



## FLEXURAL RESISTANCE OF TL-1 AND TL-2 CONCRETE BRIDGE BARRIERS USING THE YIELD-LINE THEORY

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**Abstract:** Bridge barrier must be designed so that it could resist impact loads of crashes with different conditions. These conditions are defined in the bridge design codes such as the Canadian Highway Bridge Design Code (CHBDC), and classified into different test levels. Test levels 1 and 2 (TL-1 and TL-2) are defined for low volume road bridges with vehicles at the speed of 50 km/h, and consequences of vehicles leaving the roadway at the speed of 70 km/h, respectively. Based on these conditions, impact loads, barrier height and loads distribution length are provided in the code as well, and the barrier analysis and reinforcement design must be conducted using these specifications. Yield-Line theory is a method suggested in AASHTO LRFD Design Manual for bridge barriers. However, literature have shown the current triangular yield-line scheme provided in AASHTO is not accurate. The trapezoidal scheme provided in recent studies provides a lower flexural resistance. In this study, a modified trapezoidal scheme for yield-line analysis has been utilized for analyzing TL-1 and TL-2 bridge barriers. The analyse procedure includes calculation of various cases such as five different spacing in both vertical and horizontal directions, interior and exterior locations of the barrier, and two cases of full height or top part of the barrier as part of the yield-line pattern. Finally, recommended barrier design and the impact resistance for each case have been provided.

### 1 INTRODUCTION

Different types and shapes of barriers can be designed and utilized for bridges in order to provide with safety and prevent overturning of vehicles passing the bridges. Based on number of conditions, barriers should be designed for different impact loads in either horizontal, vertical or longitudinal directions. The conditions and criteria for evaluating the impact loads are given in the design code, which is discussed in the next section. The barriers should be designed for the moments produced from these impact loads. The analysis can be done using different methods, however, LRFD Bridge Design Specifications suggests using the yield-line theory for analyzing bridge barriers, which is used in other studies for concentrated loads as well (Fadaee et al. 2013). The yield-line pattern used in LRFD Bridge Design Manual includes a triangular pattern for the analysis which in recent studies proved to be different than experimental results (Figures 1 and 2). In studies conducted by Jeon et al. in 2008 and 2011, the authors resulted in a trapezoidal yield-line scheme to be more valid for the analysis of bridge barriers. In another study by Khederzadeh and Sennah in 2014, the same scheme pattern was confirmed, and new equations were provided for analysis of bridge barriers using yield-line theory. In the latter literature, it was also indicated that the trapezoidal pattern can be also revised to be more accurate. In this study, the revised trapezoidal yield-line pattern was evaluated based on previous literature, and the equations were used to analyze two of the test level barriers provided in the Canadian Highway Bridge Design Code 2014 (CHBDC). The analysis includes evaluating the moment capacity and impact load resistance of the Reinforced Concrete (RC) bridge barrier with 5

different spacing of both vertical and horizontal reinforcement in the barrier. The spacings were considered from 100, 150, 200, 250, 300 mm. Moreover, 84 different trapezoidal yield-line schemes were considered, and the analysis was performed for both exterior and interior locations. For each case the impact load resistance based on the revised trapezoidal yield-line pattern is calculated. The impact load resistance resulted from the analysis is based on the moment capacity provided by the reinforcement and the barrier dimensions required by CHBDC. The final design recommendations of RC bridge barriers for the two test levels are provided based on the minimum impact load resistance required by CHBDC.

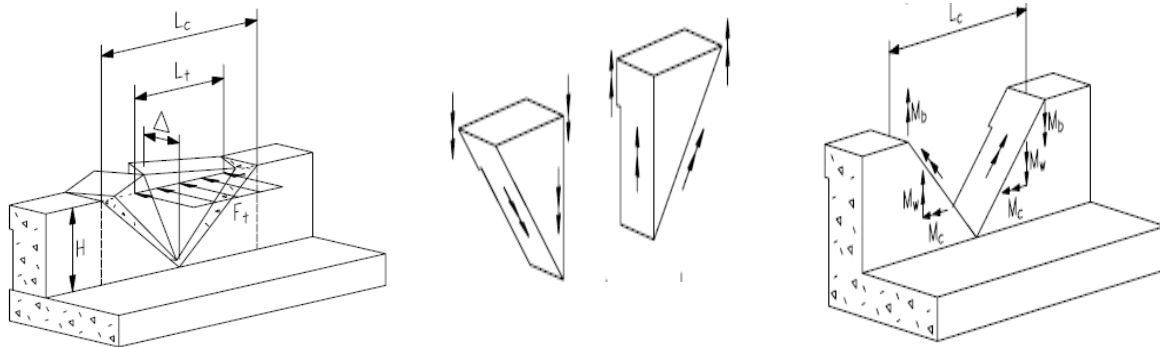


Figure 1: AASHTO-LRFD yield line failure pattern for interior region of barrier (adopted from LRFD Bridge Design Specifications, 2016)

