EFFECT OF MATERIAL DAMPING ON THE DYNAMIC AXIAL RESPONSE OF PILE FOUNDATIONS

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Abstract: Novak's elastic model is commonly used to predict the dynamic response of pile foundations. The primary source of energy dissipation within the idealized elastic model (i.e. damping) is of the geometric type as deformation waves propagate throughout the surrounding soil media. In addition to geometric damping, there is simultaneous internal material damping that occurs within the soil and pile materials, which may be incorporated as an out-of-phase compliment to the material stiffness values. Material damping is generally neglected from the analysis (or is assumed to be near-zero) when predicting the foundations dynamic response with Novak's model. The present study demonstrates the potential consequences of neglecting material damping when performing dynamic analysis of pile foundations. This paper focuses on the axial vibration of an individual pile, and the influence of material damping on the dynamic response is investigated.

1   INTRODUCTION

Pile foundations often experience dynamic loads, such as those produced by vibrating machinery. The designing engineer must use an appropriate soil-pile formulation to adequately predict the dynamic response. Experimental data has shown that the theoretical formulations reported in the literature can produce satisfactory results, provided that the appropriate material properties are used for analysis (Novak and Griggs, 1976; Puri, 1988; Elkasabgy and El Naggar, 2013).

It is common practice in Canada and the United States to use the elastic formulation developed by Novak (1974) for the dynamic design of pile foundations (Canadian Geotechnical Society, 2006; U.S. Naval Facilities Engineering Command, 1983). The focus of the present study is on the axial vibration case, for which Novak's model was later improved to account for the pile tip condition (Novak, 1977), and simplified design charts were developed for use in practice (Novak and El Sharnouby, 1983). Bryden et al. (2017) reformulated Novak's (1977) mathematical expressions and presented explicit solutions that are easily programmed in spreadsheet software.

A key component of the theoretical analysis involves the damping characteristics of the soil-pile system. When a pile foundation is subjected to dynamic loading, two forms of energy dissipation occur simultaneously: 1) geometric damping, and 2) internal material damping. Geometric damping refers to the reduction in local energy caused by the increase in wave front surface area during propagation away from the source, and is included in Novak's (1977) formulation. Material damping, on the other hand, refers to the energy loss induced by particle deformations, inter-particle friction, and heat generation, and is included with material damping parameters. In practice, material damping is generally neglected from analysis for

simplicity; geometric damping is the dominant form of energy dissipation, and material damping is assumed negligible (Novak, 1977).

The focus of the present study is to assess the implications of neglecting material damping in the design of pile foundations subject to axial vibration. The commonly employed elastic model for the axial vibration of an individual pile is summarized, and material damping values reported in the literature are reviewed. Parametric studies are then performed using Maple software to investigate the effect of material damping on the dynamic response. It is shown that the exclusion of material damping can significantly over-estimate the resonant amplitude, particularly for piles embedded in soft and loose soil.

2   NOVAK'S (1977) ELASTIC THEORY

It is assumed that the pile is elastic, vertical, circular in cross section, and maintains perfect contact with the soil. The soil media is divided in two elastic regions: the adjacent soil, and base soil, as indicated in Figure 1.
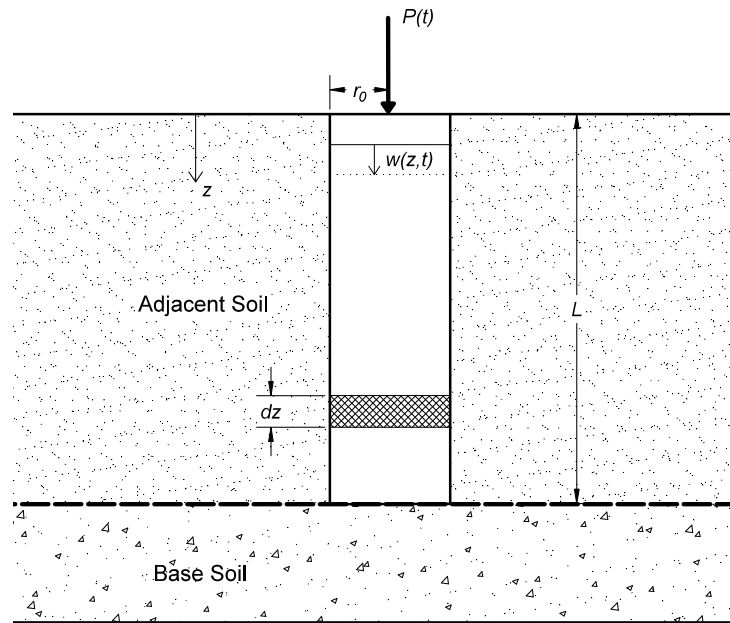

