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EFFECT OF INPUT EXCITATION CHARACTERISTICS ON EXTRACTING MODAL PARAMETERS USING OPERATIONAL MODAL ANALYSIS

MacLeod, Ethan[1,3], Arjomandi, Kaveh[2]
[1] Research Assistant, Department of Civil Engineering, University of New Brunswick, Canada
[2] Assistant Professor, Department of Civil Engineering, University of New Brunswick, Canada
[3] emacleo1@unb.ca

Abstract: A central assumption in classical operational modal analysis (OMA) is that the excitation is a perfect Gaussian white noise. This assumption is difficult to validate and, in most practical situations is not correct. In practice, most excitations rarely are Gaussian and often take on other distributions such as Rayleigh for wind loading or poorly defined distribution for sporadic loading generated from low traffic volumes. This paper investigates the influence of excitation distributions other than Gaussian white noise on the accuracy of commonly used OMA methods. Characteristic parameters such as the input probability density functions, spectral content, and magnitude of signal to noise ratios are considered in this study. It was shown that although natural frequencies may be accurately estimated, the estimated mode shapes when the input excitation is other than Gaussian white noise contain significant errors, and in most cases, accurate estimation is not feasible.

1 INTRODUCTION

Modal analysis is a well-developed and active field beginning in the mid 20th century and is employed in many applications today. Experimental Modal Analysis (EMA) is modal identification using known input excitations and has been successfully proven in many fields such as automotive, aerospace, and industrial testing (Ewins 1984). In relatively small structures such as those in aerospace or manufacturing industries it is possible to excite the structure with a known excitation via an oscillator or impact. This greatly simplifies the analysis procedure as both the input and output are deterministic. In large civil structures this is not feasible as the mass of the shaker or the size of the impact needed to excite the structure would be too large and possibly damaging. The ambient operational environment is instead used as a source of excitation originating from wind, waves, traffic, or occupants. This is called Operational Modal Analysis (OMA) and shares common theoretical background to EMA except it becomes a stochastic problem where only the output is deterministic. Although analytically more complex, OMA presents many advantages as tests are relatively inexpensive and do not interfere with day to day operations of the structure. The tests are also actually representative of the operational conditions that the structure experiences day to day (Rainieri and Fabbrocino 2014).

OMA can primarily be divided into two categories, Time Domain (TD) and Frequency Domain (FD) identification methods.TD methods are typically parametric models obtained by Least Square (LS) fitting. By formulating an overdetermined set of equations, it is possible to find a solution by using the pseudo inverse of the equation matrix. The Poly Reference (PR) method uses free decays established from correlation functions to find Auto Regressive (AR) coefficients through a LS solution. Modal parameters are then estimated by forming a companion matrix and performing an eigenvector decomposition (Vold et al.

1982). A different approach is taken in the Ibrahim Time Domain (ITD) method where a block Hankel matrix is formed with 4 block rows an split down the middle. The system matrix is again solved by LS with modal parameters being estimated from an eigenvector decomposition of the system matrix (R. Ibrahim 1977). The Eigensystem Realization Algorithm (ERA) forms two block Hankel matrices and performs a Singular Value Decomposition (SVD) of the first, enabling the use of an observability and controllability matrix to find the system matrix. An eigenvector decomposition is performed on the system matrix to estimate the modal parameters (Juang and Pappa 1985). The above methods all use the free decays to create block Hankel matrices, however, in the Stochastic Subspace Identification (SSI) technique the time response is used to construct the matrix instead. A projection matrix is formed by LS solution and a SVD of the projection matrix is used to find the system matrix. Finally, an eigenvector decomposition of the system matrix is used to estimate modal parameters (Peeters 2000).