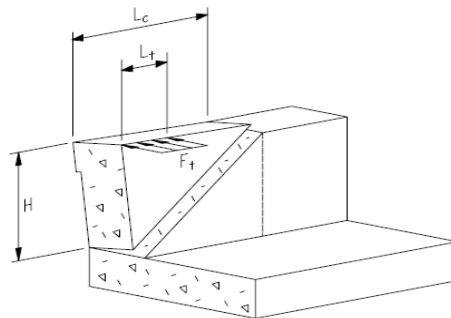


Figure 2: AASHTO-LRFD yield line failure pattern for exterior region of barrier (adopted from LRFD Bridge Design Specifications, 2016)

## 2 CHBDC BRIDGE BARRIER DESIGN AND TEST LEVELS

Chapter 12 of CHBDC 2014 provides with details and criteria for the design procedure of bridge barriers. Barriers are classified into number of categories in CHBDC known as test levels, based on different conditions that are defined. These conditions include speed, mass and angle of the crash impact of the vehicle. There are five basic Test Levels (TL) defined in CHBDC for design of bridge barriers which previously were known as Performance Levels (PL). Although in CHBDC traffic conditions are defined for the test levels, the aforesaid detailed criteria are referred to NCHRP Report 350 or the AASHTO Manual for Assessing Safety Hardware (MASH). As discussed in previous studies (Fadaee et al. 2017), the latest changes in these criteria can be seen in AASHTO MASH 2009 for different test levels. Test Level 1 or 2 (TL-1 or TL-2) is selected for this study in order to provide with analyzing with the modified yield-line pattern and design recommendations. The criteria for TL-1 and TL-2 are given in Table 1. Although different traffic conditions are defined for TL-1 and TL-2 in CHBDC, the criteria are the same. Thus, same design can be utilized for both TL-1 and TL-2 concrete bridge barriers.

Table 1: Details for test levels 1 and 2 (AASHTO MASH 2009)

Test Level	Vehicle	Mass (kg)	Speed (km/h)	Impact Angle (degrees)
1	1100C	1100	50	25
	2270P	2270		
2	1100C	1100	50	25
	2270P	2270		

### 3 MODIFIED TRAPEZOIDAL YIELD-LINE PATTERN

Recent studies (Jeon et al. 2008, 2011; El-Salakawy et al. 2012, Khederzadeh et al. 2014) show that a trapezoidal failure scheme for yield line analysis of bridge barriers is more accurate rather than the AASHTO-LRFD triangular shape. Thus, studies were carried out to provide with a developed trapezoidal failure pattern to provide with a proper barrier resistance for both interior location and exterior location of a barrier. For the interior location, it was assumed that two diagonal yield lines are extended from top of the barrier wall down to the deck-wall junction including a horizontal yield-line at the joint connection as shown in Figure 3. As resulted from experimental tests in the literature (Jeon et al. 2011), it can be assumed that two plastic hinges form at front face of the wall and two plastic hinges develop at back face of the wall. From the proposed yield-line pattern in Figure 3, three difference scenarios have been considered in this investigation, namely:

- (a) The length of horizontal yield-line at the base,  $X$ , to be greater than the loaded length,  $L_t$  ( $X > L_t$ )
- (b) The length of horizontal yield-line to be equal to the loaded length ( $X = L_t$ )
- (c) The length of horizontal yield-line to be less than the loaded length ( $X < L_t$ ).

