Building Tomorrow's Society
Bâtir la Société de Demain

Fredericton, Canada
June 13 – June 16, 2018/ *Juin 13 – Juin 16, 2018*

PERFORMANCE OF SIMPLIFIED BIAXIAL CONCRETE MATERIALS BASED ON DAMAGE FORMULATIONS

Salazar, Jesús R.[1], Cruz-Noguez, Carlos[2,4] and García, Bernardo[3]
[1] MSc Student, University of Alberta, Canada
[2] Assistant Professor, University of Alberta, Canada
[3] Research Assistant, University of Alberta, Canada
[4] cruznogu@ualberta.ca

Abstract: An accurate prediction of the response and strength of concrete in elements subjected to biaxial stresses is important to assess their safety and serviceability. Continuum damage mechanics has been used to develop damage models of concrete, which aim to describe the nonlocal behavior of this material. The objective of this research is to study the performance of simple, yet accurate biaxial concrete materials that are amenable for FE analysis and that have the capacity to account for stiffness recovery in reversal loading (crack closing), permanent strains, and confinement. Two existing damage models will be used to create two new OpenSEES biaxial concrete materials: the "μ" Model (Mazars 2013) and "PRM Model" (Mazars et al 2010). A comparison between analytical and experimental data is used to evaluate the performance of the new materials.

1 INTRODUCTION

Reinforced Concrete is one of the most important and widely used materials. The ultimate goal of structural design is to achieve economic and safety structures. Determining an adequate safety margin requires an accurate prediction of the ultimate capacity of concrete, which exhibits nonlinear behavior even under moderate loading. Continuum damage mechanics have been used in the last decades to develop damage models for concrete, which aim to describe the nonlocal behavior of this material.

Finite element analysis has been widely used in the last decades as a tool in the analysis of RC structures. OpenSEES is an open source code software framework for simulation in earthquake engineering using FEA techniques. It was developed by PEER (Pacific Earthquake Engineering Research), University of California, Berkeley; with the objective to be a mechanism for exchanging and building upon research accomplishments (Fenves, 2001).

An isotropic scalar damage model with multiple damage variables was introduced for the first time by Mazars (1984), it uses a combination of elastic damage mechanics and linear elastic fracture Mechanics within the framework of thermodynamics. This model is adept at predicting the nonlinear behavior of concrete elements without requiring complex solution formulations. Although this model has rendered accurate results, it should not be employed when the material is confined or subjected to alternate loading.

Garcia Ramirez (2017) developed a new OpenSEES concrete material using the "Mazars Scalar Model" (Mazars, 1984) and compared the analytical results with experimental data for an RC beam under monotonic loading, an RC shear wall under reverse-cyclic loading, and a full-scale building subjected to earthquake loading. This software material accurately predicted the failure load of the beam, but it was not capable of determining the failure deflection, nor the cracking and yielding moments. It exhibited moderate agreement in the hysteretic analysis of the shear wall but failed to predict the energy dissipation capacity and residual displacements.

The objective of this research is to assess the performance of biaxial concrete materials capable of determining stiffness recovery in reversal loading (crack closing), permanent strains, and confinement. Two existing damage models will be used to develop two independent OpenSEES concrete materials: Mazars' damage "μ Model" (2013) and the "PRM Coupled Model" (Mazars et al 2010). The analytical analysis generated by these new materials will be compared with experimental tests to check their accuracy and performance. In contrast with most FEA software, this project will provide a tool for modeling large-scale 3D or 2D concrete structures using 2D nonlinear elements, without having any restrictions on the number of nodes or elements that can be used. This project will also provide some hints concerning the performance of each material for different types of analysis.

Experimental research is driven by the need to develop more accurate analysis and design methods for RC structures. The ultimate objective of this investigation project is to provide the research community a new and sophisticated, yet simple tool to implement precise concrete nonlinear models, allowing them to compare their analytical and experimental data for any concrete element they are testing.

2 CONCRETE MODELS FOR FEA

The basic information needed for reinforced concrete FE calculations is the multi-dimensional stress-strain relationships of the material, which describe the material response under monotonic, cyclic, and dynamic loading. In the last decades, the framework of continuum mechanics has been used to create multiple models to describe concrete behavior. Some approaches are plasticity models (Argyris, 1981), fracture-based models (Bazant, 1994), damage models (Mazars, 1984), or plastic-damage models (Lee, 1998).

Due to its simplicity for modeling, a damage-based approach is selected for this study. These models can explicitly solve the material constitutive, equilibrium and compatibility equations without needing an iterative procedure to calculate stresses from a given set of strains.

