



## SEISMIC RESPONSE SENSITIVITY OF CONTROLLED ROCKING STEEL BRACED FRAMES

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**Abstract:** Controlled rocking steel braced frames have been demonstrated recently as self-centering systems that can reduce permanent deformation of structural members in a major earthquake and thus minimizing repair costs. The design of these frames allows for rocking action at the base of columns, thereby limiting demands on frame members. The overturning resistance is provided by gravity loads and high-strength post-tensioned steel strands, while energy dissipation is primarily provided by easily replaceable steel plate fuses. In this paper, the sensitivity of seismic response of controlled rocking steel braced frames is investigated with respect to design parameters, including the yield strength, initial lateral stiffness, and strain hardening ratio of the fuse, the initial post-tensioning force and modulus of elasticity of the strands, as well as rocking column gravity load. Furthermore, the effects of frame aspect ratio and earthquake intensity level are assessed. The study is based on conducting nonlinear response history analysis of controlled rocking steel braced frames as well as utilizing the design of experiment method for effective and reliable sensitivity analyses. Based on the results, the peak roof drift response is dominated by the effects of initial post-tensioning strand force and rocking column gravity load, while the residual roof drift and peak floor acceleration are influenced by several factors and interactions.

### 1 Introduction

Steel buildings designed as per current seismic design codes provide a minimum margin of safety against collapse under major earthquakes (FEMA 2009). However, these structures are susceptible to structural damage due to large permanent deformations, which increases financial losses.

In order to minimize the permanent structural damage in steel frames, researchers have developed controlled rocking steel braced frames (CRSBFs) (Eatherton et al. 2014a; b; Eatherton and Hajjar 2010; Ma et al. 2011). Figure 1 shows a representation of CRSBFs. In these frames, rocking of columns are allowed so as to avoid damage in the main frame elements such as beams and columns. Therefore, the damage is localized at the energy dissipating steel fuses, which can be easily replaced following earthquake. High-strength post-tensioned steel strands are used to provide overturning resistance along with gravity on the rocking frame.

Past research has demonstrated the efficiency of using CRSBFs for minimizing permanent structural damage in steel buildings. Among others, results of hybrid simulation tests by Eatherton and Hajjar (2014) show that CRSBFs sustain no residual storey drift under ground motions with intensities greater than the maximum considered earthquake (MCE) hazard level. Based on analysis of single degree of freedom models, Eatherton and Hajjar (2011) reported that the self-centering response of rocking frames is reliable even with small amounts of restoring force. A parametric study by Hall et al. (2010) evaluates the influence

of three design variables on the seismic response of CRBFs. The results of a study by Steele and Wiebe (2017) show that CRSBFs have acceptably low probabilities of collapse.

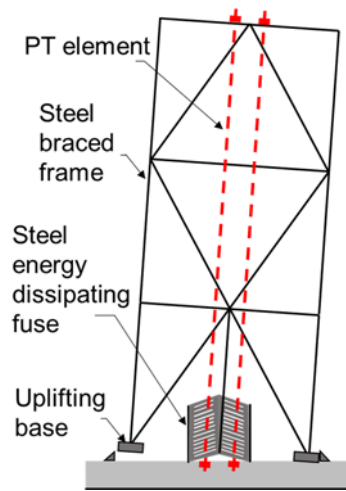


Figure 1: Schematic view of CRSBFs (adapted from Ma et al. 2011)

In this paper, we present a parametric study on the seismic response sensitivity of CRSBFs. Nonlinear response history analyses are performed using OpenSees (ver. 2.4.3) (Mazzoni et al. 2013) to evaluate the influence of several factors on the seismic demand parameters including peak roof drift, residual roof drift, and peak floor acceleration. The design factors that are considered in the sensitivity study are factors related to the frame configuration, and PT and fuse properties. By using a statistical design of experiment methodology, the sensitivity study is conducted more efficiently and the interactions between parameters are evaluated in addition to the main factor effects. An interaction between factors A and B is active when the effect of A on the response parameter will change at different levels (or values) of factor B.

