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MODAL ANALYSIS AND DAMPING OF BRIDGE LIGHT POLES

Desjardins, Serge L.[1,3], Poitras, Gérard J.[1] and McGinn, Donald[2]
[1] Département de génie civil, Faculté d'ingénierie, Université de Moncton, Moncton, NB, Canada
[2] Strait Crossing Bridge Ltd., Borden-Carleton, PE, Canada
[3] serge.desjardins@umoncton.ca

Abstract: This study aims to provide a better understanding of the dynamic behaviour of light poles installed on bridge decks subjected to significant winds and to propose a damping solution to attenuate the large amplitude vibrations that contribute to fatigue failure. The subject of this study is the light poles installed on the Confederation Bridge in eastern Canada. The unique nature of this 12.9 km bridge and its location in the Northumberland Strait provides an opportunity to study light pole vibrations in harsh environmental conditions. The full-scale vibration testing of a light pole was carried out on the deck of the Confederation Bridge. Four wireless tri-axial accelerometers installed on the light pole captured the free vibrations of the pole subjected to pull and release excitations. Results were obtained via two experimental modal analysis techniques: the free vibration method (in the time and frequency domain) and a variant of the stochastic subspace identification technique. The extracted modal properties were compared to the analytical modal properties obtained from structural component and finite element modelling. The experimentally obtained modal properties such as frequency, and mode shapes, correlate well with those obtained analytically (within 1.4%). To attenuate the large amplitude vibrations, the second part of the study focused on pole-top impact dampers by comparing the damping performance of a custom developed prototype impact damper and a commercial option. A prototype impact damper using two lead balls within a pipe cylinder added to the top of the pole was found to exhibit comparable damping behaviour to the commercial damper under large displacements.

1 INTRODUCTION

The study is undertaken to better understand the dynamic behavior of the light poles installed on the deck of the Confederation Bridge and to propose a damping solution to attenuate the large amplitude vibrations observed during significant wind events. The objectives of the current study are the proper identification of the modal properties of light poles currently installed on the bridge, and the identification of a cost-effective solution to attenuate the pole vibrations during strong wind events, and thus extend the remaining service life of these important bridge components.

1.1 Pole dynamics

Light poles are simple vertical cantilever structures that vibrate under certain excitations conditions. They vibrate in different modes at different frequencies. Natural wind is the most common excitation force for these pole structures when installed on bridges. Once the excitation force is removed, the vibration decays and the pole structure returns to rest. The rate of this decay is the damping coefficient or damping ratio. Very tall and slender structures such as light poles have inherently low damping coefficients. When certain conditions exist, these poles may vibrate strongly for long periods of time.

The most significant effect of these vibrations is the inducement of stresses at the pole base. The greater the vibration amplitude (or pole-top displacement), the greater the stress. When these stresses are continually repeated over time, they represent fatigue stresses which may lead to stress cracks and premature failures of the light poles. It is therefore advantageous, in the case of fatigue stress, to either limit the amplitude of vibrations or to limit the number of high amplitude cycles or both. For the modal analysis of cantilever poles, it is important to distinguish between first mode vibrations and second mode vibrations. First mode vibrations, more commonly referred to as 'pole sway', starts at moderate wind speeds, but when gusts occur at very high wind speeds, more violent behavior such as whipping, and pulsing may occur resulting in very high repetitive stresses at the base. First mode vibration movement occurs generally in the direction of the wind. The second mode vibrations are caused by vortex shedding, where small eddies are alternately spinning off the sides of the pole under moderate wind (12 to 32 km/h) (Briden 2016). When the vortex detaches off the pole, it creates a pressure collapse that drives the pole in the direction of the vortex. This happens continually and alternately on both sides of the pole normal to the wind direction. Second mode vibration therefore occurs at a 90° angle to the direction of the wind.

The service life of light poles installed in wind prone areas is not typically measured temporally, but rather by the number of stress cycles it experiences while in service. According to fatigue theory, the higher the stress amplitude of the cycles of oscillations, the lower the number of cycles until the end of service life (Hosch and Fouad 2009, Lee and Taylor 2004, AASHTO 2015). The service life of poles can be increased (sometimes significantly) by limiting the high amplitude stress cycles. Conversely, light poles can suffer sudden failure during a single event if extreme oscillations occur.

