Building Tomorrow's Society
Bâtir la Société de Demain

Fredericton, Canada
June 13 – June 16, 2018/ *Juin 13 – Juin 16, 2018*

ENHANCED MODEL FOR CALCULATING THE REQUIRED LENGTHS OF ACCELERATION LANES AT FREEWAY INTERCHANGES

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Abstract: Many traffic collisions occur every year due to inadequate lengths available for merging drivers to accelerate to the design speeds of the freeways they are merging with at freeway interchange ramps. Current US geometric design guide provides design tables to calculate the required lengths of those acceleration lanes that are based on research studies conducted in the 1950's and 1960's. The Canadian geometric design guide provides design domains for the required lengths of acceleration lanes that are based on those older editions of the US geometric design guide. Furthermore, the design domain provided by the Canadian geometric design guide for each speed is significantly large. For example, the recommended length of the acceleration lane that is required to accelerate from 60 km/h to 100 km/h is 140 m to 325 m. This paper provides an enhanced model for calculating the required lengths of acceleration lanes at freeway interchanges based on actual drivers' behavior and vehicles' mechanical characteristics. Acceleration profiles are established based on field data collected using GPS data loggers that recorded the positions (latitudes, longitudes and altitudes) and the instantaneous speeds of different vehicle types piloted by different drivers at 1-s intervals. Design tables are presented to help designers select the required length of an acceleration lane based on the design speed of the freeway and the entering or exit speed of the interchange ramp.

1 Introduction

An acceleration lane is provided to help drivers entering a highway, from an entrance curve, to accelerate to the design speed of the highway they are merging with (AASHTO 2011). Previous research studies show that highway interchanges are usually associated with an increased frequency and severity of traffic collisions resulting from drivers who are unable to safely merge with mainline traffic stream (Fatema et al. 2014, Lord and Bonneson 2005, McCartt et al. 2004, Twomey et al. 1992). According to data obtained from Fatality Analysis Reporting Systems (FARS 2018) and General Estimates System (GES 2018), approximately 18% of all interstate collisions, 17% of injury collisions, and 11% of fatal collisions occurred at highway interchanges that cover only less than 5% of the total highway space. These statistics indicate the increased frequency and severity of collisions at highway interchanges when compared to mainline highways. One way to improve safety at highway interchanges is by improving design standards of acceleration lanes at those interchanges. The current US geometric design guide (AASHTO 2011) provides design tables for the required lengths of acceleration lanes based on the difference between the operating speeds on the entrance curve and the mainline highway. Those design tables are based on research studies conducted in the 1950's and 1960's that investigated kinematic and dynamic capabilities of vehicles with minimal consideration given to drivers' behavior (Fitzpatrick and Zimmerman 2007). It is established that most drivers do not use the maximum kinematic and dynamic capabilities of their vehicles while accelerating unless in emergency situations (Dabbour and Easa 2016, Dabbour 2015, Long 2000, Perco et al. 2012, Wang et al. 2004). Furthermore, the design tables provided by the current design guide assume that the reached merging speed (V_a) is approximately 26% lower than the design speed of the mainline highway. For example, for a freeway design speed of 120 km/h, the reached merging speed of 88 km/h is assumed; and therefore, the needed length of the speed-change lane is also less than that if the lane was designed for a merging speed of 120 km/h. There is a virtual consensus in the literature that the current operating speeds on most highways are usually very close to (or exceed) the speed limit and the design speed (Fitzpatrick et al. 2003, Transportation Research Board 2011). Long (2000) and Hassan et al. (2012) provided in-depth analyses on the different limitations related to the current methodology of calculating the lengths of acceleration lanes as found in the current US design guide (AASHTO 2011). Based on these limitations, several modern research studies found that increasing the lengths of acceleration lanes from those suggested by the design guide would reduce the frequency and severity of collisions at highway interchanges by addressing drivers' needs and meeting their expectations (Fatema et al. 2014, Hassan et al. 2012, Brewer et al. 2011, Waard et al. 2009, Wang et al. 2009, Chen et al. 2009; Sarhan et al. 2008; Sarhan et al. 2006; McCartt et al. 2004; Harwood and Mason 1993; Twomey et al. 1992). The Canadian geometric design guide (TAC 1999) provides design domains of the required lengths of acceleration lanes that are based on the same old editions of the US geometric design guide. Furthermore, the design domain provided by the Canadian geometric design guide for each speed is significantly large. For example, the recommended length of the acceleration lane that is required to accelerate from 60 km/h to 100 km/h is within the design domain of 140 m to 325 m. Therefore, more specific design lengths may be needed for practical applications.

