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OPTIMIZATION OF STEEL UNBRACED FRAME SYSTEMS IN MULTI-STOREY BUILDINGS

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Abstract: A Particle Swarm Optimization algorithm (PSO) was used to find the optimal design of unbraced frame configuration systems in multi-storey buildings. The unbraced frames were optimized by minimizing the weight of their members while respecting the design requirements of the National Building Code of Canada (NBC 2010) and the Canadian standard for the Design of Steel Structures (CAN/CSA S16-09). The algorithm finds the optimal member sizes of the lateral force resisting system of a rectangular building consisting of four unbraced frames. Since a symmetrical rectangular building is implied, only two distinct frames are optimized, perpendicular to one another. The frames with the smallest weight are found by selecting appropriate sections from a commercially available set of wide flange steel sections. The algorithm accounts for the serviceability and strength constraints as specified in CSA S16. Two major changes to the standard PSO were used in this study. During the iteration process, a fly-back method was implemented for members that do not meet all design criteria. Furthermore, the particle velocity of the PSO is limited to a minimum value to avoid premature stagnation of the solution. This prevents the PSO from being stuck in a non-optimal solution. For a four-story building and 100 trials, the difference in weight between the optimal and worst solutions obtained by the PSO is less than 2.2% and 2.5% for the two frames, respectively. The weights of the best solution obtained by the algorithm for the two frames were validated with a commercial software.

1 INTRODUCTION

In recent years, the use of heuristic algorithms for the design of structures has become a more common practice. The Particle Swarm Optimization (PSO) algorithm is an optimization method to solve complex optimization problems. This algorithm was first introduced by Kennedy and Eberhart (1995). Furthermore, Kennedy and Eberhart (2001) demonstrated that this type of algorithm converges to the optimal solution quicker than other types of heuristic algorithms at that time. The PSO algorithm is based on the social behaviour of groups of animals such as bees, insects and birds. When these groups of animals and insects are looking for food, there is normally a leader who influences the other group members. In addition, each member is able to memorize its position in the group. The displacements made by each member of the group are based on their personal knowledge and the behaviour of the other group members. The PSO algorithm operates in a similar manner. Many studies have used the PSO algorithm for different types of optimization problems, including design optimization of structures. In addition, Eberhart and Shi (2001) showed the developments and potential areas of application of the PSO algorithm.

Even though the PSO algorithm was developed for specific applications such as the optimization of a mathematical function, it was used by Fourie and Groenwold (2002), Schutte and Groenwold (2003) and

Perez and Behdinan (2007) to optimize members of truss structures. The PSO algorithm was also used for the design of different types of structures. Lefrançois et al. (2011) used the algorithm to optimize composite and non-composite floor systems. The purpose of their study was to minimize the cost of each floor configuration. They demonstrated that the algorithm was effective and found that the PSO algorithm regularly found the optimum solution for each floor configuration.

Saka and Kameshki (1998) used a genetic algorithm for the design of multi-stories steel frames subjected to multiple loading cases. The design included serviceability and strength constraints according to the British Standards BS5950. Lateral torsional bucking of beam columns was also considered for the design of these frames. Kameshki and Saka (2001a) used a genetic algorithm to optimize multi-storied steel frames with semi-rigid connections. A nonlinear empirical model was used to include the moment–rotation relation of beam-to-column connections. Kameshki and Saka (2001b) also used a genetic algorithm based optimum design method for multi-storied non-swaying steel frames with different types of lateral load resisting systems. The design method obtains a lateral load resisting system with the least weight by selecting appropriate sections for beams, columns and bracing members from a standard set of steel sections. The algorithm accounts for serviceability and strength constraints. A similar type of genetic algorithm was used by Kameshki and Saka (2003) for nonlinear multi-storied steel frames with semi-rigid connections. The algorithm accounts for the effect of the flexibility of the connections with a polynomial model and for the geometric non-linearity of the members.