## 2 Frame structures and computational modelling

Three- and nine-storey CRSBFs designed by Ma et al. (2011) per ASCE 7-05 (2005) for site class D in Los Angeles are used in this study. The design details of CRSBFs are not influencing the results of this research because these details will be the same for all the rocking frames analyzed. Figure 2 shows the frame configurations and member sizes. The computer program OpenSees are used to develop two-dimensional structural models (shown in Figure 3). Elastic beam-column elements (with modulus of elasticity = 200 GPa) are used for steel beams, columns, and braces, which are expected to remain elastic during the rocking action of CRSBFs. As shown in Figure 3, the OpenSees model also includes leaning columns to consider  $P-\Delta$  effects associated with the gravity frames, which are not explicitly modelled. The gap opening at the column base of CRSBFs is modelled using compression-only gap springs, which are stiff in compression, but have zero tensile stiffness. The damping ratio is set to 2% using Rayleigh damping.

PT strands are modelled using a truss element with a bilinear material model and in-parallel combination of elastic and elastic-perfectly plastic material models (Ma et al. 2011). All the rocking steel braced frames analyzed in this study have sixty PT strands with yield and ultimate strengths of 1675 MPa and 1862 MPa, respectively. The energy dissipating steel plates (fuses) are modelled using the assembly model proposed by Ma et al. (2011).

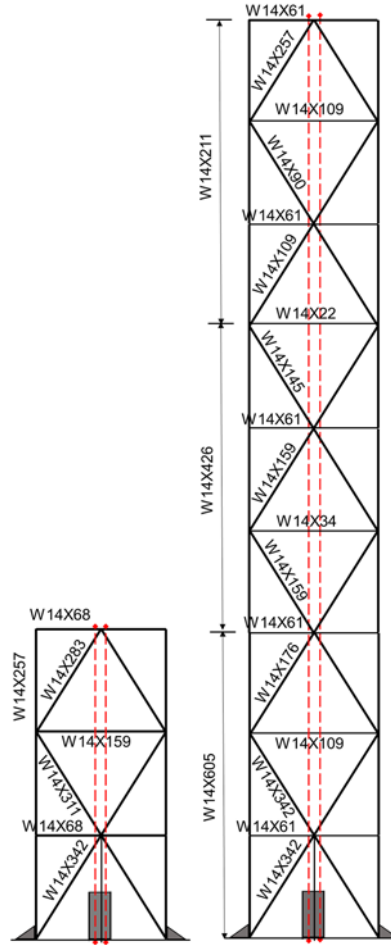


Figure 2: Frame member sections (adapted from Ma et al. 2011)

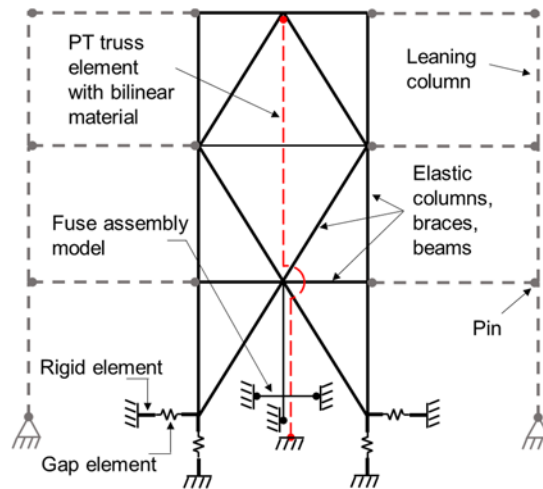


Figure 3: OpenSees model of a three-storey frame structure

### 3 Sensitivity analysis

The sensitivity analysis is conducted to statistically determine the influence of design factors and their interactions on the seismic demands of CRSBFs. Table 1 summarizes six factors and their (low and high) values that are considered in the sensitivity study. The factor ranges are chosen to consider a practical but broad range for each factor. The low level for  $P_D$  is set to zero to include buildings where the rocking system is isolated from the gravity system. Additionally, to evaluate the influence of frame aspect ratio (defined as frame height to bay width) and earthquake intensity level, nine different analysis cases are considered (Table 2). The analysis case 3d15, for example, represents a three-storey CRSBF with 15 in (4.57 m) bay width subjected to earthquake records scaled to a design-based earthquake (DBE) hazard level ( $Sa_{T1} = 0.65g$ ). The MCE level corresponds to  $Sa_{T1} = 0.98g$ .