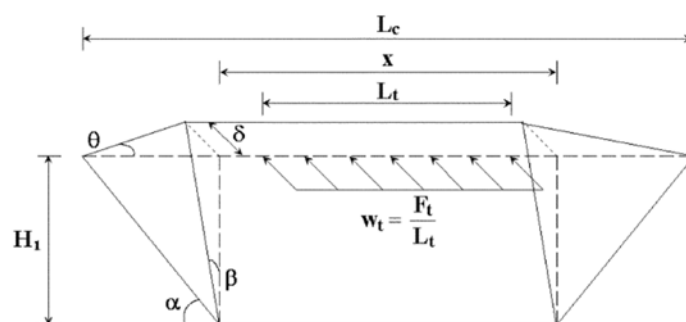


Figure 3: Proposed trapezoidal yield line pattern for bridge barrier by Jeon et al. (2011)

The corresponding equations can be derived based on the above-mentioned scenarios (Jeon et al. 2011). However, as recommended in the study carried out by Jeon et al. (2011), the developed trapezoidal failure is not yet fully accurate as the moment capacity for the horizontal yield lines of the wall ( $M_c$ ) is taken as an average from bottom to top of the wall. Thus, the equations can be modified by calculating different moment capacities of the base of the barrier ( $M_{c, base}$ ) and the moment capacity of the wall throughout the height of the barrier ( $M_c$ ).

Later, in a study carried out by Khederzadeh et al. (2014), the modified yield line was developed for the same scenarios as mentioned in the previous section, and the equations were provided as follows:

### 3.1 Yield Line Pattern with $X \geq L_t$ (Interior Location)

In the study it is assumed that the length of the horizontal yield line ( $X$ ) to be a function of the loaded length (i.e.  $X = n_1 \cdot L_t$ ); where  $n_1$  is a magnification factor to  $L_t$ . Since in this case  $X$  is smaller than  $L_t$ ,  $n_1$  is considered any value between 1 and 2 (i.e.  $1 \leq n_1 \leq 2$ ). The external work is produced by the applied load and the lateral displacement. The deformed shape is shown in Figure 4. The shaded area represents the total external work due to applied distributed load of  $w_t$  ( $w_t = F_t / L_t$ ). Consequently, the external work can be calculated as  $W_E = W_t \cdot (\text{Deformed area}) = F_t \cdot \Delta$ . Given the fact that the external work is equal to the internal work (produced from yielding moment of the reinforcement at yield lines), the authors concluded the following formulas for calculation of the impact load ( $F_t$ ) and the critical length ( $L_c$ ) as shown in the figures and the minimum transverse load ( $R_w$ ):

$$[1] \quad L_c = n_1 \cdot L_t + \sqrt{(n_1 \cdot L_t)^2 + \frac{8M_b \cdot H + 8M_w \cdot H^2 - M_{c,w} \cdot (n_1 \cdot L_t)^2}{M_{c,w}}}, \text{ for } 1 \leq n_1 \leq 2$$

$$[2] \quad R_w = \left( \frac{1}{L_c - n_1 \cdot L_t} \right) (8M_b \cdot H + 8M_w \cdot H + \frac{M_{c,w} \cdot (L_c^2 - n_1 \cdot L_t \cdot L_c)}{H} + \frac{(M_{c,base} - M_{c,w}) \cdot (n_1 \cdot L_t \cdot L_c - n_1^2 \cdot L_t^2)}{H}),$$

for  $1 \leq n_1 \leq 2$

where the variables are defined as follows (Khederzadeh et al., 2014; LRFD Bridge Design Manual, 2016):

$M_b$ : Flexural capacity of the cap beam (if applicable)

$M_w$ : Flexural capacity of the barrier about its vertical axis

$M_{c,w}$ : Flexural capacity of the barrier about its horizontal axis at the wall

$M_{c,base}$ : Flexural capacity of the barrier about its horizontal axis at the base

$H$ : Height of the transverse impact load application

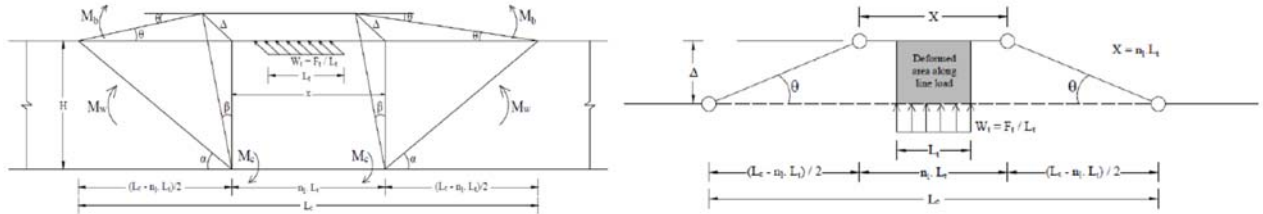


Figure 4: Trapezoidal yield-line failure pattern (interior location,  $X \geq L_t$ ) (Khederzadeh et al. 2014)