1.2 Background

Wind accelerations as it flows over the 15 m deep structure of the Confederation Bridge and the absence of pole foundation damping result in a system sensitive to wind-induced vibrations. Light pole failures have occurred during several strong wind events, the strongest of which had sustained winds of 126 km/h with gusts of 151 km/h. During these strong wind events, a few poles experienced structural failures and following these events, during special inspections, more poles were replaced as a precautionary measure as they showed signs of structural weakness such as surface micro cracks and/or surface discoloration at the pole-base interface. The new replacement poles have an increased wall thickness and an improved weld detail at the base plate interface.

From anecdotal evidence, the pole movements observed during very strong wind events on the Confederation Bridge, were generally in the direction of the wind and correspond with first mode vibration (sway) with the maximum displacements occurring at the top of the pole (and maximum stresses at the pole base). From this anecdotal evidence, the second objective of the current study focuses on the first mode vibrations of the bridge light poles. The proposed solutions to attenuate these vibrations focus on pole top impact dampers which, according to literature, is very effective in reducing the effects of first mode vibrations. Pole top impact dampers have shown to reduce displacement amplitudes and are very effective in reducing the number of high amplitude vibrations (Caracoglia and Jones 2007, Cook, et al. 2001, Cheng and Xu 2006). Section 6 describes the dampers considered for this study.

A previous internal study carried out in 2002 focused on the second mode vibrations of the Confederation Bridge light poles (Lau, Chang and Londono 2002). This study was mandated after repeated occurrences of premature light bulb and fixture failures (incandescent) were thought to be due to pole vibrations in the second mode. The subsequent technical report stated that the rubbing-pipe "chain" configuration is the most effective in increasing the damping ratio for the 2nd mode vibrations. Following this study, all light poles on the bridge were outfitted with a rubbing pipe type chain damper and since, the rate of light fixture failures has drastically diminished.

2 ANALYTICAL MODELLING

To better understand the vibration characteristics of the light pole and to verify the experimental modal analysis results, two analytical models of the cantilevered pole structures were created. First a simple structural component model (SCM) was created which idealized the pole cross-section and was analysed in a structural analysis software program. Second, for added precision, a finite element model (FEM) of the light pole was created and analysed within a finite element analysis program.

2.1 Structural component model

The structural analysis software program SAFI® was used to create a simple structural component model and to perform a frequency analysis. The structural component model of the light pole was created based on the manufacturer's shop drawings. The structure was modeled as a variable section cantilever beam with a fixed end support. The octagonal cross section of the light pole was idealized with a circular cross section with an outside diameter equal to the across flats (A/F) dimension of the light poles. The 9.1 m length was subdivided into 200 elements of linearly varying diameters for improved mode shape resolution. Table 1 summarizes the poles physical properties that were used as input parameters for the structural component model. The current LED light fixtures were installed on the light poles in December 2011. The LED light fixture was weighed and measured in the laboratory and is modeled as an eccentric load at the top of the light post. Results of the frequency analysis are shown in Table 2.

Table 1: Physical properties of the light pole for structural modelling

	Light pole
Length (mm)	9100
Wall thickness (mm)	3.038
A/F base (mm)	203
A/F top (mm)	83
Steel	300W
Light fixture weight (kg)	13.23
Light fixture lever arm (mm)	290

2.2 Finite element model

A three-dimensional model of the light pole was also created, and a finite element analysis was carried out using Solidwork® commercial software. To ensure convergence of the results, the mesh control option was used. The runs were realized with increasingly fine mesh sizes until constant results were obtained (43064 solid mesh elements, 4 Jacobians points). The final mesh control size used was set at 30 mm for the pole and 10 mm for the light fixture (60 mm diameter tubing, 3 mm thick, 331.5 mm in length from the center of the pole and weighs 1.25 kg). A load of 11.98 kg was placed at the end of the light fixture tubing (center of gravity of the light) for a total mass of the light and light fixture of 13.23 kg. An elastic support was used at the pole base in the vertical direction (740 MN/m) calibrated to match the experimental results obtained on site since perfect fixity at the pole base is not always achievable. In fact, when using a perfectly fixed base, the finite element analysis results in a first mode frequency of 2.04 Hz compared to 1.92 Hz obtained experimentally. This type of behaviour was also found in the laboratory setting when varying the base fixity of the pole to the strong floor. Modal results of the finite element analysis are presented in Table 2.