Frequency Domain (FD) methods are popular because they produce intuitive plots that allow insight into the system but often suffer from bias problems due to leakage when calculating PSD (Brincker and Rune 2014). The most simplistic method is the classical FD technique in which the natural frequencies are estimated from the peaks of the considered PSDs, with the damping being estimated by the width of the peak and mode shapes from any row or column of the PSD matrix (Bendat and Piersol 1980). This method only works well in the case of well-separated modes and can be difficult to perform if there are numerous PSD plots. The Frequency Domain Decomposition (FDD) method overcomes these shortcomings by performing a SVD of the PSD matrix from which the modal parameters are estimated as in the classical FD method. This creates a single PSD plot that is easier to interpret and allows for the identification of closely spaced modes (Brinker, Zhang, and Andersen 2001). The Polymax method is an extension of the PR method to the frequency domain. Two block Hankel matrices are formed, and a companion matrix is solved for by a LS solution with eigenvector decomposition then being performed to estimate modal parameters (Peeters and Van Der Auweraer 2005).

To account for the unknown nature of the input excitation several overarching assumptions must be made. First, in most practical applications the system of interest is assumed to be a linear dynamic system. Second, the system and source of vibrations is assumed to be stationary. This means that the dynamic characteristics of the system and input excitation do not change with time. The third assumption is observability which means that the sensor layout is properly designed to observe all the modes of interest. Finally, due to the stochastic nature of the input excitation, further assumptions must be made about its characteristics. In most OMA methods it is assumed that the input excitation is a Gaussian white noise, therefore equally exciting all modes of interest (Rainieri and Fabbrocino 2014).

The assumption of the input excitation to have a Gaussian white noise characteristic is rarely the case in practice. This is evident if one considers the ambient loading conditions for large civil structures. For example, ambient vibration of bridges is mainly due to wind and traffic, both of which have well defined characteristics that are not Gaussian or white in nature. Wind velocities are typical characterized by a Weibull distribution (Altunkaynak et al. 2012) and the spectral content has two main regions separate by a large spectral gap. The spectrum is characterized by high frequency turbulent flow which also takes the shape of a Weibull distribution and low frequency long term weather systems (Koloušek 1984). Traffic loading follows a bimodal probability distribution caused by the separation in size of passenger vehicles and large trucks (Quan 2004) and the spectral content can vary considerably depending on traffic volume. Modes are weighted by the spectrum of the input therefore the output is a combination of response from properties of both the input and structures modal parameters making modal identification more difficult in these cases (Herlufsen, Gade, and Møller 2005) .

This paper investigates the influence that input distribution has on the accurate estimation of modal parameters. This is accomplished by conducting a parametric study through exciting a simulated system with different generated signals. The generated signals are representative of turbulent wind flow with varied input probability density functions, spectral content, and magnitude of signal to noise ratios. It was shown that although natural frequencies may be accurately estimated, when the input excitation is other than Gaussian white noise the estimated modes shapes contains significant errors, and in most cases, accurate estimation is not feasible.

2 TECHNICAL BACKGROUND AND MOTIVATION

When one considers the derivation of one of the fundamental equations of OMA, the assumptions of white noise plays a critical role. The equation states that the spectrum of the output contains the modal information of the system by expressing the PSD matrix of the output in pole-residue form. Beginning with a state space representation of a dynamic system, it is necessary to first define its state space transfer function as well as a representation of the white noise spectrum. It is then possible to relate the two and express the result in pole-residue form. It is show by (Ljung 1987) that the input and output spectra of a process are related by their transfer function as:

[1] $$S_y(s) = H(s)S_u(s)H^T(s^*)$$

Where $S_y(s)$ is the spectral content of the input, $H(s)$ is the system transfer function and $S_u(s)$ is the spectral content of the input, which in the case of a white noise input is constant. Assuming a white noise input equation [1] can be expressed in pole residue form as:

[2] $$S_y(j\omega) = \sum_{r=1}^{N} \frac{\{\phi_r\}\{\gamma_r\}^T}{i\omega - \lambda_r} + \frac{\{\phi_r\}^*\{\gamma_r\}^H}{i\omega - \lambda_r^*} + \frac{\{\gamma_r\}\{\phi_r\}^T}{-i\omega - \lambda_r} + \frac{\{\gamma_r\}^*\{\phi_r\}^H}{-i\omega - \lambda_r^*}$$

