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OPTIMIZATION OF SINGLE-LANE ROUNDABOUT GEOMETRIC DESIGN: ENVIRONMENTAL SUSTAINABILITY

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Abstract: Intersections are a significant source of congestion which produces the majority of vehicle emissions; therefore, environmental sustainability is the primary objective of intersection design. Research has proven that roundabouts provide an effective solution to calm traffic and improve the environmental impact of intersections. This study presents a model that directly identifies the optimal geometric parameters of a single-lane roundabout in order to minimize vehicle emissions. The model is based on an optimization technique which uses environmental sustainability as a design objective. The Vehicle Specific Power (VSP) methodology was used to model the vehicle emissions of approaching vehicles based on an operation speed model. The traffic conditions, site limitations, and guideline recommendations were used as inputs and the optimal geometric parameters that minimize vehicle emissions were the decision values determined by the model. A sensitivity analysis was conducted and revealed that the model improved environmental sustainability, providing reductions in vehicle emissions.

1 INTRODUCTION

A roundabout is a type of intersection in which the traffic circulates counter clockwise around a central island, and the entering traffic must yield to the circulating traffic (Rodegerdts et al., 2010). Roundabouts play a key role in the improvement of the environmental impact of traffic compared to other forms of at-grade intersections. Roundabouts improve mobility by reducing the total delay compared to other controlled intersections. This improvement leads to environmental benefits and savings in fuel consumption (Federal Highway Administration, 2010). Mandavilli et al. (2008) examined the effect of converting AWSC intersections into modern roundabouts on vehicle emissions. The researchers evaluated 6 intersections indicating the environmental improvement of installing roundabouts. They reported reductions of 42%, 59%, 48%, and 65% for CO, CO2, NOX, and HC emissions, respectively. Another study conducted in Sweden indicated that converting a signalized intersection into a roundabout reduced vehicle emissions by 29% and 21% for CO and NOX (respectively), and fuel consumption by 28% (Várhelyi et al., 2002). Roundabouts are an effective traffic calming approach for intersection congestion, reducing delay and average queue length (Rao et al., 2014). Congested intersections are a major source of vehicle emissions. According to the Government of Canada, passenger cars are a major contributor to air pollution; these vehicle emissions account for almost 21% of the nitrogen dioxide (NO_x), 51% of the volatile organic compound (VOC), and 4% of the fine particle matter (PM 2.5)(Canada, 2017). Road designers need to include vehicle emissions as an important objective in order to achieve improved environmental impact and air quality. This research project involves the development of an optimization model based on minimizing vehicle emissions at single-lane roundabouts. Single- lane roundabouts have the highest safety performance due to the minimizing of conflict between traffic and because of their simplicity. The single-lane roundabouts that have been constructed in the U.S until the year of 2013 represent approximately 71% of the total number of roundabouts (Pochowski et al., 2016). The geometric features of roundabouts are linked to several design measurements such as speed, safety, and mobility and significantly affect the environmental impact. It is therefore necessary to develop a geometric design optimization model for single-lane roundabouts that is able to determine the optimal geometric parameter values that satisfies specific conditions and objectives of each site.

In the planning stage, designers determine the design objectives based on the unique conditions of each site. Each site has its own issues and specific objectives that require improvement. If the location is within a sensitive environmental area, more weight will be applied on minimizing vehicle emissions to improve the air quality in that location. Sometimes the implementation of a roundabout is required in order to improve the safety of a particular intersection due to increases in annual crashes. In this case, safety performance would be the primary design objective. The purpose of this study was to develop an optimum design model. The proposed model can determine the geometric design parameters of single-lane roundabouts that minimize vehicle emissions in order to improve the environmental impact of the intersection. This model improves the geometric design process for roundabouts and improves environmental sustainability. The model can also be expanded by adding more design objectives such as safety performance, mobility, cost-effectiveness, energy consumption, and sight distances to become a complete design tool.

2 METHODOLOGY

Geometric data and traffic data are required as inputs when modeling vehicle emissions. The data are used to model the operating speed of the roundabout, second by second. The vehicle specific power is determined for each speed profile using the modeled operating speed of the roundabout and the vehicle emissions are modeled. Using Solver, the model is solved minimizing the total vehicle emissions subjected to all the defined constraints to provide optimal geometric parameters including the inscribed circle diameter, the circulated width, and the entry width. The methodology is described in more detail in the following section.