3 FULL SCALE TESTING

The full-scale testing of a light pole was carried out on the deck of the Confederation Bridge. This section describes the site setup, the instrumentation and the manipulations. Prior to site testing, verifications of the equipment and manipulations was carried out on a truncated pole of 6.1 meters mounted to a strong floor in a laboratory environment.

An original light pole on the north side of the bridge deck approximately 5 km from the New-Brunswick shore was identified for testing. For this study, the light pole was instrumented with four tri-axial accelerometers. While damping of the first mode is of primary interests, the accelerometers are positioned to properly capture the first four modes of vibration while avoiding modal node positions. Sensors are numbered 1 to 4 starting at the top (Figure 1). The measured x-direction is in-line with the luminaire and follows the pull direction. The z-direction is horizontal and normal to the x-direction, y-direction is upwards.

The installed sensors are wireless tri-axial MEMS accelerometers with ±2g measurement range and 16-bit resolution by LORD Microstrain (GLINK2™-LXRS®). This wireless setup permits quick and efficient installation of the sensors on the light poles on site and in the laboratory. Custom removable mounting brackets were created to facilitate the installation of the accelerometers to the light poles using band clamps. The data acquisition was controlled by the specialized software by LORD Microstrain (Node Commander®). Data capture is synchronous across sensors at a sampling rate of 128 Hz. Data was captured in datasets of 60 second duration and stored in an ASCII file for further data processing and analysis described in section 4.

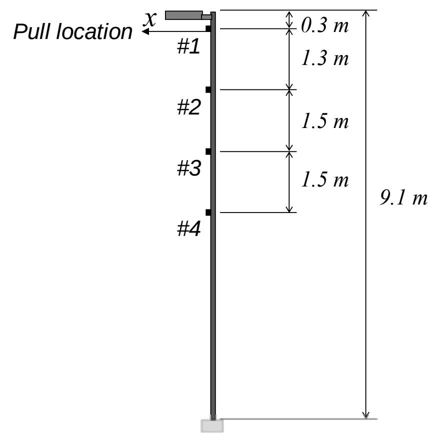


Figure 1. Accelerometer stations and pull location for on-site testing.

Since first mode vibrations were determined to be of primary importance, the light pole is subjected to free vibrations via a pull and release load applied at the top of the pole (see Figure 1 for pull location). The pull and release is carried out manually with a force gauge and a pull string. At the proper force level, the string is cut, and the pole is permitted to oscillate freely until its return to rest. Testing was completed in the months of April and August 2017. Wind conditions during testing were low to moderate (10 to 25 km/h). For each tested configuration of Table 3, three pull tests were performed at a 222 N pull force (50 lb).

4 MODAL ANALYSIS

This section describes the data processing and the analysis operations performed on the raw vibration data obtained from experimental testing.

4.1 Data processing

The data processing operations include removal of duplicate records (if present), de-trending by mean removal, scaling of data and the double integration of acceleration data with filtering to obtain displacement data. The filters are high-pass sixth order Chebyshev type 2 filter (0.1 Hz cut-off for filtering of acceleration data and 0.5 Hz cut-off for filtering of velocity and displacement data).

4.2 Free vibration – time domain

Since it is not possible to determine the damping ratio analytically, this modal property is typically determined experimentally by testing on practical structures. Free vibration tests provide an efficient means with which to determine damping (Chopra 2017). The natural logarithmic of the ratio of two peaks (u_i and u_{i+j}) of an underdamped free vibration system is called the logarithmic decrement, δ. For lightly damped systems where the damping ratio, ζ, is less than 20%, we can state in Equation 1 that:

[1] $$\delta = 2\pi\zeta$$

The damping ratio, ζ, can be determined from Equation 2:

[2] $$\zeta = \frac{1}{2\pi j} ln \frac{u_i}{u_{i+j}}$$

where u_i is the displacement amplitude of the i^{th} peak and u_{i+j} is the displacement amplitude of the $(i+j)^{th}$ peak.

The vibration frequency of the free vibration signal is determined by measuring the period between peaks in the displacement time history of the pole top signal. Results of the free-vibration time domain analysis for the undamped pole are shown in Table 2. Damping results for all testing configurations of Table 3 are shown in Table 4.