Camp et al. (2005) performed the optimization of steel frames using an Ant Colony Optimization (ACO) algorithm. The frame is minimized with a penalty function to enforce strength and serviceability constraints. A comparison was presented between the ACO frame designs and designs developed using a genetic algorithm and classical continuous optimization methods. Saka (2009) used a harmony search method algorithm for the optimum design of unbraced steel frames. Member grouping was used so the same section can be adopted for each group. Again, the selection is carried out so the design limitations are satisfied while minimizing the weight of the steel frame. Kaveh and Talatahari (2010) proposed a two-phase ACO algorithm to optimize unbraced steel frames. The first phase was executed with a sub-optimization mechanism to reduce the search space and the computation time of the algorithm. During the second phase, searches were concentrated on the regions neighbouring the best solution obtained in the first phase. They compared their results with those of a conventional ACO, a genetic algorithm and a harmonic search algorithm. They demonstrated that the results of the algorithm were achieved faster than other methods.

The design of unbraced frame configurations using a PSO algorithm was presented by Dogan and Saka (2012). The purpose of their study was to minimize the weight of the members of each unbraced frame that met the requirements of the American Institute of Steel Construction (AISC) Load and Resistance Factor Design standard (LRFD). They demonstrated that the PSO algorithm was an effective method for the design of rigid frames.

All these studies have used heuristic algorithms for the design of swaying or non-swaying two dimensional frames that are part of a building structure. They do not incorporate in their optimal design algorithms the three-dimensional lateral load resisting system for the whole building in one optimization calculation. For this study, a PSO algorithm is used to optimize a complete lateral load resisting system for a rectangular type building consisting of unbraced frames on each side of the building and where two symmetrical frames are facing each other. There is no restriction on the building's number of floors. The design satisfies all the requirements of the National Building Code of Canada (NBC 2010) and the Canadian Steel Construction Standard (CSA S16-09). Lateral load resisting systems ensure stability when the building is subjected to wind loads and earthquakes. Unbraced frames are often used but are limited to low rise and medium height buildings since they are usually associated with higher lateral deformation compared to concentric braced frames. However, they can be very effective to dissipate energy in high seismic areas. One of the difficulties with unbraced frames is the iterative nature of the calculations and design since the inner forces in the members are a function of the size of the members themselves. The design of these lateral load resisting systems are thus time consuming. As a result, they are often oversized to reduce the computational time and an optimum design is seldom obtained. An effective optimizing algorithm will provide a physically realizable solution where additional constraints can be specified by the user (e.g. minimum and maximum size of members, classes and types of members, etc.). This is easily implemented in the PSO algorithm.

2 DESIGN PROBLEM

The design of a building's steel lateral load resisting system is a potentially long and complex process that involves many calculation steps. This process can become even more challenging when trying to get an optimized solution. For this project, the procedure can be divided into six stages, namely the identification of the geometric parameters of the building, the identification of relevant climate data, the identification of constraints, the analysis of potential solutions and finally the optimization of the solution.

2.1 Building's geometric parameters

For this study, a typical building of rectangular shape was used as shown in Figure 1. This form is commonly used for commercial and industrial buildings. The locations of the unbraced frames are illustrated by the dotted lines. The number of stories is specified by the user and may differ from what is shown in Figure 1 (four stories were used for this work).

The user must specify the general data of the building in a file that can be read by the algorithm in order to start the optimization process. The data specified for this study are presented in Table 1. The last column shows the values used for this study while the previous column shows the range of values that can be used in the algorithm. The user is free to impose the size and number of stories of the building. However, an even number (e.g. 4, 6, 8, etc.) of frames must be specified for the lateral load resisting system configuration. This means that for a building with more than four frames, the user must ensure that the frames are positioned so there is symmetry between them. For this study, four unbraced frames were imposed, one for each side of the building. Other parameters to be provided are the length of the frames, the orientation of the frames, the type of members (i.e. beam or columns) and the type of restraints (i.e. free or restrain). Furthermore, for this work, all columns are considered unique sections. They can be different sizes per story and only two different perpendicular seismic load directions are considered. This condition was imposed in order to validate the robustness of the algorithm.