2.1 Roundabout Data

2.1.1 Geometric Data

The maximum ranges of the geometric parameters are retrieved from an aerial photograph of the proposed location and verified through the site survey which includes utility information such as poles and drainage lines. The minimum ranges of the geometric parameters are retrieved from the guidelines that are used based on the design vehicle. In this study, "Roundabouts - An Informational Guide (2nd Edition)" was used as a design guideline (Rodegerdts et al., 2010).

2.1.2 Traffic Data

The actual traffic data are used if the model is used to evaluate an existing roundabout or convert an existing intersection into a roundabout in order to improve any of the design objectives. The predicted traffic data are used if the model is used to design a new roundabout. The total approach entering traffic is the sum of all movements at this approach while the conflicting traffic or the circulating traffic is the sum of all of the traffic that passes in front of this approach, given by Equation 1.

[1] $$Q_{conf,j} = Q_{UT,(j+1)} + Q_{UT,(j+2)} + Q_{LT,(j+2)} + Q_{UT,(j+3)} + Q_{LT,(j+3)} + Q_{TH,(j+3)}$$

Where, $Q_{conf,j}$ = the circulating traffic for approach j, Q_{UT} = the U-turn movement entering approach j (pce/hr.), Q_{LT} =the left turn movement entering approach j (pce/hr.), Q_{TH} = the through movement entering approach j (pce/hr.), and j = the approach number.

2.2 Modeling Vehicle Specific Power

Numerous researchers have used models based on vehicle specific power to measure vehicle emissions (Coelho 2006; Federal Highway Administration, 2010; Salamati at al., 2013; and Song, 2015). Vehicle specific power (VSP) is the engine power needed to move a vehicle unit mass. Researchers have simplified the VSP equation to be expressed in terms of the instantaneous speed and acceleration of the vehicle as given by Equation 2.

$$[2]\ \mathrm{VSP} = \mathrm{v} * (1.1\mathrm{a} + 0.132) + 0.000302 * \mathrm{v}^3$$

Where, VSP = the vehicle specific power, v = the vehicle instantaneous speed or second by second speed (m/s), a = the vehicle acceleration or deceleration (m/s^2),

In this study, the acceleration and deceleration values were 2.1 m/s^2 and 1.3 m/s^2 , respectively, as recommended by Rodegerdts (2010). The acceleration and deceleration are considered constant for simplicity and due to the unavailability of acceleration/ deceleration models for roundabouts.

The speed is the operating speed modeled by Bassani and Sacchi (2011) and given by Equation 3.

$$[3]\ \mathrm{V}_{85} = 0.4433 * \mathrm{D}_{\mathrm{INT}} + 0.8367 * \mathrm{C} + 3.2272 * \mathrm{E}$$

Where, *v* = 85th percentile speed (km/hr), *C* = the width of the circulatory roadway (m), *E* = the width of the entry lane (m), and $\mathrm{D}_{\mathrm{INT}}$ = the central island diameter (m).

2.3 Modeling Speed Profiles

The speed profile of roundabouts based on vehicle trajectories is divided into three cases with three different speed profiles, as modeled by Coelho et al. (2006).

1. Unstopped vehicles

The percentage of vehicles that cross the roundabout without stopping is given by Equation 4.

$$[4]\ \%(Unstopped\ vehicles) = 100 - 0.0000611(Q_{conf} + Q_{en})$$

2. One-Stop vehicles

The percentage of vehicles that stop only once at the yield line of the roundabout while waiting for an acceptable gap is given by Equation 5.

$$[5]\ \%(\mathrm{one-stop\ vehicles}) = 100 - \%(Unstopped\ vehicles) - \%(more\ than\ one - stop\ vehicles)$$

3. More than one stop vehicles

The percentage of vehicles that experience more than one stop through go and stop cycles before crossing the roundabout is given by Equation 6.

$$[6]\ \%(more\ than\ one - stop\ vehicles) = \exp[0.00123\left(Q_{conf} + Q_{en} - 300\right)^{1.2} - 1$$

Where, Q_{conf} = the conflict traffic or the corresponding circulating traffic of the calculated approach,

Q_{en} = the total traffic entering the approach entering traffic

The stop and go cycles experienced by the vehicles while stopping more than once were characterized into short stop and go cycles (SSG) and long stop and go cycles (LSG) based on the queue length. The numbers are given by Equations 7 and 8.