4.3 Free vibration – frequency domain

Frequency estimates of the first four modes of vibrations were obtained by peak picking in the frequency domain. The power spectral density (PSD) function represents the power content of a signal in an infinitesimal frequency band. An improved estimator of the PSD by Welch is employed in the analysis of the light pole acceleration data (Welch 1967). The method consists of dividing the time series data into windowed overlapping segments, computing a modified periodogram of each segment, and then averaging the PSD estimates. These segments are subjected to a windowing operation using a Hamming window to prevent leakage in the Fast Fourier Transform (FFT) from time domain to frequency domain of the data. An n-point FFT is applied to the windowed data where n is approximately equal to the number of samples in the segment, 8192 in this case. The periodogram of each windowed segment is then computed to obtain the segment's power spectrum. The set of periodograms is averaged to form the spectrum of all segments which is then scaled with the sampling frequency to compute the power spectral density. Results of the free-vibration frequency domain analysis for the undamped pole are shown in Table 2.

4.4 Stochastic subspace identification (SSI)

Among the different system identification techniques proposed for civil engineering monitoring applications, the stochastic subspace identification method has been found to be a reliable output-only identification technique which compares favourably to other available methodologies (Rahman 2012). The data-driven technique is derived from the work of B. Peeters and is the preferred operational modal analysis method for the Confederation Bridge superstructure (Peeters 2000, Londono 2006). As with all the system identification methods, a model of the underlying system is needed. For the SSI method, a stochastic state-space model is used. Since it is generally impossible to measure the input forces of in-operation structure, these terms are modeled as stochastic white noise. The SSI method uses data correlations which compresses the data while still preserving vibration information. The data correlations also eliminate the uncorrelated noise and can be factorized into state space matrices. Taking advantage of these correlations, data at different time lags are assembled into a matrix, that is then decomposed into factors using single value decomposition. From these factors, the state space matrices can be extracted and the modal parameters such as modal frequency, modal damping and mode shapes can be directly identified. Where the original method uses data from a subset of sensors, the variant of the method implemented for the analysis of the light poles uses data from all sensors to improve resolution and reduce uncertainty. The signal processing platform for analysis of structural health (SPLASH), which was developed at Carleton University, is a sophisticated computer application for rapid and efficient bridge vibration monitoring data management, signal processing, and system identification operations (Desjardins, et al. 2006). The application has been in continued use since its initial development in 2003 for the Confederation Bridge vibration monitoring program. The platform is currently undergoing an update (version 3.0) and among other notable improvements, it now has the capability to monitor any type of structure. Consequently, the light pole vibration data of the current study was processed, analysed and visualized using the SPLASH application to obtain the dynamic properties of the first four modes and to confirm the first mode results obtained from the log decrement method.

5 MODAL PROPERTIES

The modal properties of the undamped light pole were determined analytically and experimentally. The results are compared in Table 2. The analytical results were obtained from the models described in section 2. The experimental determination of the modal properties was carried out by picking peaks from the free vibration signals (in the time domain and in the frequency domain) and by the stochastic subspace identification method. While only data from sensor #1 (in the x direction) was used for the free vibration analysis, data from four sensors (12 channels) were used for the stochastic subspace identification which allows proper identification of the first 4 modes as shown in Table 2

Table 2: Modal frequencies (Hz) of the undamped light pole structure.

	Analytical		Experimental		
			Free vibration analysis		
Mode	Structural component model	Finite element model	Time domain	Frequency domain	Stochastic subspace identification (SSI)
1	1.95	1.92	1.92	1.92	1.92
2	9.38	9.13		9.27	9.27
3	25.2	24.0		23.6	23.7
4	49.7	42.6		51.9	51.9

Experimental results correlate well with the theoretical results from the analytical models, especially for the lower modes with an average relative error of 0.8% and 1.4% for mode 1 and 2 respectively. Although the manual pull and release method strongly excites the first mode of vibration, the SSI technique identifies the higher vibration modes, albeit with greater uncertainty. While the frequency domain analysis provides modal frequencies, the SSI technique identifies all modal properties; including frequency, damping and mode shapes. Figure 2 shows a qualitative comparison between the analytical mode shapes from the frequency analysis of the structural component model and the experimental mode shapes extracted using the SPLASH application with it's integrated SSI module.