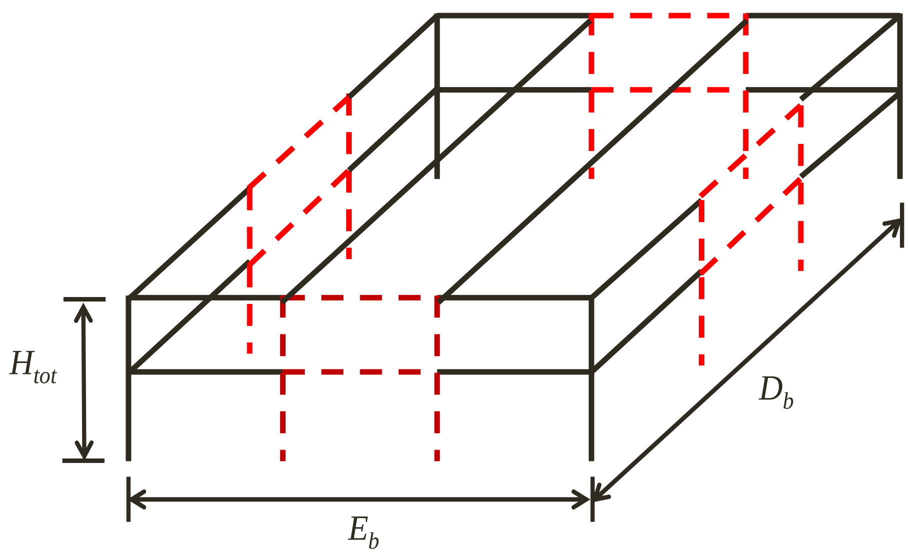

Figure 1: Unbraced steel frames of a typical commercial or industrial type building

Table 1: Geometric parameters of the building

Building parameters	Allowed values	Used values
Length, D_b	All	30 m
Width, E_b	All	21 m
Height, H_{tot}	All	16 m
Number of stories (+1)	All	5
Number of frames	Even number	4
Frame length, L_{CD}	≤ D_b	10 m
Frame length, L_{CE}	≤ E_b	7 m

The National Building Code of Canada (NBC 2010) requires that all lateral load resisting systems designed to resist earthquake forces must take into account their ability to dissipate seismic energy. This requirement is satisfied through dissipation coefficients, R_d and R_o. Each possible level of ductility is defined by a different combination of R_d and R_o values. For example, the lateral load resisting system shown in Figure 1 has four possible levels of ductility. These levels are ductile (R_d = 5 and R_o = 1.5), moderately ductile (R_d = 3.5 and R_o = 1.5), limited ductility (R_d = 2 and R_o = 1.3) and low ductility (R_d = 1.5 and R_o = 1.3). Low ductility represents conventional constructions which can be used for all lateral load resisting systems. Since there are fewer requirements specified in CSA-S16 for this case (e.g. capacity design of members is not required), they are often used by engineers for the design of lateral load resisting systems. For this study, the algorithm was tested by considering the level of ductility corresponding to conventional constructions. In future studies, the algorithm will be modified to consider other ductility levels with the requirements (i.e. capacity design) that accompany them.

2.2 Loads

Loads can be divided into two categories, gravity loads and lateral loads. The gravity loads include dead loads (D), live loads (L) and snow loads (S) while the lateral loads include wind loads (W) and earthquake loads (E). The earthquake loads are often responsible for higher forces in a building's lateral load resisting system than wind loads. Earthquake loads were calculated by the algorithm as required by the NBC 2010. For the unbraced system example used in this study, wind loads did not control any part of the design.