3 DAMAGE-BASED MODELS USED TO IMPLEMENT NEW MATERIALS FOR OPENSEES

Concrete damaged-based models are formulated using continuum damage mechanics and aim to describe the nonlinear behavior of concrete. Many materials, including concrete, can exhibit internal failures at the micro- and macro- scale produced by different types of effects such as creep, fatigue, constant load and chemical reactions. These internal failures are produced in form of microcracks, their propagation and coalescence in concrete elements is known as "damage" (Kachanov, 1958).

Concrete is a composite material formed by granulates in a hydrated cement paste or brittle matrix. Damage mechanics is able to describe the interface between the aggregate grains and the cement matrix under loading. Stiffness degradation of concrete is presented due to the failure of the cement matrix which leads to the apparition and propagation cracks. Damage mechanics is a simplified strategy to describe the behavior of concrete upon its complex microstructure.

The concrete models used in this research are based on the work done by J. Mazars through the last three decades. The first model that will be discussed was formulated in 1984, which is an isotropic scalar damage model implemented in OpenSEES by García Ramírez in 2017. The second one is the "PRM" model, which is capable to account for permanent strains. And finally the "μ" model, which includes effects that weren't contemplated in other models as unilateral effects.

These concrete damage models assume the material being elastic, isotropic and with constant stiffness. The stiffness of the material is modified using a scalar damage parameter (D), which ranges from 0 for the undamaged material to 1 for the complete failure of the material. Mazars' models account for all the micro and macro effects of loading, the rearranging of concrete particles, collapse of the micro-voids in the mixture, and the interaction of the cement matrix with the aggregates. Figure 3.1 describes the behavior of stress-strain (σ-ε) curves for concrete elements using damage models, where the parameter Γ is defined as the stiffness matrix.

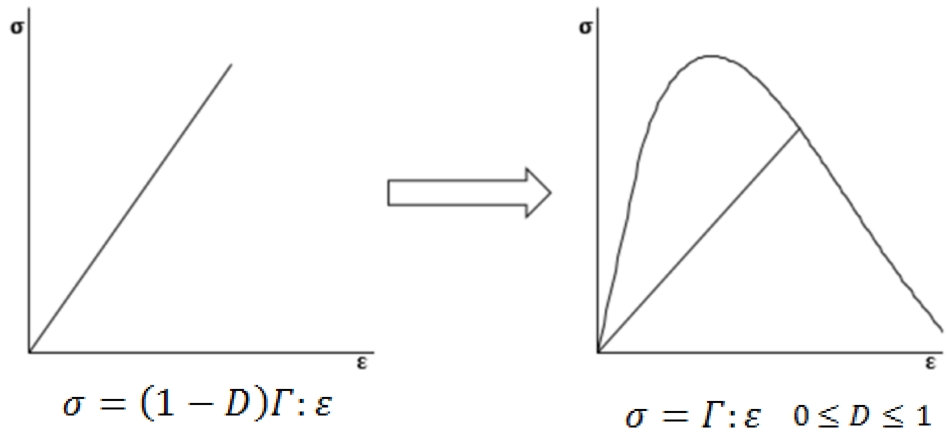

Figure 3.1. Stress-strain relationship of concrete using damage models.

3.1 Mazars Scalar Damage Model

Mazars (1984) formulated a scalar damage model to predict the triaxial behavior of concrete. This model describes the behavior of concrete as isotropic, elastic-damageable. The damage parameter (*D*) is calculated starting from an equivalent strain, which is the average of the tensile principal strains of the element, meaning that all compressive strains can be represented as tensile strains in the orthogonal direction.

The calculation of the stresses uses elastic theory, reducing the elastic stiffness matrix with a damage parameter (*D*).

[3.1.1] $\sigma = (1 - D)\Gamma : \varepsilon \quad (\Gamma : Stiffness\ matrix)$

The total damage of the element is composed by the weighted sum of the damage given by the tensile stresses and the compressive stresses. The weights have a modification factor (β) that accounts for the presence of shear resistance in the interaction of compression and tension.

[3.1.2] $D = {\alpha_t}^{\beta} * D_t + {\alpha_c}^{\beta} * D_c; \qquad 0 \leq D \leq 1$

The formulation of the damage in tension and compression depends either on the tensile or compressive material parameters that can be obtained from tests (*Ac*, *Bc*, *At*, *Bt*). All the calculations are made with the equivalent strain of the element (ε_{eq}), and damage starts only when the strain crosses the initial damage strain threshold (ε_{D0}).