Where $\{\gamma_r\}$ is the operational reference vector associated with the rth mode and serves as a modal participation vector that depends on all the modal parameters of the system, the input locations, and the input correlation matrix (Peeters 2000). The poles hold the information about the frequencies and damping ratios and the residues hold the modal information. The assumption that $S_u(s)$ in equation [1] is a constant value makes the derivation of the pole residue representation possible, however, in practice the spectral content is changing with frequency which presents the possibility of causing errors in modal parameter estimations.

In order to account for input excitations with different characteristic behaviour than white Gaussian noise, Ibrahim et al. (1996) proposed to assume an OMA model as shown in Figure 1 (Ibrahim, Brinker, and Asmussen 1996). In this method the structure is loaded by unknown forces that are the output of a so-called excitation system loaded with Gaussian white noise. The measured response can be interpreted as the output of the combined system as shown in Figure 1. In this figure, $N(\omega)$ is the white noise input, $F(\omega)$ is the output of the excitation system, $Y(\omega)$ is the output of the structure and $H_f(\omega)$ and $H_s(\omega)$ are the Frequency Response Functions (FRF) of the excitation system and the structure respectively. In this approach, a fictitious excitation system that acts as a filter is considered to transform an ideal white noise input to an excitation with characteristics other than Gaussian white noise.

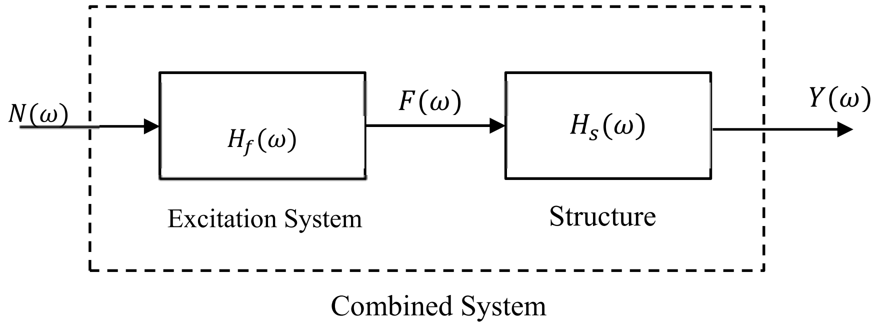

Figure 1: Combined system to account for excitations with non-Gaussian non-white characteristics

Since the systems are in series the FRF of the combined system is the product of the excitation system and the structure.

[3] $$H_c(\omega) = H_f(\omega)H_s(\omega)$$

The output and input for each subsystem are related by the following responses:

[4] $$F(\omega) = H_f(\omega)N(\omega)$$

[5] $$Y(\omega) = H_s(\omega)F(\omega)$$

It is clear that the measured response includes information about the excitation and the structure, however the modal parameters of the system are preserved and identifiable according to the authors (Ibrahim, Brinker, and Asmussen 1996) based on the following proof. $H_s(\omega)$ and $H_f(\omega)$can be represented in pole residue as:

[6.a] $$H_s(s) = \sum_{i=1}^{2n} \frac{a_i}{s - \lambda_i}$$

[6.b] $$H_f(s) = \sum_{j=1}^{2m} \frac{b_j}{s - \alpha_j}$$

Substituting equations [6.a] and [6.b] into equation [3] and simplifying yields:

[7] $$H_c(s) = \sum_{i=1}^{2n} \sum_{j=1}^{2m} \frac{a_i b_j}{(s - \lambda_i)(s - \alpha_j)}$$

Using partial fractions [7] becomes:

$$H_c(s) = \sum_{i=1}^{2n}\sum_{j=1}^{2m}\left\{\frac{a_i b_j(\lambda_i - \alpha_j)}{(s-\lambda_i)} + \frac{a_i b_j(\alpha_j - \lambda_i)}{(s-\alpha_j)}\right\}$$