There are two unbraced frames analyzed by the algorithm in this study, one on the E_b face of the building and one on the D_b face of the building. Load types on the beams forming the unbraced frames were either uniform distributions (w_d, w_l, w_s) or concentrated loads (P_d, P_l, P_s), depending on the type of floor system. The load distribution for each configuration is illustrated in Figure 2. In both configurations, part of the loads from adjacent bays can be transferred directly to the frame columns by specifying additional axial loads at each level of the frames (P_{ad}, P_{al}, P_{as}, P_{bd}, P_{bl}, P_{bs}).

The equivalent static seismic load method defined in the National Building Code of Canada (NBC 2010) was used to determine the lateral loads on the building. The lateral seismic loads are calculated for each floor level where the relative stiffness of each frame, the centre of gravity relative to the centre of rigidity and the required minimum eccentricity limit were all considered in the calculations. Lateral earthquake loads obtained by the algorithm were distributed on each floor of the frames according to the scheme of Figure 3. Furthermore, fictional lateral loads can be specified to account for a structure's erection defects for the dead, the live (floor levels) and the snow loads (roof level).

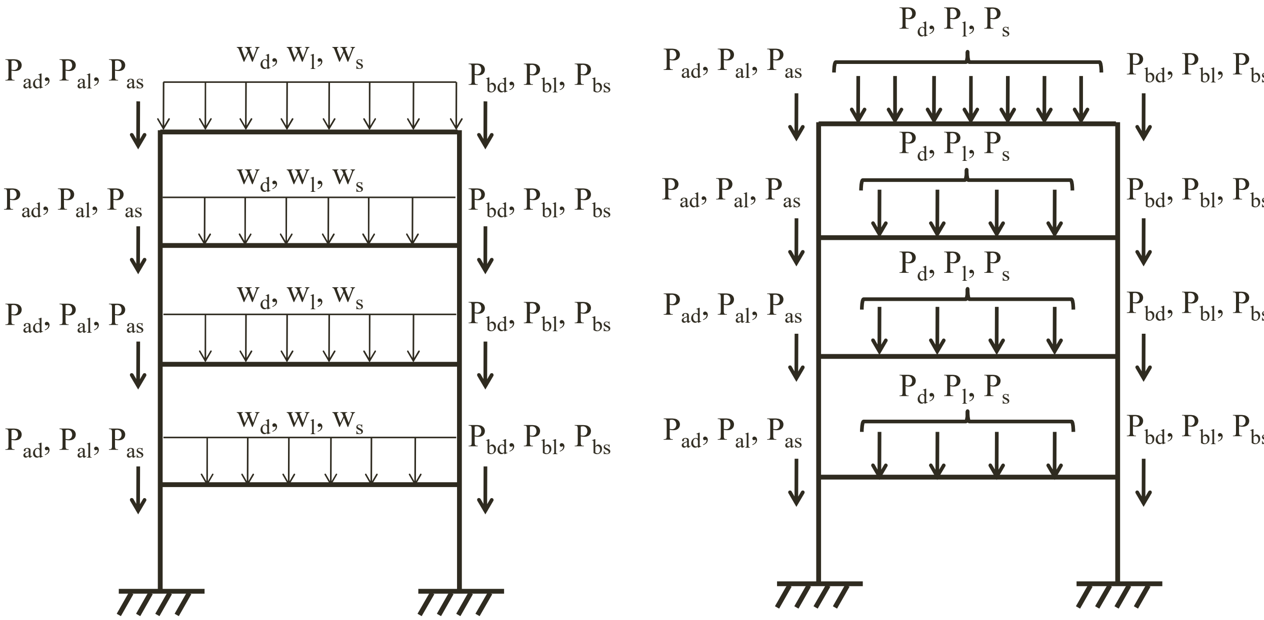

Figure 2: Gravity loads on frames with a) uniform loads on beams and b) concentrated loads on beams

