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A COMPARATIVE STUDY OF METHODS FOR ANALYZING ALUMINUM PONY TRUSS STRUCTURES

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Abstract: Truss bridges constructed without overhead bracing between the top chords, also known as "pony trusses" are particularly common in pedestrian bridge applications, where the bridge top chords can also serve as the handrails or barriers. The lack of overhead bracing allows pedestrians and bicyclists to traverse the bridge unimpeded but results in a unique failure mode due to out-of-plane or lateral buckling of the compression chord members. Various methods have been proposed to analyze this phenomenon, including the ones by Holt in 1952, Timoshenko and Gere in 1961, Alberta Transportation in 2016, and the British Standards Institution in 2000. These methods vary significantly in terms of their implementation; some use an equivalent stiffness based on the 'U-frame' stiffness provided by each bay's verticals and diagonals, while others rely on 3D modelling to determine an elastic buckling load. In the current study, all four methods are compared across a range of section properties that affect top chord compression capacity using the example of a 46' (14.0 m) aluminum pedestrian bridge. From the initial comparison, it was found that the Timoshenko and Gere and Alberta Transportation methods gave similar results, while the Holt and BS 5400 methods were relatively conservative. The Timoshenko and Gere and Alberta Transportation methods do not generally align across the investigated range of section properties, however. Given these results, further study is recommended to better assess which method can be considered the most accurate or consistently conservative across the broadest range of bridge configurations.

Keywords: aluminum, pony truss, pedestrian bridges, lateral buckling

1 INTRODUCTION

Trusses can generally be categorized into through, half-through, and deck trusses. In a through trusses the deck sits on floor beams at the level of the bottom chord, which transmit forces to the truss, while deck trusses consist of a deck on top of the top chords of the truss. Through trusses and most deck trusses also make use of bracing along the length of the top chord to prevent a lateral buckling failure of the compression members. A half-through truss, or "pony truss", has a deck at the level of the bottom chord, but does not include bracing along the top of the structure. Typical pony truss examples are shown in Figure 1. Pony trusses have the same failure modes as a through or deck truss, but also require additional design considerations for the top (compression) chord. The top chord of a pony truss may fail due to out-of-plane buckling, which can occur suddenly and at a much lower load level than estimated by assuming planar behaviour. This behaviour is affected not only by the top chord stiffness, but also by the out-of-plane bending stiffness of the diagonal web members and the truss floor beams.

Aluminum has a number of benefits as a building material, in particular its light weight and strong corrosion performance. This makes it particularly useful in harsh environments, areas with poor access, when weight is critical, and when accelerated bridge construction techniques are required. Aluminum has been used as a building material for both vehicular and pedestrian bridges for many years. Prominent vehicular examples include the 1950 Arvida Bridge in Saguenay, QC, and the 1933 rehabilitation of the Smithfield Street Bridge in Pittsburgh, PA (Walbridge and de la Chevrotière 2012). Examples of pedestrian bridges using aluminum include the 1950 Tummel River Bridge in Pitlochry, Scotland, UK and the 1953 Dusseldorf Bridge in Germany (Das and Kaufman 2007), as well as the Daigneault Creek bridge in Brossard, QC (see Figure 1 (right)), which has a span of 44 m and was installed in a day with a single crane.

Figure 1: Typical steel (left) and aluminum (right) pony truss pedestrian bridges.

2 BACKGROUND

Top chord compression failures have been studied for many years, including by Engesser in 1885, Holt from 1951 through 1957, Timoshenko in 1961, and others (Ziemian 2010; Timoshenko and Gere 1989). Design codes generally make little if any mention of specific methods to analyze the capacity of pony truss top chords, and the few that do typically refer to or are based upon the British Standard BS 5400. The Canadian Highway Bridge Design Code (CSA S6) is currently developing an annex based on BS 5400 for use in the next code publication. Holt, Timoshenko, and BS 5400 all make use of a “U-Frame” stiffness directly to calculate the effective length, and from there the failure load of the top chord, while an alternative method proposed by Alberta Transportation uses the elastic buckling load of a structure obtained from a 3D model to calculate the equivalent slenderness of the chord (Alberta Transportation 2016).