[8]

$$H_c(s) = \sum_{i=1}^{2n}\frac{a_i(A\lambda_i - B)}{s-\lambda_i} + \sum_{j=1}^{2m}\frac{b_j(C\alpha_j - D)}{s-\alpha_j}$$

Where

[9] $$A = \sum_{j=1}^{2m} b_j \quad ; \quad B = \sum_{j=1}^{2m} b_j\alpha_j \quad ; \quad C = \sum_{i=1}^{2n} a_i \quad ; \quad D = \sum_{i=1}^{2n} a_i\lambda_i$$

As was proven in this approach by the authors (Ibrahim, Brinker, and Asmussen 1996) the modal parameters of the structural system and the excitation system are preserved and separable. The poles that are the roots of the denominator in equation [8] are unaffected by considering a combined system as shown in Figure 1. This means that using OMA methods the natural frequencies and damping ratios can be accurately identified for a system subjected to a non-Gaussian non-white noise. The residues are simply scaled by a constant for each mode and are clearly shown in equation [9] to be a combination of both the structural system and the excitation system. This theory was tested in a 2 degree of freedom verification problem where one system was excited with a white noise and one system was representative of the combined system of Figure 1. The natural frequencies showed close agreement between the theoretical and proposed method, but the mode shapes showed large discrepancies indicating a need for further research in this topic.

3 PARAMETRIC STUDY

To determine the effect of the input excitation characteristics on the accuracy of modal parameter extraction a parametric study was performed. In this study the focus is on the effect of the spectral distribution and the presence of noise in the output signal. A 4 DOF system as seen in Figure 2 with stiffness k of 400 N/m and 5% modal damping was used. The system is excited with a series of 36,000 seconds long acceleration time histories. The time histories were sampled at 10 Hz that were representative of a general turbulent wind flow with 5 different PDFs and frequency distributions and 4 levels of signal to noise ratios.

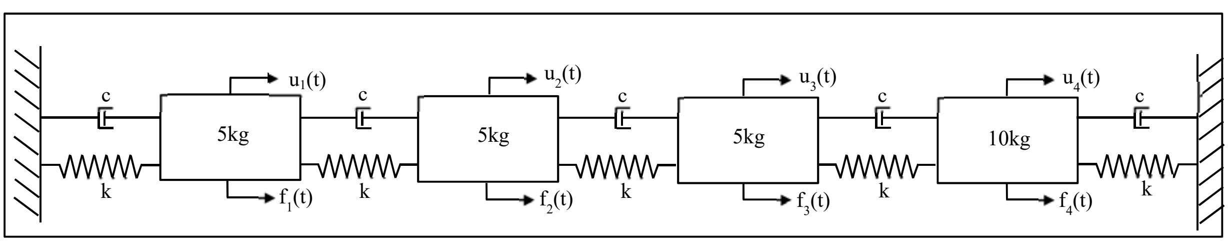

Figure 2: 4 DOF test system used for parametric studies

The input excitation signals were generated using the flow chart found in Figure 3. A Weibull velocity PDF is created and randomly sampled to create a velocity time signal with white spectral content. The signals are then converted to frequency domain and filtered to the shape of the PDF to achieve the desired spectral distribution. After the filter is applied the time domain PDF of the signal had become Gaussian, therefore, the signal is corrected in the time domain to have a Weibull PDF. This is done by calculating the inverse CDF of the signal (red line) and adjusting it to a theoretical Weibull inverse CDF (blue line). Next the resulting signals are differentiated to acquire an acceleration time history. Every acceleration signal is normalized to unity and the spectral contents were all scaled to have the same total power. Signals with varying PDFs and PSDs covering the bandwidth of the system were then produced by varying the alpha and beta parameters as shown in Figure 4. The results of this process for a single sample are shown in Figure 5.