[3.1.3] $D_t = 1 - \frac{\varepsilon_{D0} * (1 - A_t)}{\varepsilon_{eq}} - A_t * exp\left[-B_t * \left(\varepsilon_{eq} - \varepsilon_{D0}\right)\right]$

[3.1.4] $D_c = 1 - \frac{\varepsilon_{D0} * (1 - A_c)}{\varepsilon_{eq}} - A_c * exp\left[-B_c * \left(\varepsilon_{eq} - \varepsilon_{D0}\right)\right]$

The weights of the tensile and compressive contribution to the total damage are calculated analyzing each of the principal strains obtained from the positive and negative principal elastic stresses. The weight is only considered if the total strain is tensile, thus the use of the *H* parameter.

[3.1.5] $\alpha_t = \sum_{i=1}^{3} H_i \frac{\varepsilon_{ti}(\varepsilon_{ti} + \varepsilon_{ci})}{{\varepsilon_{eq}}^2} \qquad where\ \ H_i = 1\ if\ \varepsilon_i = \varepsilon_{ci} + \varepsilon_{ti} \geq 0, otherwise, H_i = 0$

[3.1.6] $\alpha_c = \sum_{i=1}^{3} H_i \frac{\varepsilon_{ci}(\varepsilon_{ti} + \varepsilon_{ci})}{{\varepsilon_{eq}}^2}$

The positive (tensile) and negative (compressive) strains are calculated from the elastic stiffness matrix, and the positive and negative elastic stresses respectively.

[3.1.7] $\varepsilon_t = \Gamma^{-1}: \sigma_+$

[3.1.8] $\varepsilon_c = \Gamma^{-1}: \sigma_-;$

The equivalent strain is calculated as the average of the tensile principal strains (ε_i) of the element.

[3.1.9] $\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3}(\langle\varepsilon_i\rangle)^2}; \quad where\ \langle\varepsilon_i\rangle = \varepsilon_i \ \ if \ \ \varepsilon_i > 0$

3.2 PRM Model

The "PRM" model (Mazars et al 2010) is a two scalar damage model formulated from works done by Pontiroli (1995), Rouquand (1995,2005) and Mazars(1986). Its particular improvement over previous models is that keeps the simplicity of being an elastic-damage model but adding the capacity of accounting for permanent strains. This model uses two damage variables, *Dt* and *Dc*, for traction and compression damage respectively. The variables σ_{ft} and ε_{ft} are the crack closure stress and strain respectively. The main equations of the PRM model are shown below.

The constitutive equation of the model:

[3.2.1] $(\sigma - \sigma_{ft}) = (1 - D)\Gamma: (\varepsilon - \varepsilon_{ft})$

[3.2.2] $\sigma_d = \sigma - \sigma_{ft}$

[3.2.3] $\varepsilon_d = \varepsilon - \varepsilon_{ft}$

Before damage in compression is presented, the crack closure stress and strain are equal to the initial material parameters σ_{ft0} and ε_{ft0}respectively. Once compressive damage (*Dc*) is presented the crack closure stress (σ_{ft}) is calculated from *Dc* as follows:

[3.2.4] $\sigma_{ft} = \sigma_{ft0}(1 - D)^2$

The PRM model uses the same equivalent strain concept as the Mazars' Scalar Damage Mode and uses the same equation [3.9] to determine it. The difference is that under traction the principal strains (ε_i) are obtained as:

[3.2.5] $\varepsilon_i = \left(\varepsilon - \varepsilon_{ft}\right)_i$

The damage evolutions were taken from the original Mazars model (1984), with the main difference that the PRM model has two different thresholds for tensile damage (ε_{t0}) and compressive damage (ε_{c0}). The damage parameter (D) remains a scalar and is obtained from the calculation of *Dc* and *Dt*. The activation factor (α_t) evolves from 0 to 1 depending on the tensor σ_d, where α_t=1 if σ_d>0 and α_t=0 if σ_d<0:

[3.2.6] $D_t = 1 - \frac{\varepsilon_{t0}*(1-A_t)}{\varepsilon_{eq}} - A_t * exp\left[-B_t * \left(\varepsilon_{eq} - \varepsilon_{t0}\right)\right]$

[3.2.7] $D_{cM} = 1 - \frac{\varepsilon_{c0}*(1-A_c)}{\varepsilon_{eq}} - A_c * exp\left[-B_c * \left(\varepsilon_{eq} - \varepsilon_{c0}\right)\right]$