2.1 U-Frame Stiffness

Pony trusses rely on the bending stiffness of the diagonals, verticals, and floor beams and their connections to prevent lateral buckling of the top chord. These three elements act together as a frame in the shape of a U as shown in Figure 2, and will henceforth be referred to as U-Frames. The stiffness of the U-Frames determines how the top chord may fail in compression. Relatively stiff U-Frames result in relatively stiff nodes in the top chord, forcing a buckling mode with multiple sine waves, while flexible U-Frames will act as spring supports along the length of the chord, resulting in a single half-sine buckling mode.

The stiffness of a U-Frame is estimated by applying a unit deflection to the top as shown in Figure 2. This is typically calculated ignoring the torsional stiffness of the top chord between adjacent panel points, and just makes use of the vertical, diagonal, and floor beam flexural stiffnesses. Equation **Error! Reference source not found.** shows the equivalent spring stiffnesses for Pratt and Howe type trusses with verticals and diagonals, and Equation **Error! Reference source not found.** shows the spring stiffness for Warren type trusses with paired diagonals. In these equations, E is the modulus of elasticity, d_1 is the height of the truss, d_2 is the length of diagonals, B is the width of the truss, I_1 is the moment of inertia of the verticals, I_2 is the moment of inertia of the floor beams, and I_3 is the moment of inertia of the diagonals. If the U-Frames are spaced consistently along the length of the top chord, these spring stiffnesses can be replaced by an equivalent continuous elastic support, which is the method used by Holt and Timoshenko and Gere. BS 5400 makes use of similar equations to calculate the stiffness of the U-Frames, but also accounts for connection flexibility as discussed in Section 2.4