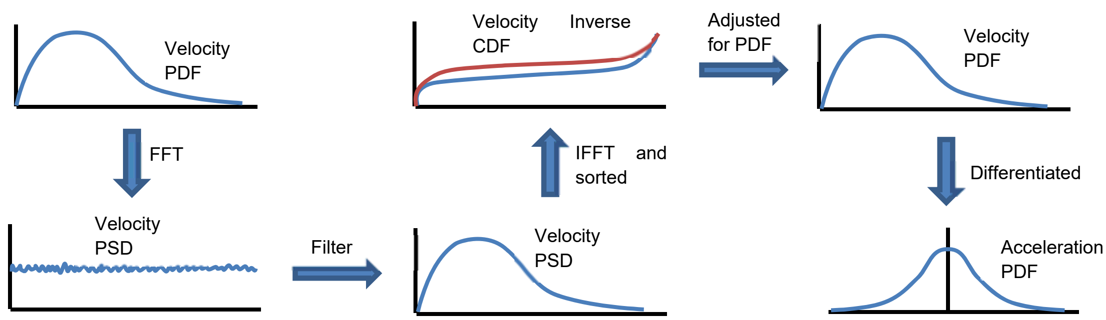

Figure 3: Calculation steps for producing simulated turbulent wind signals.

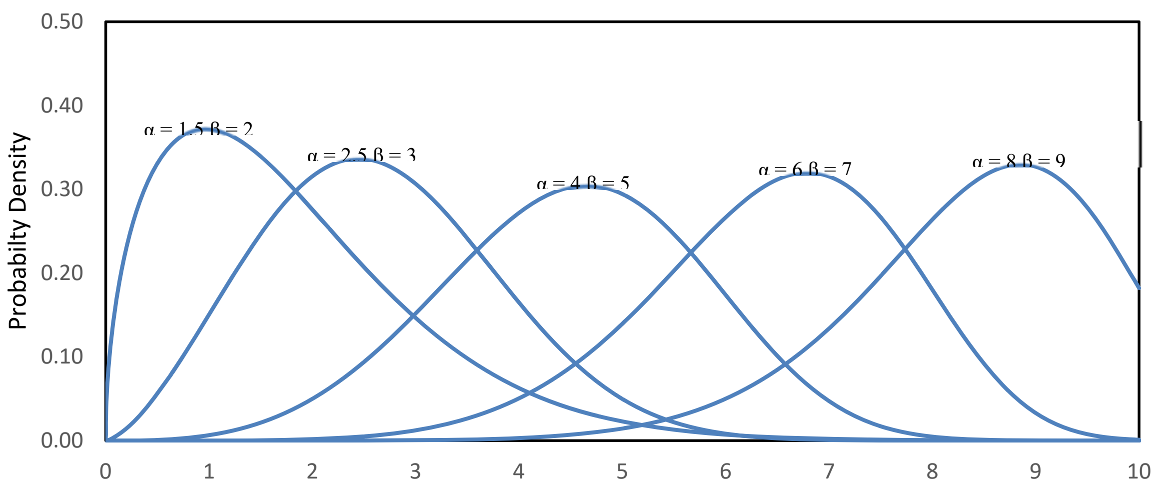

Figure 4: Varying α and β valued Weibull distribution used in parametric study.

The effect of the presence of noise in an output signal of a system excited with non-white excitation was also investigated. In this study, the system was excited with the standard Weibull distribution used for wind where α=2 and β=2.2, also known as a Rayleigh distribution; white noise was added to the output to produce signal to noise ratios (S/N) of 5 dB, 10 dB, and 15 dB.

The test system was analyzed using a modal time history method in SAP2000 under the input excitations with Weibull PSD and various levels of noise. The system response was analyzed using PolyMAX which is an OMA frequency domain analysis method. PolyMAX performs autoregressive poly-reference in the frequency domain (Peeters and Van Der Auweraer 2005). FEMtools was used for the analysis. The extracted modal parameters from each of the tests were then compared to evaluate the effects of input PDF and spectral content as well as noise have on the extraction of modal parameters.