[3.2.8] $D_c = \frac{\varepsilon_{c0}*(1-A_c)}{\varepsilon_{eq}}$

[3.2.9] $D = \alpha_t D_t + (1 - \alpha_t) D_c$

3.3 "μ" Model

The Mazars' "μ" Model (2013) was created to include the damage effects related to monotonic and cyclic loading that were not incorporated in previous models, such as unilateral effects. It has proved to be capable of describing a broad range of nonlinear behavior: monotonic, cyclic, and dynamic loading (Mazars, 2013). It follows the following assumptions:

- Describes the behavior of concrete as the combination of damage and elasticity.
- The damage behavior is assumed as isotropic.
- Two damage modes are assumed: cracking (tension) and crushing (compression). This leads to having two independent equivalent strains, one for tension and another for compression.
- In contrast with the PRM and SDM models, the effective damage parameter (*d*) describes the damage on the stiffness activated either by compressive or tensile loading.
- *d* is able to describe the unilateral effects (crack opening and closure).

The calculations needed to determine the damage parameters are shown below. The stress is obtained using the same equation [3.1.1] as the Mazars' Scalar Model (1984).
The equivalent strain for cracking (ε_t) and crushing (ε_c) are defined as follows, where v is the Poisson's ratio:

[3.3.1] $\varepsilon_t = \frac{I_\varepsilon}{2(1-2v)} + \frac{\sqrt{J_\varepsilon}}{2(1+v)}$

[3.3.2] $\varepsilon_c = \frac{I_\varepsilon}{5(1-2v)} + \frac{6\sqrt{J_\varepsilon}}{5(1+v)}$, where $I_\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ and $J_\varepsilon = 0.5[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2$

Y_t and Y_c are the maximum values reached during the loading path, while ε_{0t} and ε_{0c} are the initial threshold of the cracking and crushing equivalent strains respectively:

[3.3.3] $Y_t = Sup[\varepsilon_{0t}, \max \varepsilon_t]$

[3.3.4] $Y_c = Sup[\varepsilon_{0c}, \max \varepsilon_c]$

The damage parameter *D* is directly related to the thermodynamic variables Y_t and Y_c though the *Y* variable. The triaxial factor (*r*) evolves from 0 for pure compressive stress to 1 for pure tensile stress:

[3.3.5] $Y = rY_t + (1-r)Y_c$

[3.3.6] $r = \frac{\sum<\bar{\sigma}_i>_+}{\sum|\bar{\sigma}_i|}$, where $\bar{\sigma} = \frac{\sigma}{(1-d)} = \Gamma : \varepsilon$

The damage evolution is defined as was in the Mazars' Scalar Model, where Y_0 is the initial threshold for *Y*. *A* and *B* define the shape of the effective damage (*d*), which are defined from the test parameters *Ac*, *Bc*, *At*, and *Bt*.

[3.3.7] $d = 1 - \frac{Y_0 * (1-A)}{Y} - A * \exp[-B * (Y - Y_0)]$

[3.3.8] $Y_0 = r\varepsilon_{0t} + (1-r)\varepsilon_{0c}$

[3.3.9] $A = A_t\big(2r^2(1-2k) - r(1-4k)\big) + A_c(2r^2 - 3r + 1)$

[3.3.10] $B = r^{(r^2-2r+2)}B_t + (1 - r^{(r^2-2r+2)}B_c$

[3.3.11] $k = \frac{0.5}{A_t}\ or\ 0.7$

3.4 Finite-element implementation

The material formulations presented above were implemented into the open-source, finite-element software OpenSEES (Open System for Earthquake Engineering Simulation), which has been under development by PEER (Pacific Earthquake Engineering Research) since 1997.

The general steps for the FEA procedure that OpenSEES.exe application follows using the new materials are shown below:

1. Within the input data provided by the user, initialize elements stiffness matrix [*K*] and determine the residual force {*ΔR*}.
2. Solve for nodal displacement increments {*Δu*}, using the equilibrium equation [*K*]{*Δu*} ={*ΔR*}.
3. Find the nodal displacement and determine strains of the element.
4. Use the new material code within the plane stress framework to determine the damaged stiffness matrix and the stresses in the original direction {σ}.
5. Obtain the principal strain direction θ_1 and the principal strain vector {ε_p}.
6. Obtain the scalar damage variable D_i of the material. Equations [3.1.1] to [3.1.9] for the Scalar Damage Model (Mazars, 1984), equations [3.2.1] to [3.2.9] for the PRM model (Mazars et al., 2010), and equations [3.3.1] to [3.3.11] for the "μ" model (Mazars, 2013).
7. Calculate the damaged stiffness matrix $[\Gamma_c]_D=[\Gamma_c](1-D_i)$.
8. Calculate the principal stress vector {σ_p} and the stresses in the original direction {σ}.
9. Establishes the element stiffness matrix [*K*]=$\int [B]^T [\Gamma_c]_D [B] dV$ and element resisting force increment {*ΔF*}=[*K*]{*ΔU*}.
10. Check for the residual force {*ΔR'*} = {*ΔR*} - {*ΔF*}. If convergence is achieved, proceed to next step of the loading, if it doesn't go back to step 2.