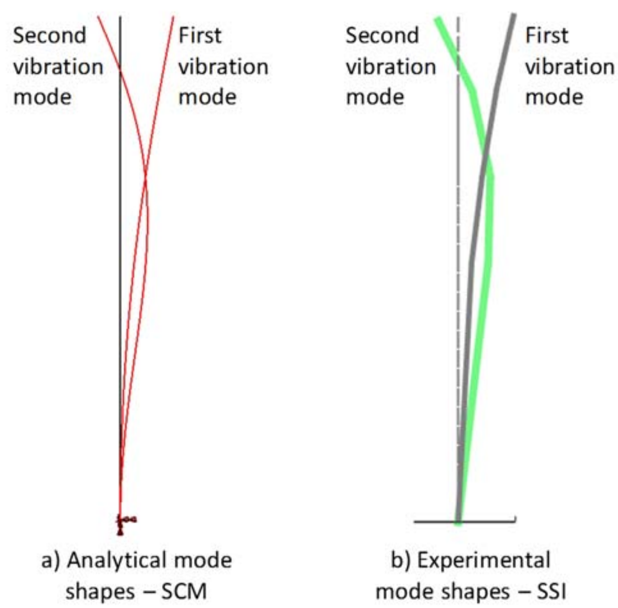


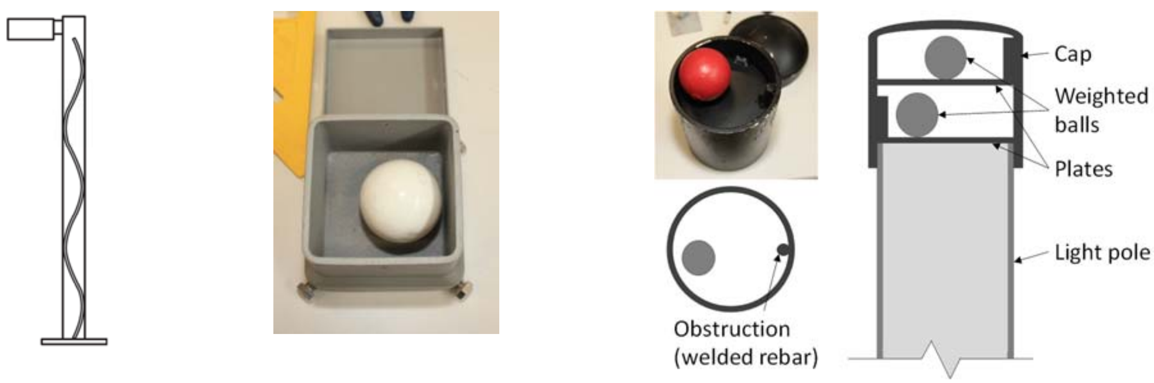
Figure 2. Analytical and experimental mode shapes.

6 COMPARISON STUDY OF POLE TOP IMPACT DAMPERS

The bridge light pole was tested in four different damping configurations summarized in Table 3. All light poles currently installed on the Confederation Bridge are outfitted with a rubbing pipe type chain damper as shown in Figure 3a. For configuration 1 this chain damper was removed, and the light pole was completely undamped. The remaining three configurations include the chain damper; either by itself (configuration 2) or in combination with a pole top impact damper (configurations 3 and 4).

Table 3: Damping configurations for site testing.

Configuration 1	**Undamped**: In this configuration, the light poles are not outfitted with any dampers.
Configuration 2	**Chain only**: In this configuration, a rubbing pipe type damper composed of a chain inside a flexible PVC pipe is installed inside the pole as shown in Figure 3a. Since 2002, all the light poles on the bridge are outfitted with this damper following the recommendations of an internal technical report (Lau, Chang, and Londoño 2002).
Configuration 3	**Chain + commercial damper**: In addition to the chain damper of configuration 2, the light pole is outfitted with a commercial pole top impact damper from Hapco Inc shown in Figure 3b.
Configuration 4	**Chain + prototype damper**: This first mode impact damper prototype encapsulates two weighted balls separated by plates welded on the inside of a schedule pipe. The damper slides on top of the pole for a quick field installation. Obstructions are added on the inside face of the cap in the form of welded rebar (Figure 3c). The chain damper of configuration 2 is also present in this configuration.



a) b) c)

Figure 3 : Pole dampers installed during testing: a) rubbing pipe (chain) damper; b) commercial first mode pole-top impact damper; c) prototype pole-top impact damper with two weighted balls.

