1, 87; node 2, 70; node 3, 70; node 4, 73; node 5, 73; and node 6, 56. For optimal service cost with two vehicles, the depot should be located at node 6 with a total service cost of 56.

Preliminary results have been generated for a larger problem consisting of 50 nodes with node 1 again assumed to represent the depot node. This is an ongoing research database where various other techniques for reducing computational time are being investigated, but to this point with a single vehicle servicing the nodes a solver engine time of about 2 minutes and 30 seconds has been achieved using a 2011 version of the GUROBI software on a 2006 DELL Precision 690 workstation. With the latest version of the software it is estimated the computational time would be under 10 seconds.

5 CONCLUSIONS

A novel method to successfully solve symmetric and asymmetric TSPs has been demonstrated in this paper by using an Assignment-Transshipment formulation of the TSP. The Assignment formulation contains the standard decision variables, while the Transshipment formulation part includes the addition of applicable constraint variables. A mixed integer linear programming solver can then be used to solve the minimization problem to an optimal solution providing the optimal sequence for servicing a given number of nodes, starting from the depot node.

With very few alterations the TSP formulation can be revised to solve the more general VRP. Basically a Transportation-Transshipment formulation is developed to solve symmetric or asymmetric VRPs. The Transportation Problem formulation contains the decision variables for the VRP, and an additional constraint is included to prevent diagonal assignments. Similar to the TSP situation, the Transshipment formulation part includes the addition of applicable constraint variables. Again a mixed integer linear programming solver can be used to solve the minimization problem, so that a set of nodes can be serviced by 2 or more service vehicles starting from the one depot node at the least total cost.

This research has developed a new way to combine constraint programming methods with linear programming to successfully solve historically difficult linear programming problems that have challenged transportation engineers in civil engineering for a number of decades. Although used to solve TSPs and VRPs here, there is potential to broaden the application of this approach within the civil engineering field and beyond.

Acknowledgements

The author wishes to acknowledge the financial support provided through Student Work Program Grants and the Retirees Research Fund Grant at the University of New Brunswick.

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