$[7]\ SSG = 1.834 * \exp(0.0759QL) - 1$

$[8]\ \mathrm{LSG} = 1.997 * \exp(0.1124\mathrm{QL}) - [1.834 * \exp(0.0759QL) - 1]$

Where, SSG= short stop and go cycles number, LSG= long stop and go cycles number, and QL = the queue length (veh.)

The typical short cycle maximum speed is 3.8 km/h, distance is 5.2m, and idle time before each cycle is 4.5 sec. The typical long cycle maximum speed is 6.6 km/h, distance is 15.1m, and idle time before each cycle is 5.2 sec. The idle time at the yield line was modeled theoretically based on the probability of accepting a gap between the circulating traffic. The average headway of the circulating traffic is given by Equation 9.

$$[9]\ \mathrm{AHW} = \frac{3600}{\mathrm{Q_{conf}}}$$

Where, AHW = the average headway time (sec), $\mathrm{Q_{conf}}$ =the circulated traffic in pce/hr

The probability of waiting for 0 gaps is calculated using Equation 10.

$$[10]\ P = \exp(-\frac{t_c}{AHW})$$

Where, P = the probability of waiting for 0 gaps, t_c = the critical gap specified in HCM6 to be 4.99 sec

The expected number of gaps is given by Equation 11.

$$[11]\ \mathrm{E_n} = (1 - \mathrm{P})/\mathrm{P}$$

$$[12]\ \mathrm{Idle\ time} = \mathrm{E_n} * AHW$$

Where, $\mathrm{E_n}$ = the expected number of gaps

The vehicles decelerate from the approach downstream speed to the roundabout operation speed defined by Equation 2, crossing the roundabout at in the operation speed for a distance that differs for in each movement, and accelerate from the roundabout operation speed to the upstream approach speed. Each speed profile includes three basic movements: through movements, right turn movements, and left turn movements. The vehicles for each movement travel different distances through the roundabout. The three basic movements with the three speed profiles provides nine different speed profiles. In this study, three movements are modeled, through movement, right turn movement, and left turn movement in each speed profile of the three speed movement specified in the previous section. The three movements are characterized by the distance or the time traveled in the roundabout until exiting. Easa and Mehmood (2004.) drew have drawn the fastest path for of vehicles in of single-lane roundabouts with different inscribed circle diameters.They found that the through movement vehicles enter and exit the roundabout at around a 60^{o} angle, the right turn movement vehicles enter and exit the roundabout at a in 30^{o} and 60^{o} angle (respectively), and the left turn movement vehicles enter and exit the roundabout at around a 45^{o} angle, as shown in Figure 2-1. The vehicles fastest path is laying 1.5 m from the concrete curb (Rodegerdts et al., 2010).The distances $\mathrm{d_{th}}$, d_r and d_L can be derived from the geometry geometric of Figure 2-1 as follows:

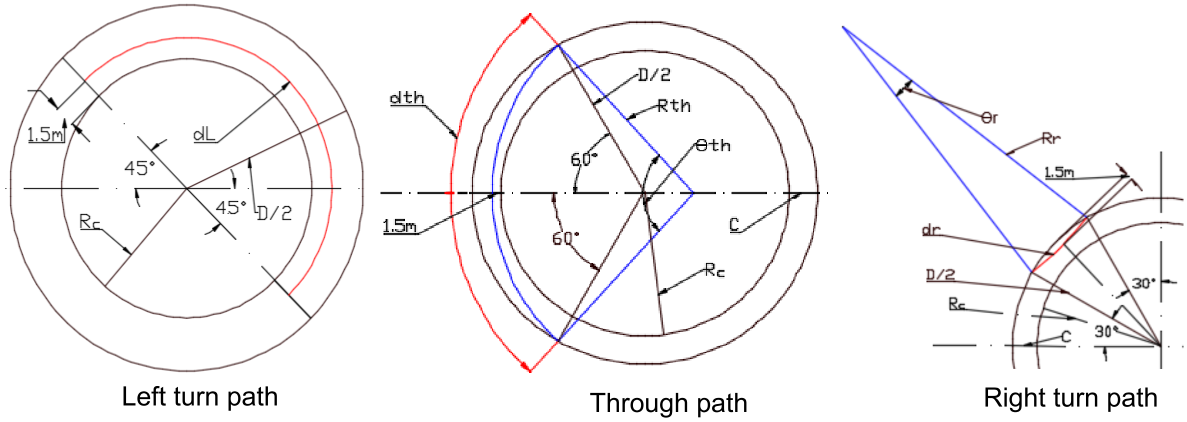

Figure 2-1: Movement paths modeling

1. Through path:

$$[13]\ \frac{D}{2}\cos 30 = R_{th}\sin(\frac{\theta_{th}}{2})$$

$$[14]\ \mathrm{R_{th}} = \mathrm{R_{th}}\cos\left(\frac{\theta_{th}}{2}\right) + 1.5 + R_c - \frac{D}{2}\sin 30$$

From Equations 13 and 14, $\mathrm{R_{th}}$ and θ_{th} can be expressed in terms of D and R_c as followings:

$$[15]\ \theta_{\mathrm{th}} = 4\tan^{-1}\left[\frac{R_c + 1.5}{0.433} - 0.57735\right]$$

$$[16]\ \mathrm{R_{th}} = \frac{0.433\mathrm{D}}{\sin\left(\frac{\theta_{th}}{2}\right)}$$

$$[17]\ \mathrm{d_{th}} = \mathrm{R_{th}} * \theta_{\mathrm{th}} * \pi/180$$

Where, D = the inscribed circle diameter (m), R_c = the central island radius (m), $\mathrm{R_{th}}$ = the through path radius (m), and θ_{th}= the through path angle (degrees).

2. Right turn path

$$[18]\ \mathrm{R_r}\sin\left(\frac{\theta_r}{2}\right) = D/2sin15$$

$$[19]\ \frac{D}{2}(1 - cos15) + R_r\left(1 - \cos\left(\frac{\theta_r}{2}\right)\right) = 1.5$$

From Equations 18 and 19 θ_r and $\mathrm{R_r}$ can be expressed in terms of D as given by Equations 20 and 21:

$$[20]\ \theta_r = 4\tan^{-1}\left(\frac{11.59111}{D} - 0.131651\right)$$

$$[21]\ R_r = \frac{0.1294D}{\sin\left(\frac{\theta_r}{2}\right)}$$

$$[22]\ d_r = R_r * \theta_r * \pi/180$$

Where, D = the inscribed circle diameter (m), R_r = the right turn path radius (m), and θ_r = the right turn path angle (degrees).

3. Left turn path

$$[23]\ R_L = R_c + 1.5$$

$$[24]\ d_L = \pi(R_c + 1.5)$$

Where, R_L = the left turn path radius (m), d_L= the distance traveled in the roundabout by the left turn vehicle (m)

The time spent in the roundabout for each movement can be determined from the distances traveled in the roundabout at with the roundabout operation speed, the time spent in the roundabout for each movement can be determined. Using a small optimizing model, the stop and go cycles in speed profile 3 are modeled based on the information given by (Coelho et al., (2006) and the Kinematic Equations considering acceleration and deceleration are the same value. The model determines the acceleration/deceleration of SSG and LSG to be 0.327 and 1.902 m/s^2 respectively.