Table 1: Factors considered in the sensitivity study

Factor	Symbol	Low level (-)	High level (+)	Unit
Rocking column load per floor	$P_D$	0.00	120	kN
Initial PT force per strand	$F_{pt}$	44	130	kN
Fuse yield stress	$\sigma_{yf}$	250	444	MPa
Fuse strain hardening ratio	$\alpha_f$	0.005	0.05	-
Fuse modulus of elasticity	$E_f$	185	212	GPa
PT strands modulus of elasticity	$E_{pt}$	175	208	GPa

Table 2: Analysis cases in this study

Analysis case	Number of stories	Earthquake intensity level	Bay width (m)
3d15	3	DBE	4.57
3d30	3	DBE	9.14
3m15	3	MCE	4.57
3m30	3	MCE	9.14
9d15	9	DBE	4.57
9d30	9	DBE	9.14
9m15	9	MCE	4.57
9m30	9	MCE	9.14

Nonlinear response history analyses are performed on CRSBFs with the factor combinations listed in Table 3. The '-' and '+' signs denote, respectively, the low level and high level for the factors, as listed in Table 1. This experimental design is a fractional factorial design created using the Design-Expert statistical analysis software (DX10 2016). A nonlinear analysis is performed for each analysis case (Table 2) with each factor combination (Table 3) under a suite of thirty-one ground motion records, which are selected from the large suite of ground motions assembled by Miranda (2000). A total of 3968 nonlinear response history analyses are performed. From each structural analysis, three response demands are obtained, including peak roof drift (PRD), residual roof drift (RRD), and peak floor horizontal acceleration (PFA). Then, the mean of each

response variable (under the suite of earthquake records) is used as the response variable in the sensitivity analyses. Further details are available in Moradi and Burton (2018).

Table 3: Experimental design (factor combinations) used for the sensitivity study

Combination	Factor level					
	$P_D$	$F_{pt}$	$\sigma_{yf}$	$\alpha_f$	$E_f$	$E_{pt}$
1	-	-	-	-	-	-
2	+	-	-	-	+	-
3	-	+	-	-	+	+
4	+	+	-	-	-	+
5	-	-	+	-	+	+
6	+	-	+	-	-	+
7	-	+	+	-	-	-
8	+	+	+	-	+	-
9	-	-	-	+	-	+
10	+	-	-	+	+	+
11	-	+	-	+	+	-
12	+	+	-	+	-	-
13	-	-	+	+	+	-
14	+	-	+	+	-	-
15	-	+	+	+	-	+
16	+	+	+	+	+	+

#### 4 Results and discussions

This section summarizes the results of the sensitivity analysis. Figure 4 presents two example half-normal probability plots, which are used to graphically identify significant factors influencing the PRD and RRD responses. Insignificant factors lie on a straight line on this plot of the absolute value of effect estimates (adjusted by their standard error values) versus the cumulative normal probabilities of the effects. From Figure 4a for example, it is observed that  $F_{pt}$ ,  $P_D$ , and their interaction,  $P_D \cdot F_{pt}$ , are the most significant factors and interaction affecting the PRD response of 9m30 frames.

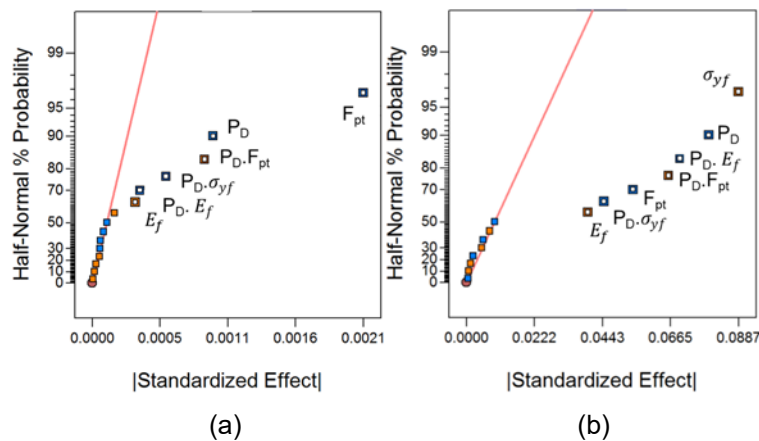


Figure 4: Example half-normal probability plots for: (a) PRD and (b) RRD response variables for analysis case 9m30

In order to statistically determine the significance of each input factor on the response, analysis of variance (ANOVA) is performed for each analysis case listed in Table 2. ANOVA is a statistical analysis to test the null hypothesis ( $H_0$ ) that the level of a particular factor has no effect on the response variable by decomposing the sources of variance (Montgomery 2013).