4 VALIDATION & DISCUSSION

Two concrete material models were introduced to OpenSEES for 3D finite model analysis: PRM model and "μ" model. The performance of these materials was studied by comparing two concrete experimental test data with analytical models, elaborated in the OpenSEES framework. The first one consists in an experimental study realized by Kupfer (1969), which describes the behavior of concrete specimens under biaxial stress states. The second one is a simply supported beam under monotonic loading tested at the University of Alberta in 2013.

4.1 Kupfer Experiment

Kupfer (1969) conducted a series of experiments to investigate the biaxial behavior of concrete. He tested concrete specimens of 200x200x50 mm under biaxial stress, for the conditions of biaxial compression, compression-tension, and biaxial tension. The experimental parameters consisted of a compression strength (*f'c*) of 32.7 MPa, a tensile strength (*ft*) of 3.2 MPa, and Young's modulus (*E*) of 30 GPa.

4.1.1 FEM Model

The biaxial tests conducted by Kupfer were simulated in OpenSEES via a "Quad" element with dimensions of 200x200x50 mm. Different displacement conditions were applied, combining compression and tensile displacements. The stresses of the element in each one of the two principal directions were obtained for each combination and normalized with the compressive strength (*f'c*). The model parameters are shown in table 4.1.

Table 4.1. Model Parameters for biaxial test.

Material	E(GPa)	ε_{0c}	$\varepsilon_{0t}/\varepsilon_{D0}$	ε_{fc}	σ_{fc}	ε_{ft0}	σ_{ft0}
SDM Mazars	30.0	-	1.0e-4	-	-	-	-
PRM Model	30.0	1.0e-4	1.38e-4	5.0e-4	37.2e6	-3.3e-5	-0.99e-6
μ Model	30.0	3.0e-4	1.1e-4	-	-	-	-

Material	Ac	Bc	At	Bt	υ
SDM Mazars	1.275	1850.0	1.0	1.0e4	0.21
PRM Model	1.29	1550.0	0.8	1.0e4	0.21
μ Model	1.7	570.5	1.0	1.0e4	0.21

*SDM stands for Scalar Damage Model (Mazars,1984).

4.1.2 Comparison of predictions with experimental results

The experimental biaxial behavior of concrete panels is compared with the calculated using each material in OpenSEES. We can observe how the three models are excellent candidates to describe the biaxial behavior of concrete under the uniaxial compression, compression-tension, and biaxial tension domains, but the PRM and Mazars' Scalar Damage models fail to accurately describe the biaxial compression domain. On the other hand, the "μ" model shows proficient results when describing each domain of the biaxial experiment. The experimental and analytical results are shown in fig. 4.1.