The purpose of this study is to introduce a new method for calculating the required lengths of acceleration lanes at highway interchanges based on realistic vehicle performance and driver behavior as well as the design speed on the mainline highway. The following section presents the methodology used to calculate the lengths of acceleration lanes in this research study, followed by other sections that provide details on the data collection and preparation, and the development and validation of the statistical models used to determine the acceleration rates used to calculate the required lengths of acceleration lanes. Design tables are also presented along with a discussion of the results and their implications. The last section provides concluding remarks and recommendations for future research.

2 Methodology

The required length of an acceleration lane depends on how vehicles accelerate on that lane. Previous research studies proposed several models that describe acceleration profiles of moving vehicles. The most common assumption used in most design guides is by assuming a constant rate of acceleration (AASHTO 2011; TRB 2010). However, this assumption was found to be unrealistic since acceleration rates usually decrease as the speed increases (Dabbour and Easa 2016, Dabbour 2015, Perco et al. 2012, Rakha et al. 2004, Wang et al. 2004). Other vehicle kinematics models were proposed based on analyzing vehicle's tractive effort and different resistance forces to calculate an average value of acceleration rate (e.g. Archilla and De Cieza 1999, Fitch 1994, Mannering and Washburn 2013). However, those kinematics-based models are microscopic in nature and cannot be applied for designing highway facilities that are used by a wide range of vehicle types (Akçelik and Biggs 1987; Bham and Benekohal 2002). A linear-decreasing model (Drew 1968) was selected by several researchers due to its simplicity and accuracy (Dabbour and Easa 2016, Dabbour 2015, Wang et al. 2004, Long 2000, Rao and Madugula 1986). The linear-decreasing model assumes that the rate of acceleration decreases as the speed increases so that the acceleration at any time may be calculated by:

[1] $$a = dv/dt = \alpha - \beta v \pm G_1 g$$

In the above equation, a is the acceleration rate corresponding to a certain speed v (m/s^2); v is the vehicle speed (m/s); α is the acceleration rate at the start of acceleration (m/s^2); β is the rate of change of acceleration with respect to speed (s^{-1}); G_1 is the grade of the highway (m/m); and g is the gravity (approximately 9.81 m/s^2). By integrating Eq. 1, the speed at any time, v, may be calculated as:

[2] $$v = \left[\left(\alpha \pm G_1 g\right)/\beta\right] - \left[\left[\left(\alpha \pm G_1 g\right)/\beta\right] - v_0\right] e^{-\beta t}$$

In the above equation, the variable t is the time elapsed from the start of acceleration (s); and the coefficient v_0 is the initial speed of the vehicle (m/s). The time, t_1, needed to accelerate from ramp speed, v_0, to the highway mainline speed, v_1, may be calculated by re-arranging Eq. 2 and substituting v with v_1 and t with t_1 as per the following:

$$[3]\ t_1 = -\ln\left[\left(\frac{(\alpha \pm G_1 g)}{\beta} - v_1\right) / \left(\frac{(\alpha \pm G_1 g)}{\beta} - v_0\right)\right] / \beta$$

The length required for the acceleration lane is the distance traveled, d_1, required to change the speed from ramp speed, v_o, to the highway mainline speed, v_1, may be obtained by integrating Eq. 2 as per the following equation (where the time t_1 is the acceleration time as calculated by Eq. 3):

$$[4]\quad d_1 = t_1\left(\frac{(\alpha \pm G_1 g)}{\beta}\right) - \left(\frac{(\alpha \pm G_1 g)}{\beta} - v_0\right)\left(\frac{1 - e^{-\beta t_1}}{\beta}\right)$$

Statistical models are developed in this research, as presented in the forthcoming sections to provide estimates of the parameters α and β, found in Eq.1 through Eq. 4, based on actual accelerating/decelerating data collected at freeway interchanges.

3 Data Collection

GPS data logging devices were used to collect actual position and speed data related to different experimental vehicles traveling along acceleration lanes at freeway interchanges. The devices recorded the latitudes, longitudes, altitudes, and the instantaneous speeds of the equipped vehicles at 1-second intervals. The records collected by the devices were extracted to a personal computer in the form of spreadsheets that show all the records collected where each record shown in a separate row of the spreadsheet represents the data collected during 1-second interval. The data were also extracted to a map so that the records would be visualized on the map as continuous tracks of the equipped vehicle. The exact locations of acceleration lanes were identified on the map and therefore the corresponding records were extracted from the spreadsheets for analysis. The date and time stamp associated with each record were used to link the map with the spreadsheet. As reported by the manufacturer of the GPS data logging devices used in this research (Holux M-1000C), the precision level in measuring the speed is 0.1 km/h (Holux 2018). A vehicle is assumed to complete its acceleration maneuver when the change in its speed is less than 1 km/h during three consecutive seconds. Given that the total number of records for one acceleration profile is n, the acceleration rate at any second (i) is estimated as the average acceleration rates of previous second (i-1) and successive second (i+1) according to the following equation:

$$[5]\ a_i = \left(v_{i+1} - v_{i-1}\right)/2 \qquad [0 < i < n]$$

Where a_i is the estimated acceleration rate at i^{th} second, v_{i+1} is the instantaneous speed at $(i+1)^{th}$ second, and v_{i-1} is the instantaneous speed at $(i-1)^{th}$ second. A dataset of observation records was then created based on Eq. 5 by calculating the acceleration rate, a_i, as a function of the instantaneous speed v_i. This dataset was then used to calibrate linear regression models that estimate the acceleration rate as a dependent variable using the speed as an explanatory variable. A linear regression model is estimated using the following relationship:

$$[6]\ a = \beta_0 + \beta_1 v + \varepsilon$$

In Eq. 6, the regression constant coefficient, β_0, is the parameter α in Eq. 1, and the regression coefficient associated with the speed, β_1, is the parameter β in Eq. 1 multiplied by -1. The error term in Eq. 6, ε, represents all unobserved factors that might influence the dependent variable. The dataset of observation records was created by collecting experimental data from 18 different participating drivers in Abu Dhabi, the capital city of the United Arab Emirates, during the period from April 2017 to September 2017. The participating drivers include 11 male drivers and 7 female drivers. The age of the participating drivers ranged from 19 years to 64 years with a mean age of 31.8 years (with a standard deviation 12.3 years), and a median age of 34 years. A GPS data logging device was installed at the private vehicle of every participating driver. Additionally, a dashboard camera was also installed to ensure that the driver's acceleration and merging behavior was not influenced by the presence of other nearby vehicles. The drivers were instructed to drive their vehicles for their normal daily routine activities for three to five days before returning the devices and the cameras to the researchers to download the data for analysis. An ethical clearance was obtained from the Institutional Review Board at Abu Dhabi University before starting the research. According to the approved research protocol, anonymity was assured to all drivers so that the downloaded driving data and video footage were not linked to specific drivers. Since the drivers were asked to drive their own vehicles, there were 18 vehicles used to collect data. They include 10 passenger cars, 5 sports-utility vehicles, 2 minivans, and 1 pickup truck. The age of the vehicles varied from less than a year (for the newest vehicle) to 12 years (for the oldest vehicle), with a mean age of all vehicles of 4.8 years (with a standard deviation of 3.4 years), and a median age of 5 years. Based on the data collected from respective vehicle manufacturers, the net horsepower (hp) of vehicle engines ranged from 106 hp @ 6000 RPM (for a 2010 Toyota Yaris) to 400 hp @ 6000 RPM (for a 2012 Nissan Patrol). The mean horsepower of all vehicles was 238.6 hp (with a standard deviation 77.9 hp), and the median horsepower was 235 hp. All the freeways, where the drivers were merging with, had low to moderate traffic volumes with near free-flow conditions at the time of the experiments so that drivers did not need to queue along the acceleration lanes to wait for proper gaps to merge with mainline traffic. There were 316 acceleration profiles used for analysis. All acceleration profiles selected for analysis were on freeways with grades less than 2%.

4 Linear Regression Model

It was previously found that if a vehicle is starting from rest, its rate of acceleration actually increases with the increase of its speed until it reaches its maximum acceleration at a speed of approximately 20 km/h, after which the rate of acceleration starts to decrease with the increase of the speed (Dabbour and Easa 2016; Dabbour 2015; Bham and Benekohal 2002; Glauz et al. 1980; Pitcher 1989). Therefore, in other research studies, where vehicles started from rest, it was necessary to develop a separate model for a vehicle starting from rest until it reaches its maximum acceleration at a speed of approximately 20 km/h, and another separate model for a vehicle accelerating for speeds higher than 20 km/h (Dabbour and Easa 2016; Dabbour 2015). However, for this research, accelerating vehicles are already in motion and therefore only one acceleration model needs to be calibrated, which is given by the following equation:

[7] $a = 2.2742 - 0.0583v$

In the above equation, the variable *v* is the speed (m/s), and the variable *a* is the acceleration rate (m/s^2). The *p*-values related to the coefficient of the independent variable (speed) is less than 0.01, indicating that the coefficient is significantly different from zero at the 95% confidence level. The value of the coefficient of determination, R^2, for the model is 0.561. The residual plot of the speed variable exhibited random dispersion around the horizontal axis, indicating a constant variance of error. It was also found that driver's age, drivers' gender, and vehicle horsepower were all insignificant explanatory variables at the 0.05 significance level with p-values of 0.317, 0.113, and 0.072, respectively.