The results of ANOVA can be expressed in terms of  $p$ -value, which corresponds to the probability that the null hypothesis is true (i.e., there is no significant effect). By assuming a significance level of 0.05, significant effects have a  $p$ -value of less than 0.05. Table 4 presents  $p$ -values associated with each response variable for the main factors in the analysis cases. Based on the  $p$ -values, the most significant factors are  $P_D$ ,  $F_{pt}$ , and  $\sigma_{yf}$ , which have low  $p$ -values for all the analysis cases and across the three response variables. It is also found that  $\alpha_f$  and  $E_{pt}$  do not significantly influence the seismic demand parameters of CRSBFs (mostly large  $p$ -values are obtained for these factors).

In order to evaluate the relative significance of each factor, we obtain the percentage contribution, which is the ratio of the sum of squares for a factor to the total sum of squares (Montgomery 2013). In fact, the sum of squares for a factor indicates the response variation due to the change in the factor level (over its range). Tables 5 and 6 summarize the percent contributions of influential factors and interactions associated with each response variable and analysis case. In these tables, the negative and positive percentage values are highlighted in red and blue, respectively. A negative percent contribution indicates that the response demand is decreased as the factor level increases.

Table 4:  $P$ -value associated with each response for the main factors

Case	Response	Factor					
		$P_D$	$F_{pt}$	$\sigma_{yf}$	$\alpha_f$	$E_f$	$E_{pt}$
3d15	PRD	<b>0.000</b> *	<b>0.000</b>	<b>0.000</b>	0.179	<b>0.031</b>	0.169
	RRD	<b>0.003</b>	0.932	<b>0.000</b>	0.737	0.186	0.098
	PFA	0.104	<b>0.013</b>	<b>0.004</b>	0.132	<b>0.000</b>	0.163
3d30	PRD	0.796	<b>0.000</b>	<b>0.000</b>	0.506	0.748	<b>0.017</b>
	RRD	<b>0.033</b>	<b>0.004</b>	0.066	0.380	0.053	0.090
	PFA	<b>0.000</b>	0.064	<b>0.000</b>	<b>0.011</b>	<b>0.007</b>	0.314
3m15	PRD	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.857	0.760	0.847
	RRD	<b>0.010</b>	<b>0.001</b>	<b>0.001</b>	0.714	<b>0.016</b>	0.678
	PFA	0.891	<b>0.006</b>	0.184	0.182	0.938	0.251
3m30	PRD	<b>0.009</b>	<b>0.000</b>	<b>0.000</b>	<b>0.046</b>	0.458	<b>0.020</b>
	RRD	<b>0.012</b>	<b>0.016</b>	0.215	<b>0.008</b>	0.246	0.119
	PFA	0.192	<b>0.000</b>	<b>0.001</b>	0.155	0.198	0.812
9d15	PRD	<b>0.000</b>	<b>0.000</b>	<b>0.001</b>	0.991	<b>0.013</b>	<b>0.034</b>
	RRD	<b>0.008</b>	0.057	<b>0.018</b>	0.367	0.093	0.083
	PFA	<b>0.000</b>	<b>0.000</b>	<b>0.001</b>	0.336	<b>0.000</b>	0.477
9d30	PRD	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.358	<b>0.000</b>	0.055
	RRD	<b>0.000</b>	<b>0.004</b>	0.369	<b>0.001</b>	0.649	0.147
	PFA	0.381	0.409	0.283	0.615	0.734	<b>0.006</b>
9m15	PRD	0.306	<b>0.000</b>	<b>0.001</b>	0.589	<b>0.009</b>	0.341
	RRD	<b>0.005</b>	<b>0.023</b>	<b>0.000</b>	0.684	<b>0.010</b>	0.270
	PFA	<b>0.001</b>	<b>0.000</b>	<b>0.035</b>	0.118	0.166	0.836
9m30	PRD	<b>0.000</b>	<b>0.000</b>	<b>0.010</b>	0.099	<b>0.000</b>	<b>0.045</b>
	RRD	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	0.910	<b>0.000</b>	0.390
	PFA	<b>0.000</b>	<b>0.000</b>	<b>0.001</b>	<b>0.042</b>	0.384	0.570

\* Boldface value: significant effect

From the results in Tables 5 and 6, it is shown that the PRD demand is most influenced by  $F_{pt}$ . For nine-storey CRSBFs, the percent contribution of factor  $P_D$  is comparable to  $F_{pt}$  whereas, in the case of three-storey frames,  $P_D$  has smaller effects. Compared to  $F_{pt}$  and  $P_D$ , factor  $\sigma_{yf}$  has a smaller effect, which is

increased at higher earthquake intensities similar to the effect of  $F_{pt}$ . Two-factor interactions are active in the case of nine-storey frames.