[1] $C_{pratt}=\frac{E}{\frac{d_1^3}{3I_1}+\frac{Bd_2^2}{2I_2}}$

[2] $C_{warren}=\sum\frac{E}{\frac{d_3^3}{3I_3}+\frac{Bd_2^2}{2I_2}}$

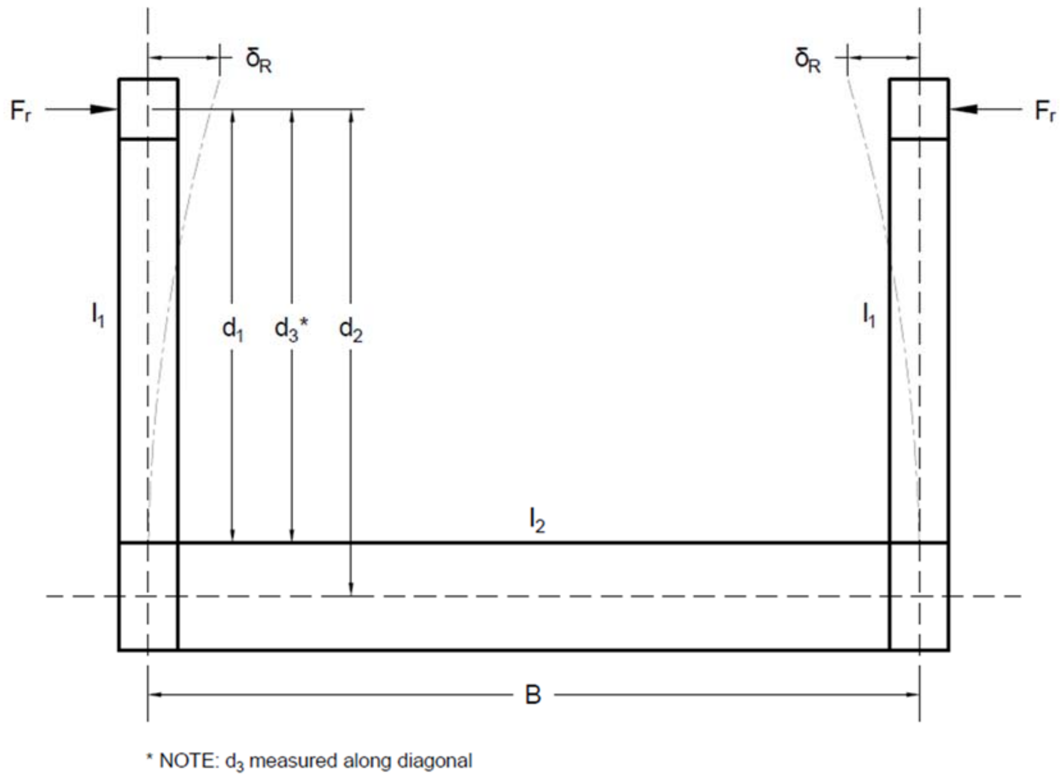

Figure 2: Typical U-Frame in pony truss.

2.2 Holt Method

The Holt method was presented in 1951, with additional work carried out through subsequent years including tests on models of pony truss structures (Ziemian 2010). It assumes that the transverse U-Frames have an identical stiffness along the length of the truss, which is true in many situations. It also assumes that the bridge carries a uniform distributed load (UDL), that the end posts cantilever up to support the ends of the chord and are connected by pins to the top chord, and that the radii of gyration of the top chord members and end posts are identical. The top chord members are all assumed to be designed to the same allowable unit stress, which results in areas and, because the radii of gyration are the same, moments of inertia being proportional to compression force applied (Ziemian 2010). This is not always the case in practice, where the top chord may consist of one single cross section for its whole length.

Holt examined other factors to determine their effect on the chord capacity, and found the effects of neglecting the torsional stiffnesses of the chord and diagonals, the lateral support given by the diagonals, the axial stresses on stiffness in the web members, the presence of non-parallel trusses, and considering the chord and end posts as a single straight member to be minimal. This allows the designer to calculate the compression chord capacity by considering the entire chord as an elastically supported column with a length equal to that of the entire top chord plus the end posts (Ziemian 2010).

To apply the Holt method, designers first design the floor beams and diagonal web members for their loads as normal. They then calculate the spring constant, C, for the U-Frame. A parameter, $C \cdot \ell / P_{cr}$, is then calculated, where ℓ is the length of a single panel and P_{cr} is the maximum chord design load. Based on the calculated parameter and the number of panels in the structure, a design table present in the *Guide to Stability Design Criteria for Metal Structures* (2010) can be used to find a corresponding $1/K$ value. This can be used as normal to calculate the slenderness, λ, of the section and the section capacity.

2.3 Timoshenko and Gere Method

Timoshenko and Gere published their work on analyzing pony trusses in 1961 in the *Theory of Elastic Stability* (1989). Their method assumes that the bridge carries a UDL, which results in a distributed axial load transmitted from the diagonals to the top chord of the structure, which varies from 0 at the centre to a maximum, q_0, at the end. The top chord of the structure is supported by intermediate U-Frames, which are approximated by a continuous elastic foundation with an elastic modulus β. For this to be true, the half-wave length of the buckled chord must be large compared to one panel length and is recommended to be greater than 3 panel lengths (Timoshenko and Gere 1989). The ends are assumed to be immovable in the lateral direction, which is often approximately true due to very stiff end frames and the supports adding stiffness to the end frame. If the elastic foundation has low rigidity, the chord will buckle in a half sine-wave that is symmetric about the middle. As the restraint from the U-Frames increases, the chord will begin to buckle in a full sine-wave (S) shape with an inflection point at the middle, and eventually into as many half-waves as there are panels. This shows that the stability of the compression chord can be increased by increasing the rigidity of the U-Frames supporting the chord laterally.

This method is used similarly to the Holt method, with designers beginning by designing the floor beams and diagonal web members. They also calculate the U-Frame spring constant, C. The method then uses C to find an elastic modulus of the continuous elastic foundation, β, given by Equation 3.