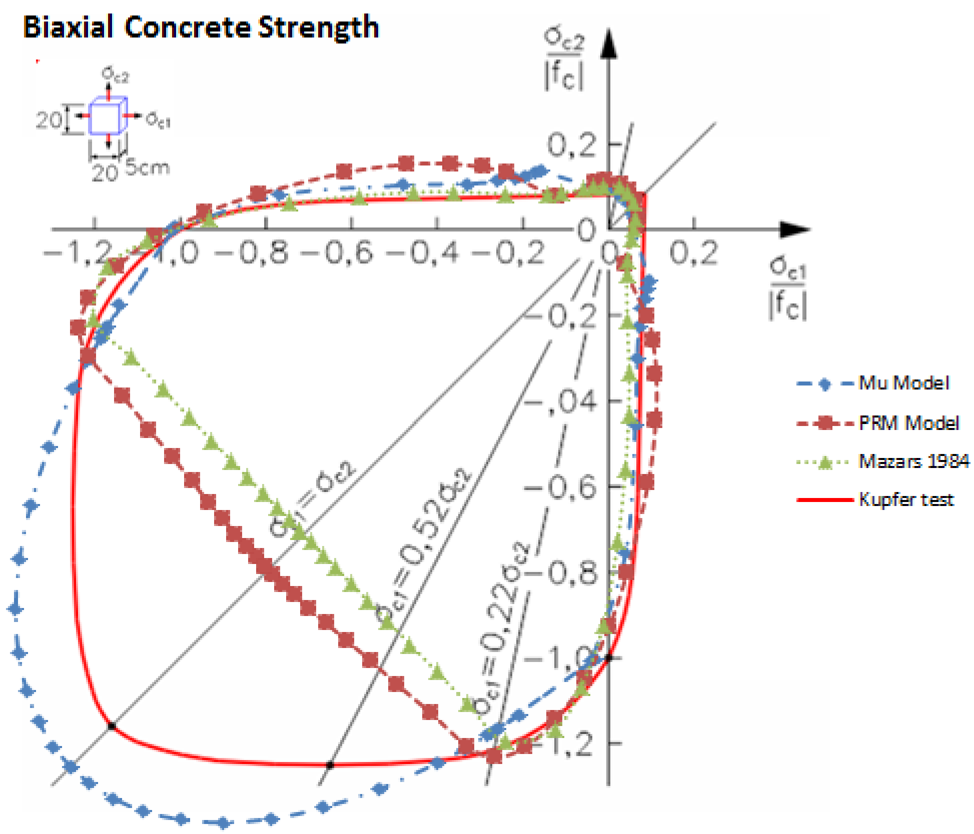

Figure 4.1. Biaxial behavior of concrete. Adapted from Kupfer et al. (1969).

4.2 Analysis of an RC Beam Under Monotonical Loading

A beam under 4-point bending tested at the University of Alberta in 2013 was selected to assess the performance of different models in OpenSEES. The experimental parameters consisted in an *f'c* of 40 MPa, and Young's modulus (*E*) of 37.2 GPa. The yield stress of the steel (*fy*) was measured as 475 MPa, with Young's modulus (*E*) of 183.33 GPa. The dimensions, loading points, and reinforcement specifications are shown in figure 4.2.

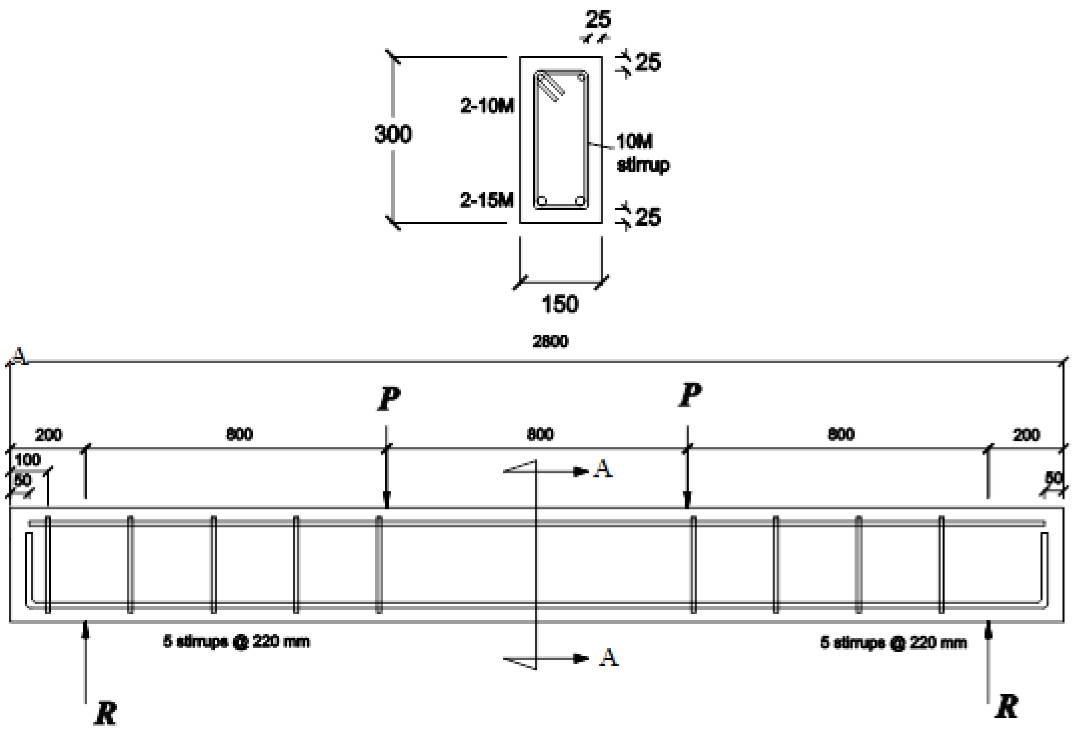

Figure 4.2. Beam specifications.