5 Design Table

Based on the findings of this research, Table 1 was created to provide designers with the minimum required lengths of acceleration lanes based on the design speeds of the main highway and the entrance curve. For simplicity, the calculated lengths of acceleration lanes shown in Table 1 are rounded up to the nearest 5-m increments. The required lengths of acceleration lanes in Table 1 are calculated as per Eq. 4 where the time needed for acceleration is calculated from Eq. 3, and the values of the parameters *α* and *β* are as given in Eq. 7. Table 2 shows the required lengths of acceleration lanes according to the minimum values suggested by AASHTO (2011) and the domain design values suggested by TAC (1999). By comparing the values shown in Table 1 with those shown in Table 2, it could be concluded that the AASHTO (2011) generally underestimates the minimum lengths needed for acceleration lanes. However, the values shown in Table 1 are within the design domain values recommended by TAC (1999). Therefore, the minimum values recommended by the AASHTO (2011) need to be updated by including the minimum required lengths of acceleration lanes suggested in Table 1 of this research study.

It should be noted that the models presented in this research study are based on actual driver behavior and vehicle mechanical characteristics; and therefore, the required lengths of accelerations lanes calculated in this research study, as presented in Table 1, are expected to be more realistic and representative of current conditions. For an acceleration lane that is currently designed according to the AASHTO (2011), without meeting the values suggested in Table 1, merging drivers may fail to comfortably accelerate to the speed of the mainline highway within the existing lengths of acceleration lanes. It was observed that accelerating drivers continued their acceleration after merging with the mainline highway in 43 cases of the 316 cases observed in this research study (representing 13.6% of the observed cases). This observation suggests that the current lengths of acceleration lanes might not be adequate for drivers to fully accelerate to the speed of the mainline highway.

Table 1: Required lengths of acceleration lanes (m)

Highway Design Speed (km/h)	Entrance Curve Design Speed (km/h)						
	20	*30*	*40*	*50*	*60*	*70*	*80*
60	80	70	55	35	–	–	–
70	125	115	95	75	45	–	–
80	180	170	150	130	100	55	–
90	250	240	225	205	170	130	75
100	350	340	325	305	270	230	175
110	495	485	470	445	415	375	320
120	715	705	685	665	635	590	540

Table 2: Required lengths of acceleration lanes as recommended by AASHTO (2011) / TAC (1999)

Highway Design Speed (km/h)	Entrance Curve Design Speed (km/h)						
	20	*30*	*40*	*50*	*60*	*70*	*80*
60	80 / 70-100	65 / 60-80	45 / 45-60	–	–	–	–
70	130 / 115-150	110 / 100-135	90 / 80-115	65 / 50-85	–	–	–
80	180 / 150-215	165 / 130-200	145 / 115-185	115 / 85-160	65 / 40-100	–	–
90	245 / 200-310	225 / 180-300	205 / 160-285	175 / 140-250	125 / 50-200	35 / 40-145	–
100	325 / 250-440	305 / 240-420	285 / 225-405	255 / 200-375	205 / 140-325	110 / 100-285	40 / 40-230
110	410 / 320-645	390 / 305-630	370 / 290-600	340 / 260-575	290 /210-525	200 / 150-475	125 / 100-410
120	530 / 400-725	515 / 375-710	490 / 370-690	460 / 340-660	410 / 285-590	325 / 250-515	245 / 195-430

6 Conclusions

In this paper, a new method was presented to determine the required lengths of acceleration lanes based on driver's behavior and current vehicle acceleration capabilities. The method is based on establishing realistic acceleration profiles using actual field data collected by GPS data logging devices. The devices recorded the positions and the instantaneous speeds of different vehicles piloted by different drivers at 1-s intervals. Realistic values were calculated for the lengths of acceleration lanes based on different highway design speeds as well as the design speeds of the entrance ramps. The findings of this research suggest that the lengths of acceleration lanes found in AASHTO (2011) might be inadequate so that approximately 13.6% of the observed drivers needed to continue their acceleration after merging with the mainline highway. By implementing the design guidelines developed in this research study, safety at freeway interchanges is expected to improve since drivers will be able to merge with mainline traffic more smoothly.

It must be noted that the model presented in this research study is based on experiments conducted in free-flow conditions so that drivers did not need to queue waiting for proper gaps to merge with the mainline highway. Further research may be needed to investigate the effect of queueing on the required lengths of acceleration lanes as calculated in this research study. It must also be noted that the model presented in this research study is based on experiments conducted on freeways with grades that are less than 2%. The model presented in this research study can calculate the lengths of acceleration lanes with any grades based on the laws of physics, as demonstrated by Eq. 1 through Eq. 4. However, further research may be needed to investigate the effect of grades steeper than 2% on the required lengths of acceleration lanes in case if the effect of grades is different from what is suggested by the laws of physics.

Acknowledgements

This research study was financially supported by a student research grant provided by Abu Dhabi University in the United Arab Emirates.

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