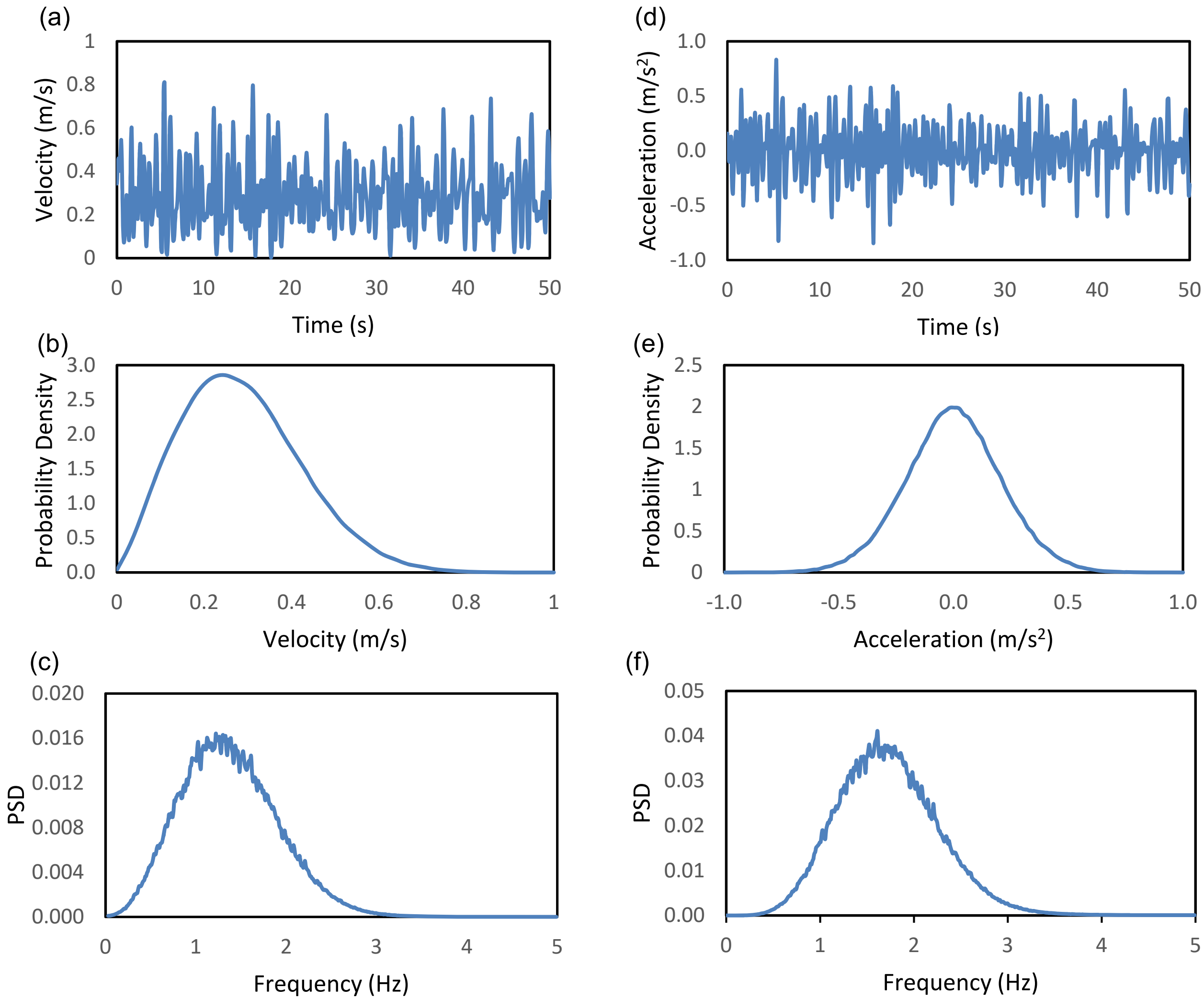

Figure 5: Example of generated wind signals: (a) velocity time history, (b) velocity PDF, (c) Velocity PSD, (d) acceleration time history, (e) acceleration PDF (f) acceleration PSD