2.4 Geometric Constraints

The geometric constraints are defined as follows:

$$[25]\ D_{min} \le D \le D_{max}$$

$$[26]\ E_{min,j} \le E_j \le E_{max,j}$$

$$[27]\ \mathrm{Max}(E_j) \le C \le 1.2 * \mathrm{Max}(E_j)$$

As recommended by "Roundabouts - An Informational Guide", the circulated width should be more than the maximum entry width of all legs and should not exceed 20% more than the maximum entry width.

$$[28]\ D = 2R_c + 2C$$

$$[29] E_{ex,min,j} \le E_{ex,j} \le E_{ex,max,j}$$

Where, D = the inscribed circle diameter (m), E = the entry width (m), C = the circulated width (m), E_{ex} = the exit width (m), R_c =the central island radius in (m), and j = the leg number (1, 2, 3 or 4)

2.5 Modeling Vehicle Emissions

The Vehicle Specific Power methodology was used to calculate vehicle emissions using Equation 1. The vehicle specific power is determined using Equation 1 (second by second) for each speed profile, and the vehicle emissions are identified based on the VSP value. Frey specified the corresponding vehicle emissions of NOx, HC, CO2 and CO (g/s) for each VSP range (Frey et al., 2002; Frey et al., 2003). The total approach emissions for NOx, HC, CO2, or CO are given by Equation 30.

$$[30]\ \mathrm{E_j} = \sum_{k=1}^{3} [\%P_k * (Q_{TH,j} * \sum_{i=1}^{n^{th}} E_{TH,j,k,i} + Q_{RT} * \sum_{i=1}^{n^{th}} E_{RT,j,k,i} + Q_{LT} * \sum_{i=1}^{n^{th}} E_{LT,j,k,i})]$$

Where, $\mathrm{E_j}$ = approach emissions, Q_{TH}, Q_{RT}, and Q_{LT} = the through, right turn and left turn approach traffic, respectively, $\%P_k$ = the vehicles percentage of vehicles from the total entry traffic that experience speed profile k, k = the profile number 1, 2 or 3, j = the approach number 1, 2, 3, or 4, I = the second number, E_{th}, E_{RT}, E_{LT} = the emissions of vehicles making through, right turn or left turn movements, respectively.

2.6 Objective Function

In this study, the design objective was maximizing environmental sustainability which was represented by vehicle emissions. The objective function is the total vehicle emissions given by Equation 31.

$$[31]\ \mathrm{OF} = \lambda_1 \sum_{j=1}^{4} NOX_j + \lambda_2 \sum_{j=1}^{4} HC_j + \lambda_3 \sum_{j=1}^{4} CO2_j + \lambda_4 \sum_{j=1}^{4} CO_j$$

Where, $\lambda1 \ldots \lambda4$ are the weight applied to each emission kind type according to the importance or the site needs, NOX represents is Nitrogen oxide emissions, HC represents is Hydrocarbon emissions, CO2 represents is Carbon dioxide emissions, and CO represents is Carbon monoxide emissions. In order to maximize environmental sustainability, the objective function, vehicle emissions, is are minimized. Solving the model provides module presents the optimal geometric parameters; such as, the inscribed circle diameter, the central island radius, the enter entry width, the exit width, and the circulated width.

3 APPLICATION EXAMPLE

The model was applied using a hypothetical intersection with assumed traffic values. The model can be used to evaluate an existing roundabout using the same stages as the design of a new one. In the planning stage, the sizing and space requirements should be assessed based on the actual traffic if it is an existing roundabout or the predicted traffic if it is a new design. The module provided the optimal approach decision variables (D, C, and E_{en}) that minimize the total vehicle emissions.

3.1 Data Preparation

The roundabout turning traffic should be calculated for a passenger car equivalent. The hypothetical traffic values are shown in Table 3-1.

Table 3-1: Application example input data

Heading	Q_e (PCE/h)	Q_c (PCE/h)	Q_{TH} (PCE/h)	Q_{RT} (PCE/h)	Q_{LT} (PCE/h)
West Approach	580	381	250	315	15
South Approach	693	315	27	450	216

East Approach	718	258	375	17	326
North Approach	105	917	35	20	50

3.2 Results and Discussion

Table 3-2 shows the optimal values for the inscribed circle diameter (D), circulated width (C), central island radius (in meters), and entering width that minimize vehicle emissions and crash rates for each roundabout approach, as shown in Table 2.

Table 3-2: Application example output data

WB-50 (WB-15)	Een (m)	NOX (g)	HC (g)	CO2 (kg)	CO (g)
West Approach	5.454	47.710	21.070	45.370	1305.644
South Approach	4.919	49.928	1177.917	839.112	1283.704
East Approach	4.919	34.250	471.440	632.835	803.352
North Approach	4.944	45.570	1175.081	836.724	1036.384

The inscribed circle diameter was 46.036 m, the circulating width was 6.760 m, and the central island radius was 16.259 m.