Figure 1: Conceptual Diagram of Novak's (1977) Soil-Pile Model

When a dynamic axial load $P(t)$ is applied at the pile head, the pile shaft experiences vertical deformation w at depth z; the governing differential equation is expressed in Equation 1.

[1] $\dfrac{d^2w(z)}{dz^2} + \left\{ \dfrac{1}{EA} \left[\mu\omega^2 - G_s S_{w1} - i G_s S_{w2} \right] \right\} w(z) = 0$

Where A, E, and μ represent the pile cross sectional area, modulus of elasticity, and mass per unit length, respectively. G_s is the shear modulus of the adjacent soil, i is the imaginary unit, ω is the frequency of applied loading, and S_{w1} and S_{w2} are the adjacent soil reaction parameters derived by Baranov (1967). The adjacent soil reaction parameters are expressed in Equation 2.

[2a] $S_{w1} = 2\pi a_s \dfrac{J_1(a_s)J_0(a_s) + Y_1(a_s)Y_0(a_s)}{J_0^2(a_s) + Y_0^2(a_s)}$

[2b] $S_{w2} = \dfrac{4}{J_0^2(a_s) + Y_0^2(a_s)}$

Where a_s, termed the dimensionless frequency, is dependent on the pile radius r_0, adjacent soil shear wave velocity V_s, and vibration frequency ω, as defined in Equation 3.

[3] $a_s = \frac{r_0 \omega}{V_s}$

Two boundary conditions are then imposed: 1) the pile head is subjected to a unit axial deformation, and 2) the soil reaction at the pile tip is defined as that of a rigid circular disk resting on an elastic half-space. The boundary condition defining the pile-tip reaction is expressed in Equation 4.

[4] $w'(L) = -\frac{G_b r_0}{EA}(C_{w1} + iC_{w2})w(L)$

Where the notation w' implies differentiation with respect to depth z, and L and G_b represent the pile length and base soil shear modulus, respectively. C_{w1} and C_{w2} are base soil reaction parameters that depend on the base soil Poisson ratio, and are defined in Equation 5 for a Poisson ratio of 0.25 (refer to Novak 1977 for additional explanations).

[5a] $C_{w1} = 5.33 + 0.364a_b - 1.41a_b^2$

[5b] $C_{w2} = 5.06a_b$

Where a_b is the dimensionless frequency expressed with base soil parameters (i.e. it is equivalent to that defined in Equation 3, except that V_s is replaced with V_b – the shear wave velocity of the base soil). The complex stiffness of the soil-pile system K^* is then defined as:

[6] $K^* = -EA\frac{w'(0)}{w(0)} = k + ih$

Where k and h represent equivalent stiffness and damping coefficients of the soil-pile system, respectively, and are obtained by isolating the real and imaginary components of the complex stiffness.

The pile-head response may then be defined by the governing equation of motion for a single degree of freedom system; the dimensionless amplitude of steady state vibration is equal to:

[7] $A_w = \dfrac{\omega^2}{\sqrt{\left(\frac{k}{M}-\omega^2\right)^2+\left(\frac{h}{M}\right)^2}}$

Where M is the inertial mass at the pile head (i.e. the superstructure).