From the results in Tables 5 and 6, it is also observed that the RRD response is influenced by several factors and their interactions. The RRD is decreased with reduction in  $\sigma_{yf}$ , whereas higher  $F_{pt}$  and  $P_D$  result in lower RRD. This finding shows the effects of  $\sigma_{yf}$ ,  $F_{pt}$  and  $P_D$  on the RRD of CRSBFs.

Further from the percent contributions in Tables 5 and 6, it is observed that PFA is influenced by several factors and their interactions similar to RRD. More notably, factors  $F_{pt}$ ,  $P_D$  and to a lesser extent,  $\sigma_{yf}$ , directly influence PFA. This is attributed to the fact that these factors positively correlate with the strength and stiffness of CRSBFs. In the case of PFA and RRD response variables, there are some active interactions present, such as  $P_D \cdot \sigma_{yf}$  and  $P_D \cdot F_{pt}$ . Further details are available in Moradi and Burton (2018).

Table 5: Percent contributions of most influential factors in three-storey frames

Case	Response variable					
	PRD		RRD		PFA	
	Factor	Contribution %	Factor	Contribution %	Factor	Contribution %
3d15	$F_{pt}$	-94.90	$\sigma_{yf}$	36.78	$E_f$	-32.06
	$P_D$	-3.20	$P_D \cdot \sigma_{yf}$	36.21	$\sigma_{yf} \cdot E_f$	-18.84
	$\sigma_{yf}$	1.18	$P_D$	10.69	$\sigma_{yf}$	14.49
			$P_D \cdot \alpha_f$	-9.02	$F_{pt} \cdot E_f$	12.64
					$F_{pt}$	8.92
					$P_D$	2.88
3d30	$F_{pt}$	-97.54	$F_{pt}$	-29.21	$\sigma_{yf}$	27.23
	$\sigma_{yf}$	1.14	$F_{pt} \cdot \sigma_{yf}$	-15.07	$\sigma_{yf} \cdot E_f$	26.74
	$F_{pt} \cdot \sigma_{yf}$	-1.11	$P_D$	-10.70	$P_D$	21.38
			$F_{pt} \cdot E_{pt}$	10.65	$E_f$	6.44
			$E_f$	-8.15	$\alpha_f$	5.49
			$\sigma_{yf}$	7.08	$P_D \cdot \sigma_{yf}$	-4.41
			$E_{pt}$	5.73	$F_{pt} \cdot \sigma_{yf}$	-3.26
			$P_D \cdot F_{pt}$	3.73	$F_{pt}$	2.11
3m15	$F_{pt}$	-85.21	$F_{pt}$	-28.92	$F_{pt}$	36.53
	$\sigma_{yf}$	7.60	$\sigma_{yf}$	26.12	$F_{pt} \cdot E_f$	18.65
	$P_D$	-4.99	$P_D \cdot F_{pt}$	11.87	$P_D \cdot F_{pt}$	-13.64
			$P_D$	-11.50	$\alpha_f$	-5.28
			$E_f$	9.30	$\sigma_{yf}$	5.23
			$F_{pt} \cdot \sigma_{yf}$	-5.41		
3m30	$F_{pt}$	-93.29	$\alpha_f$	-22.56	$F_{pt}$	87.75
	$\sigma_{yf}$	4.42	$P_D$	-19.72	$\sigma_{yf}$	5.04
			$F_{pt}$	-17.02	$F_{pt} \cdot \sigma_{yf}$	3.06
			$P_D \cdot E_f$	-13.90	$P_D \cdot F_{pt}$	-1.72
			$E_{pt}$	5.67		
			$P_D \cdot \alpha_f$	3.94		
			$\sigma_{yf}$	3.37		
			$E_f$	2.90		