2.3 Analysis and design

A stiffness analysis method (matrix method) is used to determine the critical internal forces in the members of the lateral load resisting system. Initial selection of the members is required before performing the analysis of the structure. Two distinct steel section lists were used by the algorithm for this project, one for the columns and another one for the beams of the frames. The steel sections are taken from the tables of the CISC Handbook of Steel Construction (2011). The user can specify the minimum and maximum sizes (or nominal masses) of the beams and columns beforehand in order to create the available steel section list. For the initial iteration of the algorithm, the largest available sections from the section list are used for

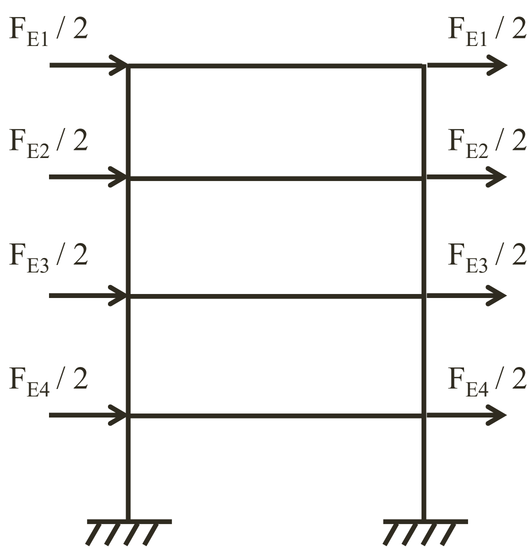

Figure 3: Lateral earthquake load distribution

each beam and column. The sections selected are then used to make a first analysis of the frames. The load combinations are created according to the requirements of the NBC 2010.

Members are designed to meet all the requirements of the CSA-S16 standard for ultimate and serviceability limit states. The ultimate limit state requirements demand that the resistance of the members must be higher than the critical factored internal forces for all load combinations. These requirements include shear resistance (V_r), compression resistance (C_r), tension resistance (T_r), flexural resistance (M_r), combined compression and flexural resistance as well as combined tension and compression resistance.

The serviceability limit state requirements imply that the lateral displacement of each floor due to seismic loads is limited to an acceptable amount (i.e. h/40). The maximum deflection of beam members is also controlled for live and snow loads (i.e. L/360). Furthermore, a slenderness limit is used for all members (i.e. L/200) and each member's class is also limited depending on the resistance criteria to verify.

3 PARTICLE SWARM OPTIMIZATION ALGORITHM (PSO)

For the lateral load resisting system of this study, the search domain for the PSO algorithm is a list of steel sections chosen by the user. The solution is found by the algorithm among the list of steel sections for each member forming the lateral load resisting system.

3.1 Basic equations

The particles of a PSO algorithm are mathematical constructs, having three main parameters: position, velocity and fitness. The position represents the unknown variables of the problem, the velocity determines the rate of change of the position, and the fitness is a measure of how well the particle solves the optimization problem. For all iterations, the PSO algorithm is based on only two equations: one to modify the velocities of the particles (Equation 1) and the other one to change their positions (Equation 2).

$$[1]\ \overrightarrow{v_{k+1}^{i}} = \omega_k \overrightarrow{v_k^{i}} + c_1 r_1 \left(\frac{p_k^i - x_k^i}{\Delta t} \right) + c_2 r_2 \left(\frac{p_k^g - x_k^i}{\Delta t} \right)$$

$$[2]\ x_{k+1}^{i} = x_k^i + \overrightarrow{v_{k+1}^{i}} \Delta t$$

In these equations, the index k represents the iterations, v is the velocity of the particle, p^i is the particle's best position since the beginning of the calculation, p^g is the best overall position of all particles of the swarm, ω is the inertial weight, and Δt is a unit time step. The variables r_1 and r_2 represent random numbers between 0 and 1. The variable c_1 is a coefficient controlling the cognitive behaviour, indicating how much a particle trust itself (i.e. local best) and c_2 is a coefficient controlling the collective behaviour, indicating how much a particle has confidence in the group (i.e. global best). To select these values, extensive testing is required with the algorithm to find the best combination. Furthermore, it is recommended to choose coefficients consistent with the equations of Ruben and al. (2007) to ensure convergence of the algorithm.