Equation 7 is used to generate the dynamic response at the pile head, where the stiffness and damping parameters k and h are obtained from Equation 6 (Novak's elastic formulation). Geometric damping is included within the formulation through the soil reaction parameters S_{w1}, S_{w2}, C_{w1}, and C_{w2}. Material damping may be accounted for by replacing the material stiffness's G_s, G_b, and E with their complex form, thereby incorporating the out-of-phase compliment to the material stiffness values. The complex form of the material stiffness's are expressed as follows:

[8a] $G_s \rightarrow G_s^* = G_s(1 + i\tan\delta_s)$

[8b] $G_b \rightarrow G_b^* = G_b(1 + i\tan\delta_b)$

[8c] $E \rightarrow E^* = E(1 + i\tan\delta_p)$

Where the parameters δ_s, δ_b, and δ_p represent the loss angles of the adjacent soil, base soil, and pile materials, respectively. The loss angle of material j is related to its material damping ratio ζ_j by Equation 9.

[9] $\tan\delta_j = 2\zeta_j$

Novak (1977) reports that soil material damping values are typically in the range of $\tan\delta_{s,b}$ = 0.10, and concludes that the omission of material damping is conservative. The design charts commonly used by practicing engineers are developed based on soil and pile material damping values of $\tan\delta_{s,b}$ and $\tan\delta_p$ equal to 0.05 and 0.01, respectively (Novak and El Sharnouby, 1983). These assumed loss factors correspond to soil and pile material damping ratios of 2.5% and 0.5%, respectively.

Typical damping ratios have been reported as: less than 1% for steel (Dassault Systems, 2018), less than 5% for reinforced concrete (Hesameddin et al., 2015), and from 2% to 10% for timber (Bremaud et al., 2009). The pile material loss factor assumed within the design charts is therefore appropriate for most

modern steel piles, but potentially underestimates the material damping characteristics of concrete and timber piles. The material damping characteristics of soils are discussed in the proceeding section.

3   SOIL MATERIAL DAMPING

It has been shown that the material damping of soil is appropriately modeled as frequency independent hysteretic damping (Meek and Wolf, 1994). During dynamic loading, the stress-strain curve differs during the loading and unloading phase, which produces a hysteresis loop. The area of the loop represents the energy loss per cycle, and is related to the material damping parameter (Stewart and Campanella, 1993). A generic hysteresis loop is displayed schematically in Figure 4.
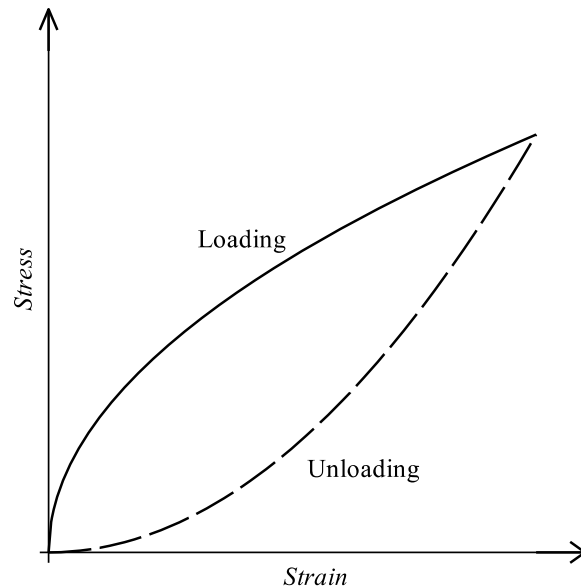