4.2.1 FEM Model

The OpenSEES beam was modeled using 312 four-node multilayer shell elements; each one was made up of three layers of 50 mm. For the steel reinforcement, 140 truss elements were needed. The pushover analysis was done using a displacement controlled analysis, applying a descending displacement at nodes 337 and 353. Boundary conditions were introduced at nodes 5 and 49, restricting the vertical displacement. The OpenSEES beam model can be observed in fig. 4.3. The parameters used for each concrete material are shown in table 4.2.

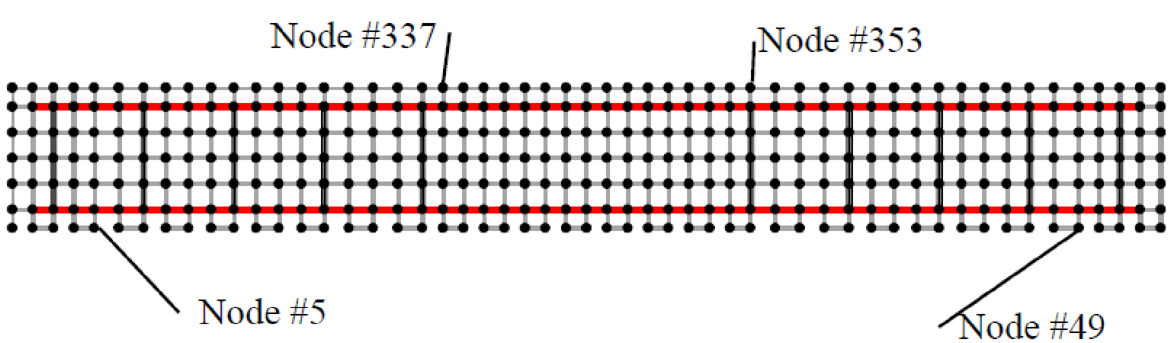

Figure 4.3. Beam OpenSEES model.

Table 4.2: Model Parameters for beam analysis.

Material	E(GPa)	ε_{0c}	$\varepsilon_{0t}/\varepsilon_{D0}$	ε_{fc}	σ_{fc}	ε_{ft0}	σ_{ft0}
SDM Mazars	37.2	-	5.0e-5	-	-	-	-
PRM Model	37.2	1.3e-4	1.25e-4	5.0e-4	37.2e6	-3.3e-5	-1.2276e6
μ Model	37.2	4e-4	5.0e-5	-	-	-	-

Material	Ac	Bc	At	Bt	υ
SDM Mazars	0.73	1065.0	0.97	1.0e4	0.18
PRM Model	0.6	1100.0	0.68	1.0e4	0.18
μ Model	0.645	280.0	0.97	1.0e4	0.18

*SDM stands for Scalar Damage Model (Mazars,1984).

4.2.2 Comparison of predictions with experimental results

The comparison between the experimental and analytical results can be found in figure 4.4. The flexural failure of the beam was due to concrete crushing on the compressive side of the beam. It can be observed that the three model materials show an accurate prediction for the force-displacement curve, but the PRM and Mazars' Scalar Damage (1984) models fail to predict the failure strain of the beam, whereas the "μ" model is able to accurately predict the strain at which the beam fails. None of the three materials accurately predicted the cracking load, but the "μ" model provided a better prediction for the yielding moment. For this experiment, the "μ" model performed best.