### 3.2 Yield Line Pattern with $X < L_t$ (Interior Location)

As shown in Figure 5 and with the similar concept, the study also provides with the equations for finding the critical length ( $L_c$ ) and minimum transverse impact load ( $R_w$ ) when the length of the horizontal yield line at the base of the wall ( $X$ ) is less than the length of the applied load. In this case, the other factor is defined ( $n_2$ ) for multiplying by  $L_t$  which is between 0 and 1 (i.e.  $X = n_2 \cdot L_t$ ,  $0 \leq n_2 < 1$ ).

$$[3] \quad L_c = 0.5L_t(1+n_2^2) + \sqrt{0.25L_t^2(1+n_2^2)^2 + \frac{16M_b \cdot H + 16M_w \cdot H^2 - M_{c,w} \cdot (n_2 \cdot L_t^2 + n_2^3 \cdot L_t^2) - (M_{c,base} - M_{c,w}) \cdot (2n_2^2 L_t^2 - n_2 \cdot L_t^2 - n_2^3 \cdot L_t^2)}{2M_{c,w}}},$$

for  $0 \leq n_2 < 1$

$$[4] \quad R_w = \left( \frac{1}{2L_c - L_t - n_2^2 \cdot L_t} \right) (16M_b \cdot H + 16M_w \cdot H + \frac{2M_{c,w} \cdot (Lc^2 - n_2 \cdot L_t \cdot L_c)}{H} + \frac{2(M_{c,base} - M_{c,w}) \cdot (n_2 \cdot L_t \cdot L_c - n_2^2 \cdot L_t^2)}{H}),$$

for  $0 \leq n_2 < 1$

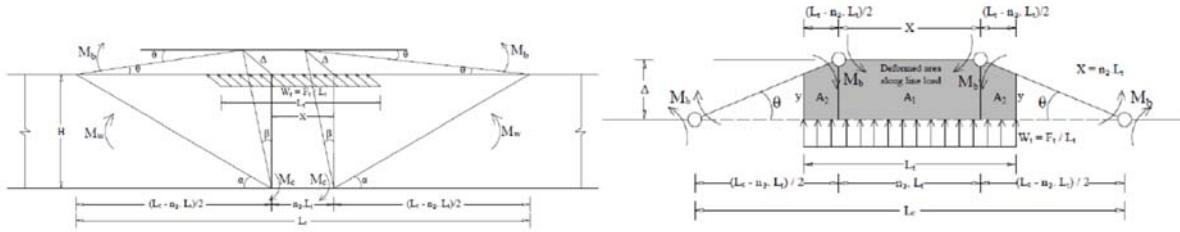


Figure 5: Trapezoidal yield-line failure pattern (interior location,  $X < L_t$ ) (Khederzadeh et al. 2014)

### 3.3 Yield Line Pattern with $X \geq L_t$ (Exterior Location)

Similarly, the yield-line pattern for barrier failure at the end location of the barrier (Figure 6) can be derived. The basis of the pattern was provided by Hirsch (1978) with a triangular scheme at the end location of the barrier. His work concluded with the AASHTO-LRFD current equations as mentioned above. However, in recent studies (Khederzadeh et al. 2014) the modified yield-line pattern and formulas for determining the barrier resistance was provided as follows:

$$[5] \quad L_c = n_1 \cdot L_t + \sqrt{(n_1 \cdot L_t)^2 + \frac{M_b \cdot H + M_w \cdot H^2 - M_{c,w} \cdot (n_1 \cdot L_t)^2}{M_{c,w}}}, \quad \text{for } 1 \leq n_1 \leq 2$$

$$[6] \quad R_w = \left( \frac{1}{L_c - n_1 \cdot L_t} \right) (M_b \cdot H + M_w \cdot H + \frac{M_{c,w} \cdot (Lc^2 - n_1 \cdot L_t \cdot L_c)}{H} + \frac{(M_{c,base} - M_{c,w}) \cdot (n_1 \cdot L_t \cdot L_c - n_1^2 \cdot L_t^2)}{H}), \quad \text{for } 1 \leq n_1 \leq 2$$