The inertial weight ω balances the current velocity against the local and global bests and his usually smaller than 1.0 in order to reduce the velocity of the particles during the iteration process. For this problem, a linear reduction method (Perez and Behdinan (2007)) was used. This prevents the particle from oscillating around the optimum point. This method is applied using Equation 3.

$$[3]\ \omega_{k+1} = \omega_{max} - \left(\frac{\omega_{max} - \omega_{min}}{k_{max}} \right) k$$

In Equation 3, ω_{max} is the maximum inertial weight (initial value), ω_{min} is the minimum inertial weight (final value), k_{max} is the maximum number of iterations and k is the number of the current iteration. The new position x^i_{k+1} is part of a continuous domain. However, steel sections are identified by using discrete numbers. Therefore, the new positions of the particles are rounded to its closest integer value.

3.2 Objective function

The objective function is the total weight of the unbraced frames. Since all the beam to column connections of the unbraced frames are of the same type (moment connections), the extra weight for these connections are not included and does not influence the results. The function, represented with Equation 4, has two variables which are the total weights of the sections constituting the two unbraced frames.

$$[4]\ f(x^1) = \sum_{i=1}^{n} x_i^1 \quad , \quad f(x^2) = \sum_{i=1}^{n} x_i^2$$

A particle is a potential solution of the lateral load resisting system. It includes all the steel sections for all the members forming the two unbraced frames. Given that the steel sections are selected individually by the algorithm, each member of the lateral load resisting system has a velocity and a position that are modified using Equations 1 and 2 during the iteration process.

The algorithm starts with a user-defined population size N (i.e. the number of initial solutions to be evaluated). All beams and columns forming the two unbraced frames are taken from the section lists where the heaviest sections in both lists (beams and columns) are assigned to all initial frames. The frames are then verified to determine if they satisfy all the ultimate and serviceability states of CSA S16. The algorithm won't start the optimization process if any of the requirements are not met. This method has been used to simplify the programming of the algorithm as it provides a solution that must respect all the constraints and design criteria as well as record the best initial solutions (i.e. p^i and p^g). For subsequent iterations, potential solutions are adjusted by the algorithm using Equations 1 and 2 as well as the imposed constraints. The population size, the initial particle velocities and the number of iterations follow the recommendations from several studies, including those of Kennedy and Eberhart (1995) and Shi and Eberhart (1998 and 1999). A minimum particle velocity is used to prevent stagnation of the particles which can result in obtaining a solution that is not optimized. The minimum velocity is applicable in all the directions taken by the particles (i.e. positive and negative displacements). The PSO algorithm parameters used in this study are: N = 20, $k_{\max}$ = 500, c_1 = 0.9, c_2 = 1.1, ω_{min} = 0.6, ω_{max}= 0.9, $v_{\min}$= ± 0.5.

3.3 Constraints

At the first iteration, the initial solutions were analyzed and compared with the imposed constraints. The execution of the algorithm will automatically stop if the slenderness, the class or any strength criterion (e.g. $M_f > M_r$) of a section of the unbraced frames does not meet the specified requirements. When this happens, the user must change the list of eligible sections in order to guarantee an acceptable solution.

During the iteration process, the algorithm calculates the weight of each unbraced frame (i.e. each particle) that satisfies all the imposed constraints and determines if they are the best solutions obtained to date. The best solutions obtained beforehand are replaced by better solutions found during subsequent iterations. Furthermore, the best overall solution is also determined by the algorithm. During the iteration process, the best overall solution previously obtained is replaced by a better solution when one is found. However, when the solutions don't meet all the specified requirements, coefficients assigned individually to each member of the unbraced frames are used. The coefficient of a member, whose initial value is zero, is increased by one each time a requirement is not met. Each section whose coefficient has a value other than zero is replaced by the corresponding section of the best solution previously obtained for that particle. The particle velocities of the modified sections are reduced by a reduction factor FR (e.g. 0.99). In addition, the modified solutions are analyzed and compared with the requirements once again before proceeding with the next iteration. The modified solutions that meet all requirements are handled in the same way that was presented earlier while the modified solutions that still do not meet all the requirements are rejected.