4 RESULTS AND DISCUSSIONS

The results of the parametric studies are presented in Figure 6, Figure 7, Table 1 and Table 2. As seen in Figure 6 the natural frequencies under each loading scenario agree very closely with the theoretical values and as predicted in equation [8] it was possible to accurately identify the natural frequencies regardless of the excitation spectrum. The lower natural frequencies appear to be more stable whereas the higher natural frequencies show some slight variability. This is expected as the first mode is very dominant making the higher modes more challenging to identify accurately as they make up a smaller portion of the response.

The effects of different input characteristics were also evident in the estimation of damping presented in Figure 7. The loading scenarios which only excite the lower parts of the frequency band lead to accurate damping estimations for the lower natural frequencies but under estimate the higher frequency damping values. This can be attributed to the upper modes not being excited well enough by the low frequency input. The high frequency inputs tend to overestimate the damping values of the higher frequencies due to the input disproportionately weighting those modes in the response. The middle inputs which had a frequency distribution that covered most of the frequencies of interest had the best estimation of damping values due to more evenly exciting all modes.

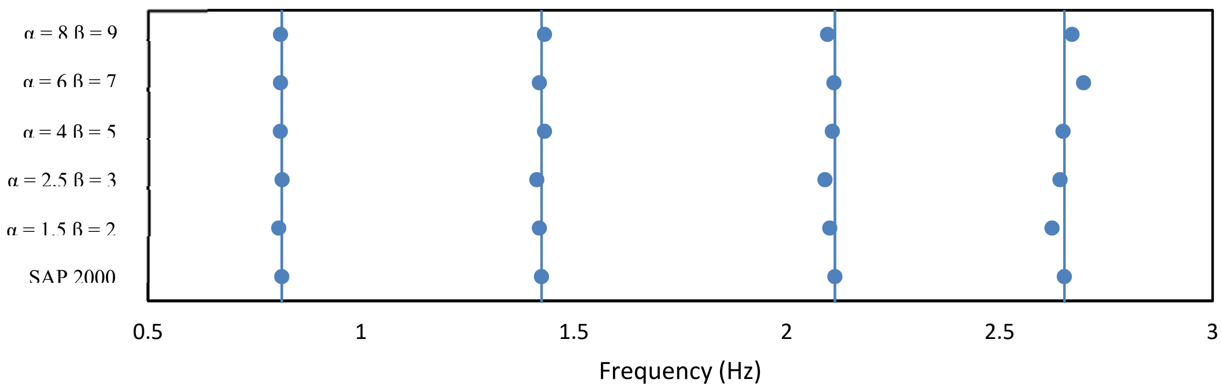

Figure 6: Identified natural frequencies estimated from each loading scenario compared to the theoretical values calculated with SAP 2000.