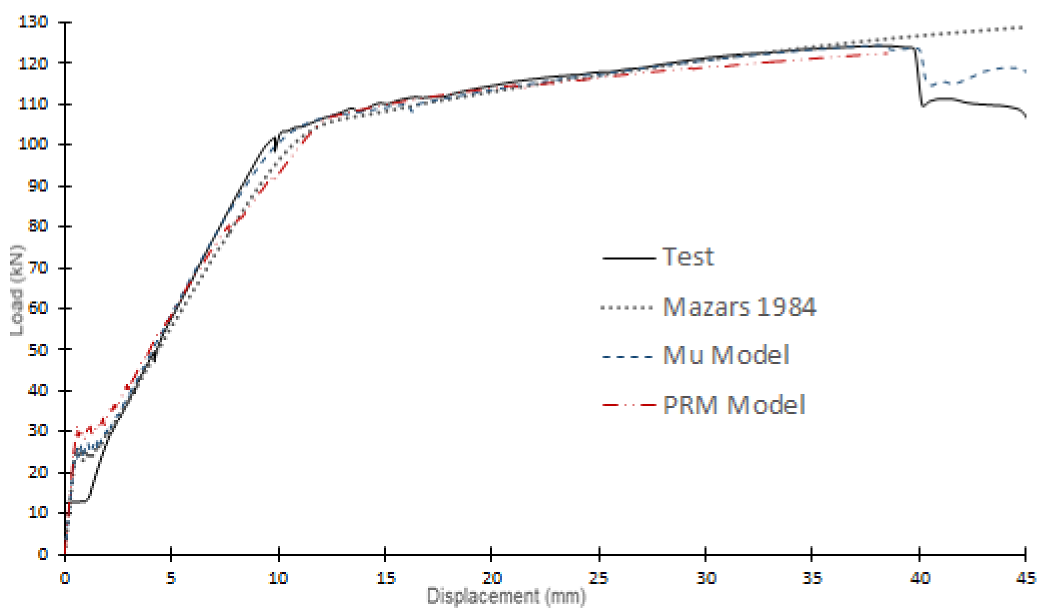

Figure 4.4. Analysis results of RC beam.

5 SUMMARY AND CONCLUSIONS

Building upon the work of García (2017), two biaxial concrete material models were implemented into the open-source finite element framework –PRM model and "μ" model –with the objective of evaluating their performance under a broad type loading for various structures. The three OpenSEES material models are capable of accurately describing the biaxial behavior of concrete, but only the "μ" model showed being able to successfully describe the biaxial compression domain. The three materials are good candidates to describe the pushover done to a simply supported beam, but only the “μ” model could predict the strain at which the beam crushed and the yielding force.

More experimental tests are being modeled and compared using these materials in order to improve our general knowledge regarding the breadth of potential applications and the most appropriate selection in individual cases. These include a four-point bending beam under cyclic loading, a squat shear wall under cyclic loading, and a full-scale concrete building under seismic loading.

References

ASCE Task Committee on Finite Element Analysis of Reinforced Concrete Structures. 1982. State-of-the-Art Report on Finite Element Analysis of Reinforced Concrete. *Special Publications*. ASCE.

Argyris, J.H., Faust, G., and William, K.J. 1981. Finite Element Modeling of Reinforced Concrete Structures. *Introductory Report, IABSE Colloquim on Advanced Mechanics of Reinforced Concrete*. Delft.

Bazant, Z.P. 1994. Nonlocal Damage Theory Based on Micromechanics of Crack Interaction. *J Eng Mech.* ASCE, **120**:593–617.

Fenves, G. 2001. *Annual Workshop on Open System for Earthquake Engineering Simulation.* (P. E. Center, Ed.) Retrieved from http://opensees.berkeley.edu/

García, B. 2017. Implementation of a Simple Biaxial Concrete Model for Reinforced Concrete Membranes. *MSc Thesis Dissertation. University of Alberta.*

Kachanov, L. 1958. Time of the rupture process under creep conditions. Izv. Akad. Nauk SSR Otd, **8**, 26-31.

Kupfer, H., Hilsdorf, H.K., and Rüsch, H. 1969. Behavior of Concrete Under Biaxial Stresses. ACI J **66**(66–62):656–666.

Lee, J. and Fenves, G.L. 1998. Plastic-damage Model for Cyclic Loading of Concrete Structures. *J Eng Mech,* **124**:892.

Mazars, J. 1984. Application de la Mecanique de L'endommangement au Compportement non Lineaire et a la Ruptura du Beton de Structure. *These de Doctorate d'Etat, L.M.T., Universite Paris, France*.

Mazars, J. 1986. A Description of Micro and Macroscale Damage of Concrete Structure. *Eng Fract Mech,* **25**:729–737.

Mazars, J. and Grange, S. 2014. Simplified Method Strategies Based on Damage Mechanics for Engineering Issues. EUROC 2014, St Anton am Arlberg, Austria.

Mazars, J., Hamon, F. and Grange, S.. 2013. A New 3D Damage Model for Concrete Under Monotonic, Cyclic and Dynamic Loading. *Internal report*. Lab **3SR** Grenoble-France.

Mazars, J., Hamon, F., and Grange, S. 2015. A New 3D Damage Model for Concrete Under Monotonic, Cyclic and Dynamic Loading. *Materials and Structures,* **48**:3779-3793.

Pontiroli, C., Rouquand, A. and Mazars, J. 2010. Predicting Concrete Behaviour From Quasi-static Loading to Hypervelocity Impact. *Eur J.* Environ Civil Eng **14**(6–7):703–727.