[3] $\beta = C / c$ (Timoshenko and Gere 1989)

where C is the spring stiffness of the U-Frame from Equation **Error! Reference source not found.** or **Error! Reference source not found.** and c is the spacing between panel points. This is used to calculate a parameter $\beta \cdot \ell^4 / (16 \cdot E \cdot I)$, where I is the lateral moment of inertia of the top chord, ℓ is the length of the whole structure, and E is the elastic modulus of the material. Timoshenko and Gere (1989) provide a table to find the equivalent L/ℓ value, where L is the effective buckling length of the chord. This can be used to then find the slenderness and capacity of the section as usual.

2.4 British Standard BS 5400

British Standard BS 5400 can be used to find an effective length using the largest of Equations 4, 5, and 6.

[4] $L_e = k_2 \cdot k_3 \cdot k_5 \cdot \ell_1$

[5] $L_e = k_3 \cdot \ell_R$

[6] $L_e = \pi \cdot k_2 \{E \cdot I_c \cdot (\delta_{e1} + \delta_{e2})/L)\}^{0.5}$ (British Standards Institution 2001)

where k_2 is a factor that accounts for whether the load is applied to the bottom or top chord of the truss, k_3 is a factor that accounts for how free the compression chord is to rotate at its ends, k_5 is a coefficient that is related to the relative lateral deflections of the U-Frame at the supports and is found via Equation 7, ℓ_1 is calculated by Equation 8, ℓ_R is the spacing of the U-Frames, I_c is the lateral stiffness of the top chord, δ_{e1} and δ_{e2} are the lateral deflections at the support at ends 1 and 2, $\delta_{e,max}$ is the larger of δ_{e1} and δ_{e2}, δ_R is the lateral deflection at an interior U-Frame, and L is the total length of the structure. If the U-Frame stiffness is consistent along the whole length of the structure, δ_{e1} and δ_{e2} will be equal to δ_R. Equation 4 finds the equivalent buckling length based on the load application point, the rotational restraint provided to the top chords at the ends of the truss, the U-Frame stiffness, and the top chord stiffness, while Equation 5 finds the equivalent buckling length based on a single panel buckling and the torsional restraint provided by the truss. Lastly, Equation 6 finds the equivalent buckling length based on the continuous elastic foundation provided by the U-Frames assuming a simply supported span. δ_R is found based on a U-Frame analysis, which is conducted similarly to the Holt and Timoshenko and Gere methods with the addition of a term that accounts for the floor beam to diagonal connection stiffness and a term that accounts for the connection between the floor beams and bottom chord. These differences are captured in Equation 10.

[7] $k_5 = 2.22 + 0.69 / (X + 0.5)$

[8] $\ell_1 = (E \cdot I_c \cdot \ell_R \cdot \delta_R)^{0.25}$

[9] $X = \frac{L_1^3}{\sqrt{2} \cdot E \cdot I_c \cdot \delta_{e,max}}$

[10] $\delta_i = \frac{d_3^3}{3 \cdot E \cdot I_3} + \frac{B \cdot d_2^2}{2 \cdot E \cdot I_2} + f \cdot d_2^2 + \theta \cdot s$ (British Standards Institution 2001)

Designers implement this method by calculating the U-Frame stiffness using Equation 10, followed by calculating the effective length factors. From there, they can calculate the slenderness and capacity of the compression chord of the structure. While it is slightly more complex than the Holt or Timoshenko and Gere methods in execution, it can be easily implemented in design spreadsheets for repeated use.

2.5 Alberta Method

Alberta Transportation publishes the *Bridge Load Evaluation Manual*, which is used as a supplement to the Canadian Highway Bridge Design Code. This document includes a method to calculate the buckling resistance of pony truss top chords that relies on a three-dimensional model of the bridge to be analyzed. The model members are rigidly connected together, and it is not permitted to use the additional stiffness that may be provided by the deck (Alberta Transportation 2016). The model is used to calculate the elastic buckling resistance of the top chord, which is used to find λ from Equation 11.