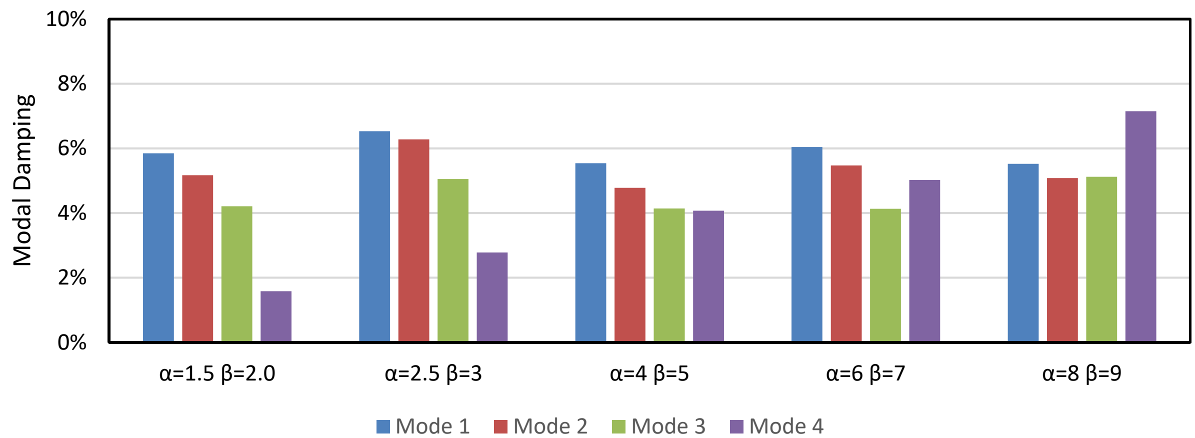

Figure 7: Estimated damping from each loading scenario.

Table 1: Summary of MAC values calculated between theoretical and experimental results.

Mode	α=1.5 β=2.0 MAC	α=2.5 β=3 MAC	α=4 β=5 MAC	α=6 β=7 MAC	α=8 β=9 MAC
1	0.998	1.000	0.984	0.808	0.998
2	0.065	0.025	0.998	0.743	0.023
3	0.015	0.015	0.170	0.944	0.065
4	0.003	0.002	0.008	0.244	0.590

The mode shape estimation shows the largest sensitivity to input characteristics. As seen in Table 1 for all cases the first dominant mode shape could be estimated as indicated by the high MAC values calculated between theoretical and estimated mode shapes. The low MAC values of the higher mode shapes indicate a lack of correlation between the theoretical and estimated mode shapes. This makes it evident that it was not possible to estimate the higher mode shapes with inputs other than Gaussian white noise. This result is predicted by [8] which clearly shows that the mode shapes estimated from the residues of the expression of the system RFR are weighted by the spectral content of the input. This means that mode shapes that are not adequately excited at their corresponding natural frequencies cannot be reliably estimated and can cause significant reduction in accuracy and effectiveness of OMA.

Table 2: Summary of results achieved by varying the output signal to noise ratio from an input with a Rayleigh distribution.

	No Noise			S/N = 15			S/N = 10			S/N = 5		
mode	Freq (Hz)	ζ (%)	MAC	Freq (Hz)	ζ (%)	MAC	Freq (Hz)	ζ (%)	MAC	Freq (Hz)	ζ (%)	MAC
1	0.813	5.26%	0.983	0.805	5.77%	0.997	0.805	5.93%	0.992	0.809	5.86%	0.997
2	1.443	6.51%	0.056	1.421	4.42%	0.051	1.42	4.94%	0.052	1.419	5.12%	0.051
3	2.111	5.55%	0.015	2.113	5.41%	0.016	2.114	5.28%	0.016	2.11	5.30%	0.016
4	2.649	3.91%	0.002	2.652	5.05%	0.002	2.653	5.14%	0.002	2.651	5.24%	0.002

Finally, the presence of noise in the output of a system excited with a Rayleigh distribution showed no marked effect on the estimation of modal parameters. As seen in Table 2 the natural frequencies were estimated quite accurately regardless of the signal to noise ratio. Though the damping estimates and mode shapes were poorly estimated, they were equally poor for the cases with no noise and those with low signal to noise ratios. That significant noise in the output signal has little effect on the modal parameter estimations supports the conclusion that the poor estimation of damping values and mode shapes is a direct result of the input distributions.

5 CONCLUSION

The OMA fundamental theories are developed on the assumption that the excitation will be a Gaussian white noise, however, this is rarely true in practice. The effects of excitation characteristics were tested through a parametric study and it was shown that the extracted frequencies are insensitive to the spectral and probability distribution of the input. The damping estimates were found to be inaccurately estimated and weighted by the input spectrum while the estimation of mode shapes was only possible for the first dominant mode shape when the input signal was not Gaussian white noise. The level of noise in the output signal did not show a significant effect on the estimation of modal parameters supporting the conclusion that the inaccuracies are attributed to the characteristics of the input signals. Further research should therefore be performed on the estimation of mode shapes when the input is not Gaussian white noise.

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