4 RESULTS

The best solutions are those which have the lowest weight while satisfying all design requirements. The weights of the potential solutions are initially high but they gradually diminish during the iteration process of the algorithm. It takes approximately 300 to 400 iterations before the best solution is found. To validate the algorithm's effectiveness, it must be executed repeatedly. This will verify if the algorithm can regularly get the same solution or a solution that almost has the same weight. For this project, the algorithm was run 100 times and a basic statistical analysis was performed. The minimum weight obtained by the algorithm was registered and the results are presented in Table 2. The weight of the best solution found by the algorithm is 47.99 kN for the unbraced frame E_b and 69.73 kN for the unbraced frame D_b. The difference between the weight of the best solution and the weight of the worst solution is less than 1.1 kN (2.2%) for the unbraced frame E_b and about 1.7 kN (2.5%) for the unbraced frame D_b. The average weight found is 48.52 kN and 70.50 kN with standard deviations of 0.24 kN and 0.62 kN, respectively.

Table 2. Results from 100 trials for the unbraced frames

Unbraced frame E_b		Unbraced frame D_b	
	Weight (kN)		Weight (kN)
Best	47.99	Best	69.73
Worst	49.02	Worst	71.39
Mean	48.52	Mean	70.50
Standard deviation	0.24	Standard deviation	0.62

The effectiveness of the algorithm is verified by determining whether the final solution obtained by the algorithm is the optimal solution. Since unbraced frames are indeterminate structures, the results of the analysis are a function of the section type of the members. When a member's section changes, the results of the analysis will change and the design requirements that have been all satisfied beforehand may now be non-satisfactory. Since there is an enormous amount of possible configurations of member sections, finding the absolute optimal solution cannot be guaranteed. Commercial structural programs also have this problem. However, an attempt to validate the results with commercial software was performed. A 3D model of the same building was created and analyzed. The same unbraced frame members (i.e. the sections constituting the best solution) found by the PSO algorithm was used (i.e. specified by the user). The best solution for the unbraced frames E_b and D_b are presented in Figure 4.

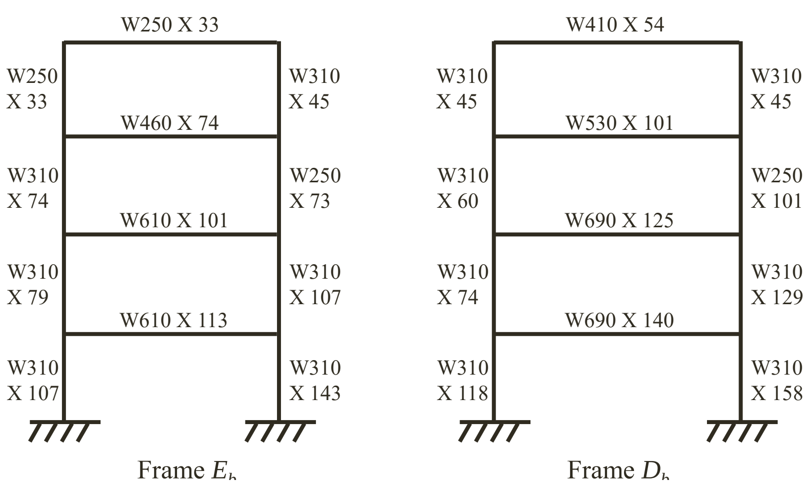

Figure 4: Sections constituting the best solutions for the unbraced frames E_b and D_b

The maximum solicitation ratios (C_f / C_r + M_f / M_r) of the selected sections for the two unbraced frames and their critical load combinations are presented in Table 3. The ratios of the sections selected by the PSO algorithm were generally greater than 90% for the beams and for most of the columns. For the unbraced frames obtained by the PSO algorithm, only two sections per frame can be replaced by smaller sections and still be an acceptable solution.