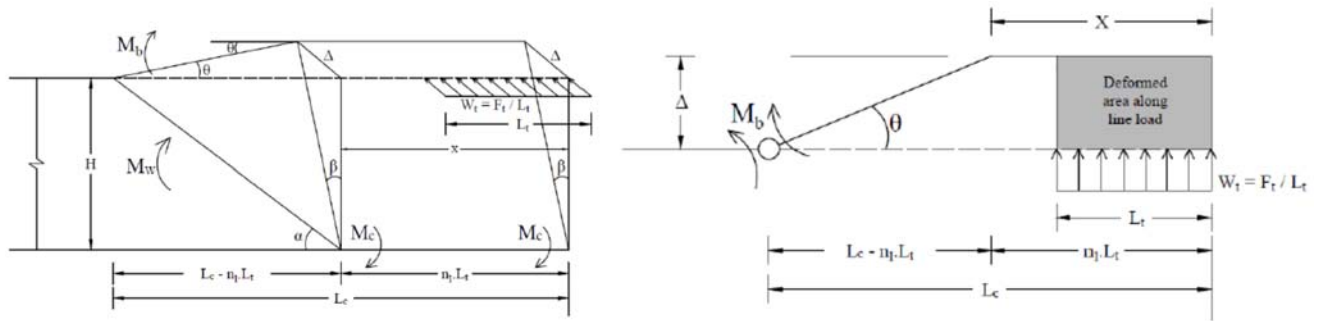


Figure 6: Trapezoidal yield-line failure pattern (exterior location,  $X \geq L_t$ ) (Khederzadeh et al. 2014)

### 3.4 Yield Line Pattern with $X < L_t$ (Exterior Location)

The final case would be the failure pattern at the end location of the barrier, when the length of the horizontal yield line at the base of the wall ( $X$ ) is less than the length of the applied load (Figure 7). In this case the following formulas can be used for finding the critical length ( $L_c$ ) and the impact load resistance ( $R_w$ ) of the barrier:

$$[7] \quad L_c = 0.5L_t(1+n_2^2) + \sqrt{0.25L_t^2(1+n_2^2)^2 + \frac{2M_b \cdot H + 2M_w \cdot H^2 - M_{c,w} \cdot (n_2 \cdot L_t^2 + n_2^3 \cdot L_t^2) - (M_{c,base} - M_{c,w}) \cdot (2n_2^2 L_t^2 - n_2 \cdot L_t^2 - n_2^3 \cdot L_t^2)}{2M_{c,w}}},$$

for  $0 \leq n_2 < 1$

$$[8] \quad R_w = \left( \frac{1}{2L_c - L_t - n_2^2 \cdot L_t} \right) (2M_b \cdot H + 2M_w \cdot H + \frac{2M_{c,w} \cdot (L_c^2 - n_2 \cdot L_t \cdot L_c)}{H} + \frac{2(M_{c,base} - M_{c,w}) \cdot (n_2 \cdot L_t \cdot L_c - n_2^2 \cdot L_t^2)}{H}),$$

for  $0 \leq n_2 < 1$

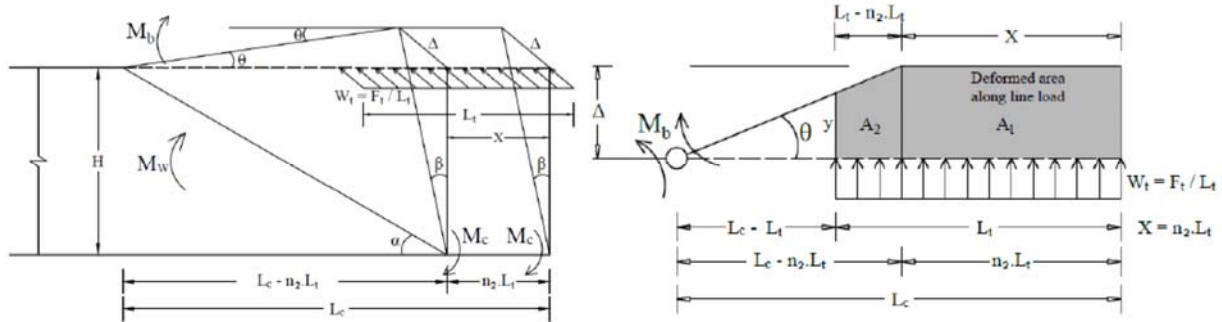


Figure 7: Trapezoidal yield-line failure pattern (exterior location,  $X < L_t$ ) (Khederzadeh et al. 2014)