[11] $\lambda = (P_y / P_e)^{0.5}$ (Alberta Transportation 2016)

where P_y is the yield resistance of the pony truss top chord and P_e is the elastic buckling resistance of the pony truss top chord. The value of λ thus obtained corresponds to $\bar{\lambda}$ in Equation 12, which comes from CSA S6 Clause 17.11.2, and can then be used to calculate the section capacity.

[12] $\bar{\lambda} = (F_o / F_e)^{0.5}$ (CSA Group 2014)

where F_o is the limiting stress, and F_e is the elastic buckling stress.

3 DESIGN METHOD COMPARISON

An aluminum pony truss structure was analyzed to compare the four methods. The structure chosen was a Warren truss structure with a length of 14000 mm, width of 1400 mm, and height of 1400 mm. It consisted of 7 bays, each spanning 2000 mm. A model of the bridge was created using the software SAP 2000 to analyze the effects of changing the top chord, diagonal, and floor beam stiffnesses.

3.1 Properties

The properties of the modelled bridge are shown in Table 1. The moments of inertia were varied to determine the effect that those properties had on the capacity of the structure. The top chord lateral moment of inertia was increased in increments of 2,500,000 mm^4 from 2,500,000 mm^4 to 20,000,000 mm^4, the diagonal lateral moment of inertia was increased in increments of 75,000 mm^4 from 75,000 mm^4 to 600,000 mm^4, and the floor beam vertical moment of inertia was varied by increments of 150,000 mm^4 from

150,000 mm^4 to 1,200,000 mm^4. The areas of the sections were not varied, and therefore the radii of gyration varied in proportion to the moments of inertia. The member torsional constants were not varied, while the shear centres were assumed to coincide with the centres of gravity. The base model properties corresponded with an intermediate moment of inertia (3rd increment) for each of the three member types that were varied. A screenshot of the SAP2000 model can be seen in Figure 3.

Table 1: SAP2000 Base Model Properties

Section	A_g (mm^2)	I_x (mm^4)	r_x (mm)	I_y (mm^4)	r_y (mm)	J (mm^4)
Bottom Chord	1750	1500000	29.277	1500000	29.277	3000000
Top Chord	3000	7500000	50.000	7500000	50.000	15000000
Diagonal	1250	225000	13.416	225000	13.416	450000
Floor Beam	2500	450000	13.416	450000	13.416	900000

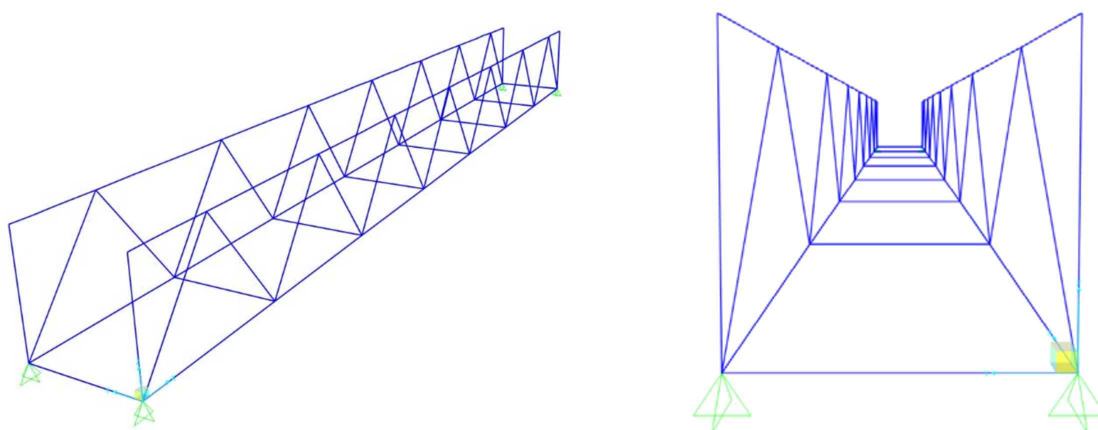

Figure 3: SAP 2000 model of investigated structure.