Figure 2: Hysteresis Loop of a Dynamic Load Cycle

Soil material damping can be incorporated in Equations 1 through 6 by simply redefining the soil reaction parameters (S_{w1}, S_{w2}, C_{w1}, and C_{w2}); all other expressions remain un-altered. Figures 2 and 3 are graphical representations of the soil reaction parameters for various material damping values.
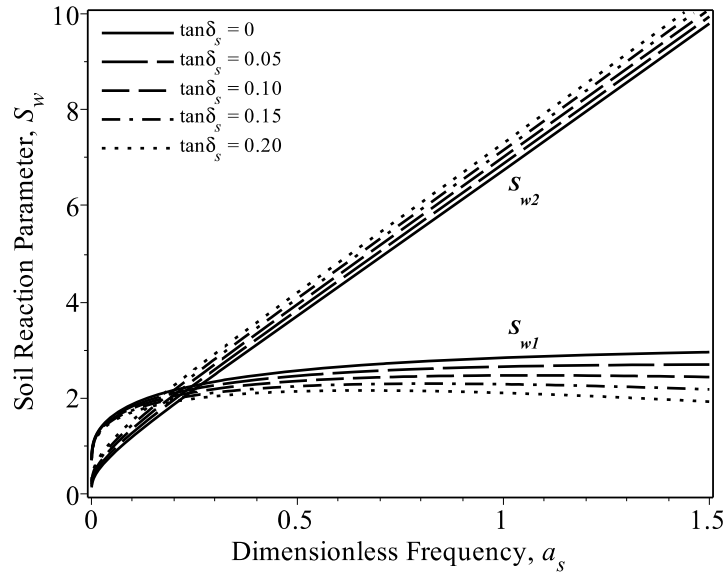
Figure 3: Adjacent Soil Reaction Parameters for Various Material Damping Values
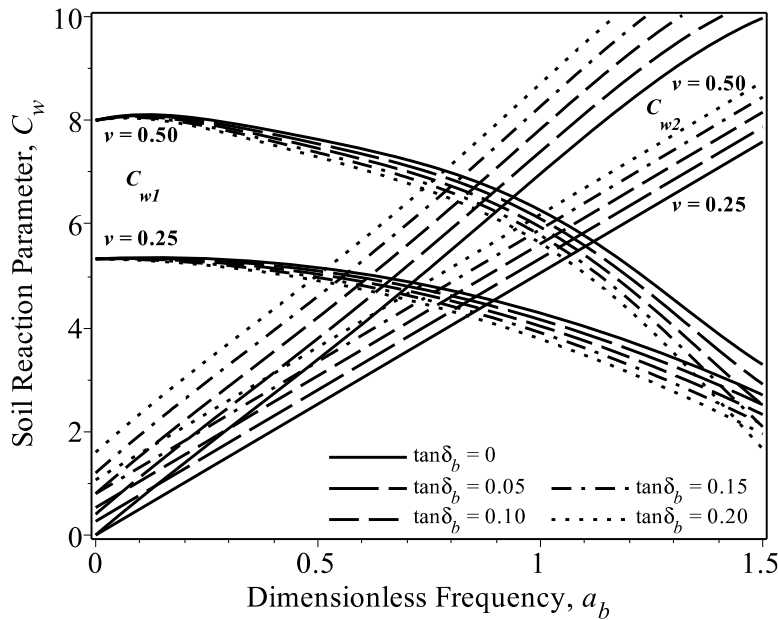

Figure 4: Base Soil Reaction Parameters for Various Material Damping Values

Material damping of soil is commonly obtained experimentally by laboratory test procedures, including: the resonant column test, torsional shear test, or cyclic triaxial test, which are performed with recovered undisturbed samples (Ashmawy et al., 1995). Typical laboratory values for soil material damping at small strains (i.e. less than 10^{-5}) have been shown to range from 0.5% to 2% for sands (Seed et al., 1986) and from 1% to 5% for clays (Stewart and Campanella, 1993). It is shown that the material-damping ratio is highly dependent on the state of strain; the material damping value of sand can increase to 9% at strains of 10^{-4}, and can exceed 20% at strains of 10^{-3} (Seed et al., 1986).

In-situ field techniques, including cross-hole seismic tests, seismic cone penetration tests, or surface wave tests, are also used to obtain dynamic soil properties (Lai et al., 2002; Stewart and Campanella, 1993).

Field techniques are often considered preferable in comparison to laboratory tests for obtaining dynamic properties; field procedures eliminate the potential for sample disturbance, while producing dynamic properties that are representative of a considerably larger sample size (Badsar et al., 2010). Field measurements of material damping ratios at small strains have generally produced values of less than 5%, however, values as high as 12% have been reported for alluvial deposits (Stewart and Campanella, 1993).