#### 4 ANALYSIS CASES AND RESULTS

In this study, TL-1 or TL-2 concrete bridge barrier as defined in CHBDC 2014 (which have the same dimensions) is analyzed using the modified trapezoidal yield-line pattern. For each of the test levels, five cases of rebar spacing in horizontal and vertical direction and at both interior and exterior locations are considered. Moreover, 84 different yield-line patterns in terms of the  $n$  value (as explained in previous section) and the height of the vertical yield line at the wall ( $H$ ) are checked to find the most critical case and evaluate the minimum impact load resistance of TL-1 or TL-2 concrete bridge barrier. The height ( $H$ ) can have two values; from top of the barrier to top of asphalt (full-height) or from top of the barrier to top of the tapered part of the barrier (top-height). All cases and parameters are given in Table 2:

Table 2: Breakdown of analysis cases and parameters

Item	Test Level	Location	Height Evaluated	$n$ ( $n_1$ or $n_2$ )	Horizontal Rebar Spacing - $S_h$ (mm)	Vertical Rebar Spacing - $S_v$ (mm)	Total Cases
	TL-1 and TL-2	Interior	Full-Height	$n_1$ : 0.05 increments from 1 to 2 (21 cases)	100	100	
					150	150	
		Exterior	Top-Height	$n_2$ : 0.05 increments from 0 to 1 (21 cases)	200	200	
					250	250	
					300	300	
<b>No. of cases:</b>	1	2	2	42	5	5	4200

The analysis is conducted for all aforesaid cases with following material properties:

Concrete compressive strength ( $f_c$ ) = 30 MPa, Rebar yield stress ( $f_y$ ) = 300 MPa and 15M rebars with 60 mm clear cover.

The results for TL-1 and TL-2 barriers (which have the same results) are shown in terms of the minimum impact load resistance and the corresponding n value ( $n_1$  or  $n_2$ ) that indicates the critical yield-line pattern (Tables 3 through 6). It was concluded that the full-height gives lower values for the impact resistance ( $R_w$ ) rather than the top-height of the barrier which is more critical. Thus, only full-height results are provided in the following tables.

Table 3: Impact load resistance ( $R_w$ ) of TL-1 and TL-2 barriers at interior location

$R_w$ (kN)	$S_h$ (mm)					
	100	150	200	250	300	
<b>100</b>	988.54	895.70	840.85	804.38	778.22	
<b>150</b>	723.39	648.38	603.91	574.13	552.69	
<b><math>S_v</math> (mm)</b>	<b>200</b>	648.84	579.52	538.38	510.79	490.89
	<b>250</b>	567.88	505.17	467.91	442.90	424.83
	<b>300</b>	478.99	424.09	391.40	369.45	353.59

Table 4: Corresponding n values of TL-1 and TL-2 barriers at interior location

n	$S_h$ (mm)					
	100	150	200	250	300	
<b><math>S_v</math> (mm)</b>	<b>100</b>	0.20	0.25	0.25	0.25	0.25
	<b>150</b>	0.20	0.20	0.25	0.25	0.25
	<b>200</b>	0.20	0.20	0.25	0.25	0.25
	<b>250</b>	0.20	0.20	0.20	0.25	0.25
	<b>300</b>	0.20	0.20	0.20	0.20	0.25

Table 5: Impact load resistance ( $R_w$ ) of TL-1 and TL-2 barriers at exterior location

$R_w$ (kN)	$S_h$ (mm)					
	100	150	200	250	300	
100	596.87	571.06	556.27	546.67	539.87	
150	401.22	379.12	366.32	358.02	352.05	
$S_v$ (mm)	200	349.51	328.64	316.56	308.64	303.06
	250	295.55	276.21	265.06	257.66	252.42
	300	238.99	221.68	211.56	204.91	200.19

Table 6: Corresponding n values of TL-1 and TL-2 barriers at exterior location

n	$S_h$ (mm)					
	100	150	200	250	300	
$S_v$ (mm)	100	0.40	0.40	0.45	0.45	0.50
	150	0.35	0.40	0.40	0.40	0.45
	200	0.35	0.35	0.40	0.40	0.4
	250	0.30	0.35	0.35	0.40	0.4
	300	0.30	0.35	0.35	0.35	0.4

## 5 DESIGN RECOMMENDATIONS FOR TL-1 AND TL-2 BARRIERS

Based on the results of the yield-line theory analysis that was conducted with the trapezoidal failure pattern, a design recommendation can be provided for TL-1 and TL-2 concrete bridge barriers. Both test levels have the same crash criteria and same dimensions are given for them in CHBDC. Therefore, one design can be utilized for both test levels.