Table 3: Maximum solicitation ratios (C_f / C_r + M_f / M_r) of the selected sections

Frame E_b			Frame D_b		
Sections	Maximum ratio	Critical load combination	Sections	Maximum ratio	Critical load combination
W310x107	0.98 *	1.0D+1.0E	W310x118	0.82	1.0D+1.0E
W310x79	0.99 *	1.0D+1.0E	W310x74	0.87	1.0D+1.0E+0.5L+0.25S
W310x74	0.92	1.0D+1.0E	W310x60	0.81	1.0D+1.0E+0.5L+0.25S
W250x33	0.59	1.0D+1.0E	W310x45	0.83	1.25D+1.5S+0.5L
W250x33	0.94	1.0D+1.0E+0.5L+0.25S	W410x54	0.90	1.0D+1.0E+0.5L+0.25S
W460x74	0.98	1.0D+1.0E+0.5L+0.25S	W530x101	0.99	1.0D+1.0E+0.5L+0.25S
W610x101	0.95	1.0D+1.0E+0.5L+0.25S	W690x125	0.98	1.0D+1.0E+0.5L+0.25S
W610x113	0.98	1.0D+1.0E+0.5L+0.25S	W690x140	0.95	1.0D+1.0E+0.5L+0.25S
W310x45	0.73	1.0D+1.0E+0.5L+0.25S	W310x45	0.83	1.25D+1.5S
W250x73	0.93	1.0D+1.0E+0.5L+0.25S	W250x101	0.97	1.0D+1.0E+0.5L+0.25S
W310x107	0.97	1.0D+1.0E+0.5L+0.25S	W310x129	0.95	1.0D+1.0E+0.5L+0.25S
W310x143	0.97	1.0D+1.0E+0.5L+0.25S	W310x158	0.97	1.0D+1.0E+0.5L+0.25S

Note: Maximum solicitation ratios mark with an * are for Tf / Tr + Mf / Mr.

They are the columns supporting the roof and have a solicitation rate of 0.83 or less. When these sections were changed in the commercial software, the next smaller sections available in the list of columns passed all of the requirements. However, only one section can be changed at a time on frame E_b since the internal forces of both sections are affected by these modifications. For this example, serviceability limits did not control the design of the sections.

5 CONCLUSION

A PSO algorithm was developed to obtain the optimal design of a steel lateral load resisting system according to the requirements of CSA S16-09, the National Building Code of Canada (NBC 2010) and any other constraints imposed by the designer (e.g. dimensions and classes of the sections).The results have determined that by using the appropriate parameters, it was possible to obtain a solution that can be considered an optimal solution. With 100 trials, the difference in weight between the best solution obtained by the PSO algorithm and the worst solution is less than 1.7 kN and the standard deviation is less than 0.62 kN. Furthermore, the efficiency of the PSO algorithm was validated by the solicitation rates of all member sections for each load combinations. The maximum solicitation rate of the members was generally greater than 90%, except for a few sections. An innovative approach used in this study for minimizing the effect of stagnation of the algorithm is the use of a minimum velocity and a reduction factor. Since the sections are referenced by integers, the minimum velocity allowed the particles to continue searching the domain once the velocities of the particles were too small (i.e. lower than 1). Furthermore, a fly back process was implemented to each individual section of the unbraced frames that did not satisfy all the design requirements. The reduction factor was then used on the velocities of those sections. These slower moving particles would try to find the best solution at the next iteration, increasing the likelihood of obtaining a feasible solution. The combination of the fly-back process and the velocity reduction factor favoured a better convergence of the PSO algorithm. Future studies will include a seismic dynamic analysis and design requirements including the capacity design requirements of the CSA S16-09 standard.

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