In practice, material damping of soil and pile materials are commonly neglected from dynamic analysis (or are assumed to be near-zero). This assumption greatly simplifies the mathematical expressions for analysis, and is a conservative design assumption as material damping acts to reduce the amplitude of dynamic deformations. However, there has been little research assessing the impact that material damping can have on the dynamic response. To assess the effect of material damping, a series of hypothetical problems are investigated analytically using Novak's (1977) elastic formulation.

4   DESCRIPTION OF CASE STUDIES

Four problems are analyzed to assess the influence of material damping on the dynamic response. Each problem involves a single pile subjected to axial dynamic loading. The problems are selected so to capture a wide range of responses, and include: 1) a stiff pile in stiff soil, 2) a soft pile in stiff soil, 3) a stiff pile in soft soil, and 4) a soft pile in soft soil. Stiff soils are those with a relative density of *dense* (SPT N-value > 30) or undrained shear strength greater than 100 kPa, while soft soils are those with a relative density of *loose* (SPT N-value < 4) or undrained shear strength of less than 25 kPa (for cohesionless and cohesive soils, respectively). Concrete and timber piles are used to represent stiff and soft pile materials, respectively. Table 1 specifies the material properties used for each problem, which are selected to represent realistic physical scenarios.

Table 1: Soil-Pile Configurations of Analysis

Material Property	Case 1	Case 2	Case 3	Case 4
Pile Properties				
Material	Concrete	Timber	Concrete	Timber
Modulus of Elasticity, E (GPa)	22.0	10.0	22.0	10.0
Unit Weight, γ_p (kN/m^3)	23.5	9.5	23.5	9.5
Material Damping, $\tan\delta_p$	0.05	0.15	0.05	0.15
Soil Properties				
Shear Wave Velocity of Adjacent Soil, V_s (m/s)	200	200	60	60
Unit Weight, γ_s (kN/m^3)	20.0	20.0	18.0	18.0
Material Damping, $\tan\delta_s$	0.05	0.05	0.24	0.24

Each pile defined in Table 1 has length and radius equal to 10.0 meters and 0.20 meters, respectively, and an inertial mass of 20,000 kg is assumed to rest on the pile head. Base soil properties are not defined in Table 1, as the two extreme cases of floating ($V_b = V_s$) and end bearing (V_b = 10,000 V_s) will be investigated for each case. For the purpose of this study, it is assumed that the Poisson ratio of the base soil is equal to 0.25. The base soil loss factor ($\tan\delta_b$) is equivalent to that of the adjacent soil for the floating case, and is selected as 0.05 for the end bearing case. Recall that the shear modulus of material j is related to the shear wave velocity as defined in Equation 10.

$$[10]\ V_j = \sqrt{\frac{G_j}{\rho_j}}$$

Where ρ_j is the mass density of material j.

5   RESULTS AND DISCUSSION

The dynamic axial response of each problem is computed with Equation 7, where the stiffness and damping parameters k and h are obtained from Novak's (1977) elastic model (i.e. Equation 6). The dynamic response of cases 1, 2, 3, and 4 (as defined in Table 1) are plotted in Figures 5, 6, 7, and 8, respectively. Each figure displays the response, both with and without the inclusion of material damping, for floating and end bearing conditions.
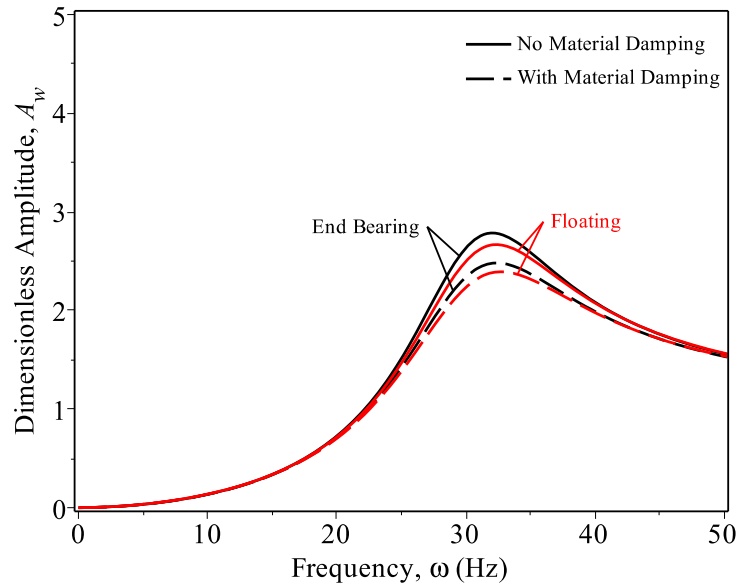