Table 6: Percent contributions of most influential factors in nine-storey frames

Case	Response variable					
	PRD		RRD		PFA	
	Factor	Contribution %	Factor	Contribution %	Factor	Contribution %
9d15	$P_D$	-36.43	$P_D$	-23.68	$F_{pt}$	93.02
	$F_{pt}$	-34.03	$\sigma_{yf}$	16.69	$P_D$	4.61
	$\sigma_{yf}$	12.14	$P_D \cdot \sigma_{yf}$	-14.70	$P_D \cdot F_{pt}$	1.35
	$F_{pt} \cdot \sigma_{yf}$	-4.84	$F_{pt}$	9.18		
	$P_D \cdot \sigma_{yf}$	-4.18	$P_D \cdot E_f$	7.69		
	$E_f$	3.70	$E_{pt}$	7.24		
	$E_{pt}$	-2.34	$E_f$	6.71		
9d30	$F_{pt}$	-56.87	$P_D$	19.91	$P_D \cdot F_{pt}$	27.75
	$P_D$	-26.00	$P_D \cdot \alpha_f$	16.89	$E_{pt}$	25.41
	$P_D \cdot F_{pt}$	8.43	$\alpha_f$	13.54	$\alpha_f \cdot E_{pt}$	23.44
	$\sigma_{yf}$	3.15	$P_D \cdot F_{pt}$	-11.92	$P_D \cdot \sigma_{yf}$	-7.25
	$F_{pt} \cdot E_f$	-2.78	$F_{pt} \cdot \alpha_f$	-11.75	$E_f \cdot E_{pt}$	4.67
			$F_{pt}$	-7.43	$F_{pt} \cdot E_{pt}$	4.42
9m15	$F_{pt}$	-47.30	$\sigma_{yf}$	33.05	$F_{pt}$	62.35
	$F_{pt} \cdot \sigma_{yf}$	-18.32	$\sigma_{yf} \cdot E_f$	15.02	$P_D \cdot F_{pt}$	25.76
	$F_{pt} \cdot E_f$	-12.08	$F_{pt} \cdot \sigma_{yf}$	-14.24	$F_{pt} \cdot \sigma_{yf}$	6.21
	$\sigma_{yf}$	12.06	$P_D \cdot \sigma_{yf}$	-13.52	$P_D$	3.31
	$E_f$	5.92	$P_D$	-8.94		
			$E_f$	6.81		
			$F_{pt}$	-4.61		
9m30	$F_{pt}$	-65.87	$\sigma_{yf}$	26.16	$F_{pt}$	51.84
	$P_D$	-13.14	$P_D$	20.79	$P_D \cdot F_{pt}$	-17.47
	$P_D \cdot F_{pt}$	11.36	$F_{pt} \cdot \sigma_{yf}$	16.13	$P_D$	15.48
	$F_{pt} \cdot E_f$	-4.91	$\sigma_{yf} \cdot E_f$	14.46	$\sigma_{yf}$	9.55
	$F_{pt} \cdot \sigma_{yf}$	-2.06	$F_{pt}$	-9.84	$\alpha_f$	1.59
	$E_f$	1.68	$P_D \cdot \sigma_{yf}$	-6.70		
			$E_f$	5.23		

## 5 Conclusions

This paper summarizes a research on sensitivity analysis of controlled rocking steel braced frames (CRSBFs) with varied storey height and bay width at two seismic hazard levels. Nonlinear response history analyses of two-dimensional frames are performed on several cases of three- and nine-storey frames subjected to a suite of thirty-one earthquake records. A design of experiment method is used to statistically and efficiently evaluate the significance of design factors on the seismic demand parameters, including peak roof drift (PRD), residual roof drift (RRD), and peak floor acceleration (PFA). The design factors considered in this study are related to the PT strand and fuse properties (including initial PT force per strand, PT strands modulus of elasticity, fuse yield stress, fuse strain hardening ratio, and fuse modulus of elasticity) as well as the rocking column gravity load. The results show that the PRD is mostly influenced by initial post-tensioning strand force and rocking column gravity load, while RRD and PFA are sensitive to several factors and interactions. The results also indicate that fuse strain hardening and PT strands modulus of elasticity do not significantly influence the response variables of PRD, RRD, and PFA. The results of this sensitivity study are used for an ongoing optimization study of CRSBFs seismic response.



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