4 SENSTIVITY ANALYSIS

In order to evaluate the sensitivity of vehicle emissions to the geometric parameters of the roundabout, the model was applied to different ranges of the inscribed circle diameter, entry width, and circulated width. The sensitivity analysis was conducted for the west approach based on increasing and decreasing the optimal geometric parameters by 5% and 10%. The results are shown in Table 4-1 and illustrated by Figure 4-1.

Table 4-1: Sensitivity analysis results

West Approach.	NOX (g)	HC (g)	CO2 (kg)	CO (g)
-10% of the optimal	51.146	24.310	48.734	1512.574
-5% of the optimal	51.492	24.478	48.828	1526.238
Optimal parameters	47.710	21.070	45.370	1305.644
+5% of the optimal	52.830	26.373	48.110	1714.412
+10% of the optimal	52.855	26.436	47.815	1726.685

As shown in Table 4-1, increasing the optimal geometric parameters by 5% resulted in an increase of approximately, 10.7%, 25%, 6%, and 31% for the NOX, HC, CO2, and CO emissions, respectively. The results also revealed that decreasing the optimal geometric parameters resulted in an increase in the vehicle emissions. The sensitivity analysis revealed that the optimal geometric parameters identified by the model improved environmental sustainability, decreasing vehicle emissions.

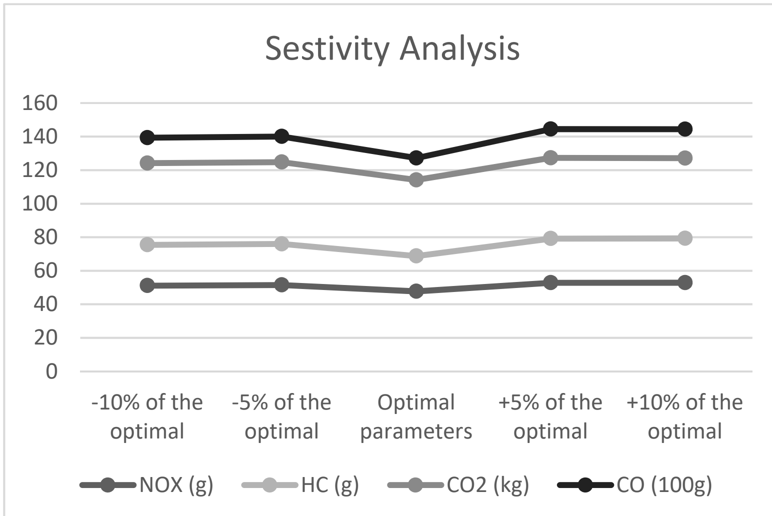

Figure 4-1: The sensitivity of vehicle emissions to the geometry

5 CONCLUDING REMARKS

Numerous researchers have linked the geometric parameters of roundabouts to important aspects of roundabout design including speed and safety performance. There is also evidence that good geometric design of roundabouts can improve environmental sustainability. This paper presents an optimization model for the design of a single-lane roundabout based on minimizing vehicle emissions in order to improve the environmental impact of the intersection. The model was applied to a hypothetical example to test its ability to determine the optimal geometric parameters. The results revealed that the model identified the geometric parameters that minimize vehicle emissions. A sensitivity analysis was conducted and revealed that using the optimal geometric parameters provided by the model improved the environmental sustainability of the roundabout. The use of the model resulted in reductions of 10.7%, 25%, 6%, and 31% for NOX, HC, CO2, and CO emissions (respectively) compared to increasing the optimal geometric parameters by 5%. The model can be improved by addressing the limitations of this study in future research. In this study, the acceleration and deceleration rates were assumed to be constant for emissions calculations, as recommended by the guidelines. Future research should model the acceleration and deceleration and relate them to the geometric parameters of the roundabout. In this study, the model used an operating speed model that was calibrated to the local conditions. Future research can feed the model with more realistic models calibrated using local data. More design objectives can be also be added to the model to suit the needs of any site. Lastly, the model can be modified for different roundabout types instead of limiting it to single-lane roundabouts.

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