Minimum impact load resistance required for bridge barrier design of different test levels are given in CHBDC 2014 and TAC Guide to Bridge Traffic and Combination Barriers based on experimental studies. The minimum impact load capacity required for TL-1 or TL-2 is 50 kN without the dynamic load allowance (DLA) and the live load factor as per CHBDC. By including the two multipliers (1.4 for DLA and 1.7 for live load factor), a minimum of  $50 \times 1.4 \times 1.7 = 119$  kN will be calculated. The results provided in previous section shows a rebar spacing of 300 mm in horizontal direction and 300 mm in vertical direction and with the assumed material properties gives the sufficient capacity for TL-1 and TL-2 barriers. Thus, the final design recommendation can be provided as shown in Figure 8.



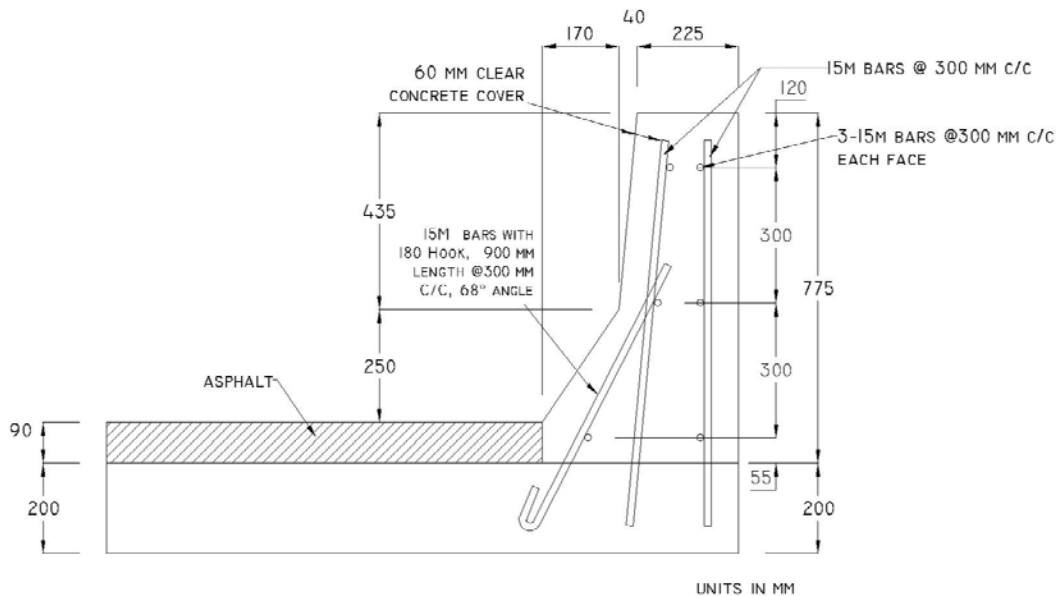


Figure 8: Design recommendation for TL-1/TL-2 concrete bridge barrier

## 6 SUMMARY AND CONCLUSIONS

In this study the analysis and design for concrete bridge barriers were investigated. Concrete bridge barriers are classified into test level (TL) categories, namely TL-1, TL-2, TL-4 and TL-5 as defined in CHBDC 2014. Each test level is utilized for different traffic conditions per CHBDC, and the criteria of the crash for the test levels are defined in AASHTO MASH 2009 and NCHRP Report 350. In this research, concrete barriers for TL-1 and TL-2 were analyzed with a modified trapezoidal yield-line failure pattern that are suggested in recent literature. The equations for the yield-line analysis were used to analyze the barrier for five different rebar spacing of 100, 150, 200, 250 and 300 mm in both horizontal and vertical directions. Moreover, as mentioned in previous studies, two locations at the interior and exterior of the barrier were considered individually for the analysis process and 84 different yield-line patterns were evaluated. The impact load resistance ( $R_w$ ) was evaluated with the analysis method and compared to the minimum impact load resistance required by CHBDC. Based on the results, the final design recommendation was provided for TL-1/TL-2 concrete bridge barriers.

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