A custom pole top impact damper was developed and fabricated with the help and input of the bridge's operations and maintenance personnel. Several iterations of the prototype were fabricated and tested. The design parameters considered for the development of the new impact damper include cost, ease of fabrication, ease of installation, durability and performance. To evaluate the performance, the various iterations of the prototype contained variations on the physical parameters such as the number of weighted balls, the ball travel distance, the overall weight and the inclusion of obstructions. Obstructions are included in the final prototype to prevent 'lock-in' of the weighted balls. 'Lock-in' describes the behavior where the ball travels in a circular motion around the inside of the circular pole or cap instead of dissipating energy by impacting the sides of the cylinder. The included obstructions force the ball out of its circular path and increase the number of energy dissipating impacts which improves performance. The performance was evaluated through free vibration testing. The pull release method was employed with a pull force of 222 N. The prototype shown in Figure 3c represents the final iteration considered. It utilises readily available stock materials, can be easily fabricated by the bridge crew, fits snugly on the pole tops for quick installation and is sealed to prevent water infiltration. The free vibration displacement time histories of the top sensors in the direction of the pull force for each damping configuration of Table 3 are shown in Figure 4.

From Figure 4 we can see the effects of the installed dampers on the duration of vibration. While the reduction of maximum displacements amplitude is minimal in some cases, the number of high amplitude cycles is significantly reduced with respect to the undamped configuration meaning the light poles comes to rest at a faster rate with the added pole top dampers.

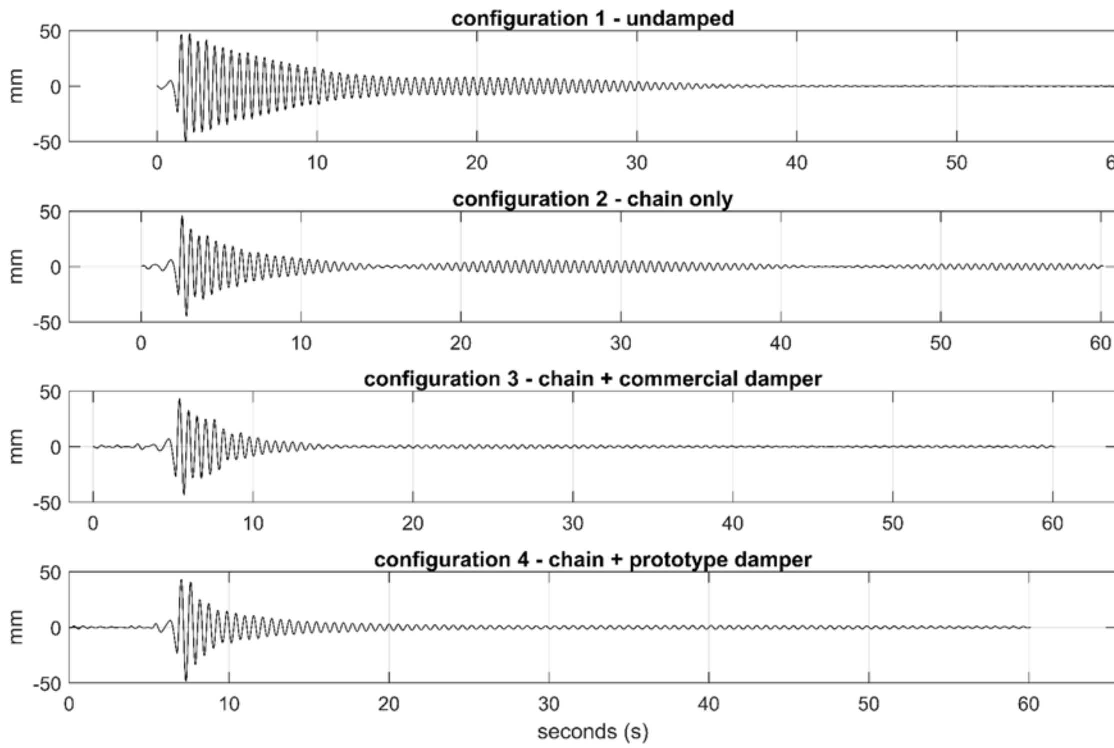


Figure 4.Free vibration displacement time histories for the damping configurations of Table 3.

Table 4 summarizes the damping results for all the tested configurations of Table 3. Results are averaged for each pull (three pull tests at 222 N). The damping results were obtained by the time domain (log decrement) method described in section 4.2. For this type of analysis, the displacement time history of the top most sensor was considered (Figure 4). The maximum peak displacement from the signal is identified and all subsequent peaks until a decay of 75% are considered for the calculation of the damping ratio, ζ (%). This number of oscillation cycles also provides an indication of the attenuating effects of various dampers.