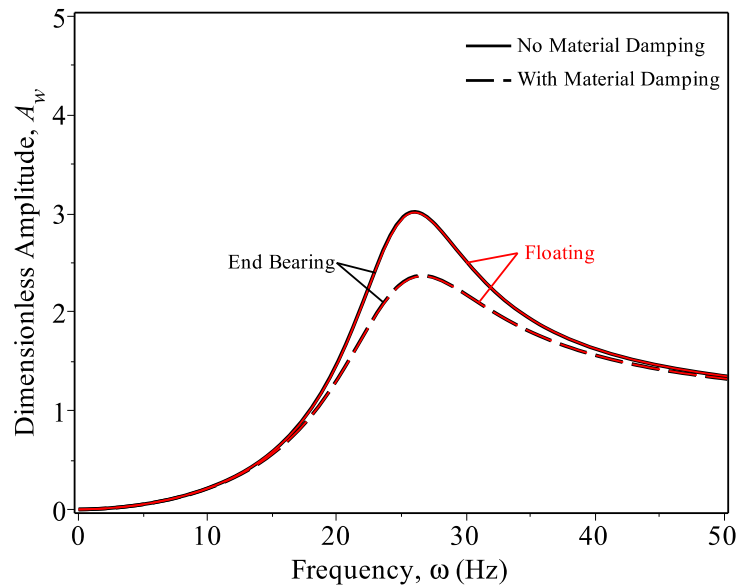Figure 5: Dynamic Response of Stiff Pile in Stiff Soil (Case 1)

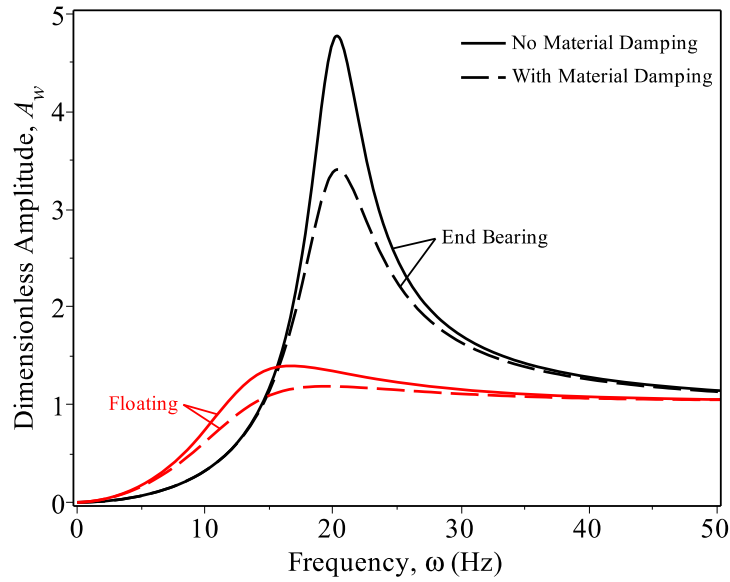Figure 6: Dynamic Response of Soft Pile in Stiff Soil (Case 2)

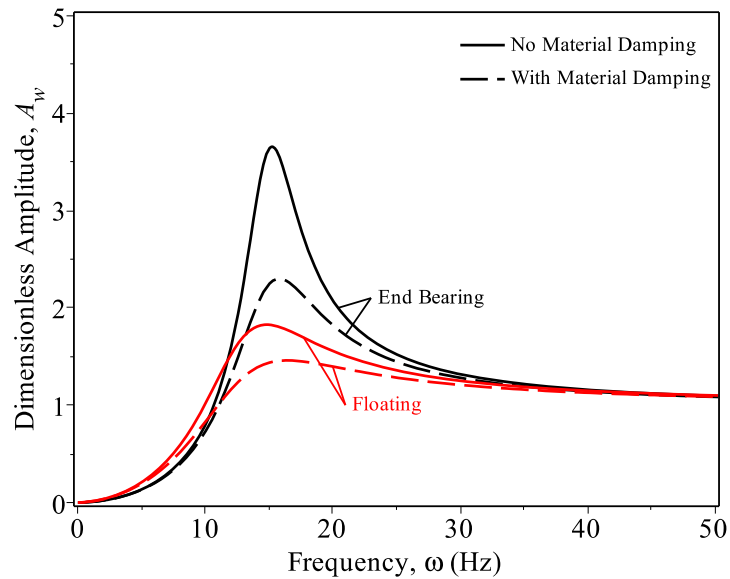Figure 7: Dynamic Response of Stiff Pile in Soft Soil (Case 3)

Figure 8: Dynamic Response of Soft Pile in Soft Soil (Case 4)

As shown in Figures 5 through 8, material damping acts to reduce the resonant amplitude in all cases, while the resonant frequency is observed to remain nearly unaffected. The reduction in amplitude ranged from 11% to 21% for the floating piles, and from 11% to 37% for the end-bearing piles. The reductions in resonant amplitude due to material damping for each case are summarized in Table 2. It is worth noting that the decrease in amplitude is more pronounced for the soft soil scenarios (cases 3 and 4) due to the higher soil loss factor. The pile tip condition is also observed to influence the degree to which material damping influences the response; the reduction in resonant amplitude is nearly doubled for the end bearing piles in soft soil compared to the floating piles.

Table 2: Reduction in Resonant Amplitude due to Material Damping

Pile-Tip Condition	Case 1	Case 2	Case 3	Case 4
Floating	11%	21%	16%	20%
End Bearing	11%	21%	29%	37%

Although it is conservative to neglect material damping for dynamic design of pile foundations, such damping should ideally be included in the analysis. This analytical study shows that the resonant amplitude can be over-estimated by neglecting material damping, particularly for piles founded in soft soils. Note that all piles assessed in the present study had a slenderness ratio (L/r_0) equal to 50. Additional analytical work is required to investigate the interaction between material damping and slenderness ratio when predicting the dynamic response.

The sample configurations investigated in the present study were selected to capture a broad range of responses, and were based on realistic mechanical properties reported in the literature. The results are for illustration purposes only, and are not intended for use in design. It is recommended that site investigations be completed to determine damping characteristics of a particular project site during the design phase. If the site-specific damping characteristics are unknown, it is recommended that material damping be neglected from the analysis. Damping characteristics of soils are highly variable, and thus its omission ensures a conservative design.

6   CONCLUSIONS

When computing the dynamic response of deep foundations, the material damping of pile and soil constituents are commonly neglected (or assumed near zero) during the analysis. The present study assesses the impact of material damping on the dynamic axial response of an individual pile foundation. Four hypothetical problems, which are selected to span a broad range of material properties, are investigated analytically using Novak's (1977) elastic formulation. It is shown that the predicted resonant amplitude is reduced when material damping is included; the amplitude reduction ranged from 11% to 21% for the floating piles, and from 11% to 37% for the end-bearing piles. The resonant amplitude can therefore be over-estimated by neglecting material damping, particularly for piles founded in soft soils. It is recommended that site-specific investigations be completed such that material damping may be properly accounted-for during the analysis and design of deep foundations subject to dynamic loads.

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