Table 4: First mode properties (frequency and damping ratio) for the damping configurations of Table 3.

Damping configurations (see Table 3)	Free-vibration analysis (time domain)			
	Freq. (Hz)	75% decay cycles (#)	Max. disp. (mm)	Damping ratio, ζ
Configuration 1 - Undamped	1.92	16	47.5	1.38%
Configuration 2 - Chain only	1.89	9	45.6	2.23%
Configuration 3 - Chain + commercial damper	1.84	7	41.1	3.00%
Configuration 4 - Chain + prototype damper	1.77	7	42.5	3.01%

According to the results show in Table 4, the prototype impact damper using two lead balls within a pipe cylinder added to the top of the pole was found to offer comparable damping properties (damping ratio, decay cycles and maximum pole top displacements) to the commercial damper.

The calculated maximum displacement for the light pole in the undamped configuration (47.5 mm) approximately matches the displacement from the static analysis of the analytical FEM model subjected to the same pull (44 mm). The SSI technique described in section 4.4 was also used to extract the first mode properties from the measured data; modal damping ratios of 0.68%, 0.79%, 2.14% and 3.06% were obtained for configurations 1 to 4 respectively. Modal damping is an inherently highly uncertain physical phenomenon. Nevertheless, a relatively marked improvement of damping performance is noted between the undamped configuration and the configurations with a pole-top impact damper. These results, while not exactly corroborating the results from the free vibration analysis (more work is required here), do confirm the suitability of the pole top impact dampers to reduce the amplitude and the quantity of fatigue stress cycles under free vibration scenarios.

7 CONCLUSION AND RECOMMENDATIONS

The modal properties of the original light poles installed on the Confederation Bridge were successfully determined following full-scale on-site tests carried in the months of April and August 2017. The extracted modal properties (frequencies and mode shapes) from the free vibration tests were found to correlate well with the analytical models (within 0.8% and 1.4% for estimated frequencies of modes 1 and 2 respectively). This confirms the suitability of the experimental modal analysis techniques employed in this study. While picking peaks in the frequency domain properly identifies the modal frequencies of the first four vibration modes, the variant of the stochastic subspace identification (SSI) technique described in this study identifies all modal properties (frequencies, damping and mode shapes) for the same four modes of vibration. Furthermore, the SSI algorithm is an output only identification technique, meaning that the modal properties can be extracted regardless of the excitation mechanism (pull-release, impact, or ambient wind conditions).

In the second part of this study, four different light pole damping configurations were tested on-site. The configurations allowed the comparison of two different pole top impact dampers; a commercially available damper and a prototype damper developed and fabricated in collaboration with the Confederation Bridge operations and maintenance personnel. The displacement time history plots show the effectiveness of the impact dampers in reducing the number of high amplitude cycles. After the pull and release, the poles come to rest at a much quicker rate in the configurations with an impact damper than in the other configurations. According to fatigue theory, this attenuating effect can increase the remaining service life of the light poles. To properly quantify the comparison between each damping configurations, a free-vibration analysis in the time-domain was carried out to quantify the damping properties of each pole configuration. Under the 222 N pull load, the prototype using two lead balls within a pipe cylinder added to the top of the pole was found to offer comparable damping properties to the commercial damper. Also, the developed prototype has the following advantages: it can be efficiently fabricated from stock material, it can be installed rapidly by snuggly fitting on the top of the pole and it can be readily sealed to prevent water infiltration and therefore increase durability. It should be noted that if optimal damping performance is desired from a custom-designed pole top impact damper, further testing and development is required to determine the optimal properties of the physical parameters of the weighted balls (weight, size, quantity, coating, etc.) and the container (size, shape, travel distance, partial obstructions, etc.).

An extended monitoring program to evaluate the performance of the damper in place on the bridge under actual environmental loading conditions is recommended. It is suggested to install the prototype pole-top impact damper on-site for a duration spanning several months (ideally in windy season) to determine the actual damping performance. Under this scenario, the light poles would be subjected to random ambient wind conditions, therefore traditional techniques such as the free vibration method in the time-domain would not be suitable to properly identify the modal properties of the light pole structure. However, more advanced analysis methods such as the variant of the SSI technique described in this study would provide proper estimates of the modal properties of the light pole. It is believed that results from such a study would better indicate the suitability of the proposed solution in extending the service life of the installed light poles.

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