3.2 Results for Changing Top Chord Stiffness

The compression capacity of the top chords was calculated using each of the four earlier-described methods. Figure 4 shows the effect of changing the lateral (out-of-plane) moment of inertia of the top chord. The Holt and Timoshenko and Gere methods display similar trends, albeit with the Timoshenko and Gere method providing larger capacities and demonstrating some non-linearity at lower stiffnesses.

The Alberta Transportation method results in a capacity that increases in a slightly non-linear fashion, but at a much lower rate than the other two methods. Lastly, the BS 5400 method increases non-linearly until the chord reaches a certain stiffness, at which time the chord gains no additional strength due to further increases in the moment of inertia. This plateau is due to the effective length becoming dominated by Equation 6, which is related to the square root of the moment of inertia. The slenderness of the column is dependent on the radius of gyration, which is also related to the square root of the moment of inertia, so the slenderness of the column remains constant at these increased stiffnesses.

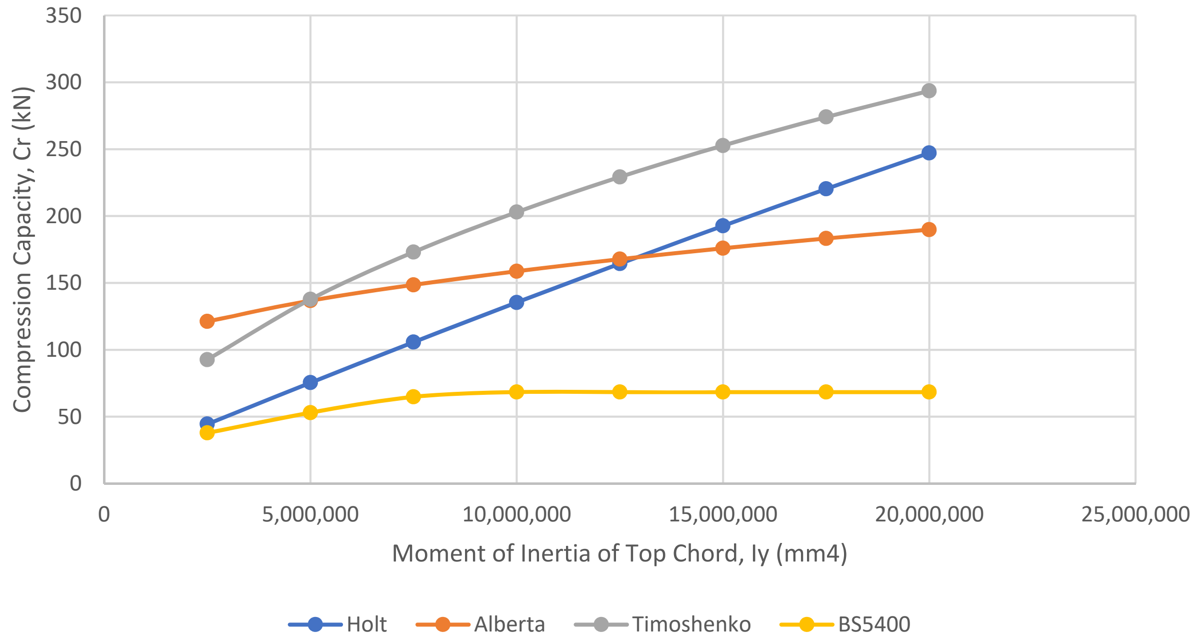

Figure 4: Compression capacity of top chord vs. lateral moment of inertia of top chord.

3.3 Results for Changing U-Frame Stiffness

The U-Frame stiffness is primarily affected by the stiffnesses of the diagonals and floors beams, which are in turn affected by their moments of inertia. The moments of inertia of each of the diagonals and floor beams were varied and the capacities were calculated and plotted against the U-Frame stiffness. Figure 5 shows the resulting plot of the capacities calculated by each method.

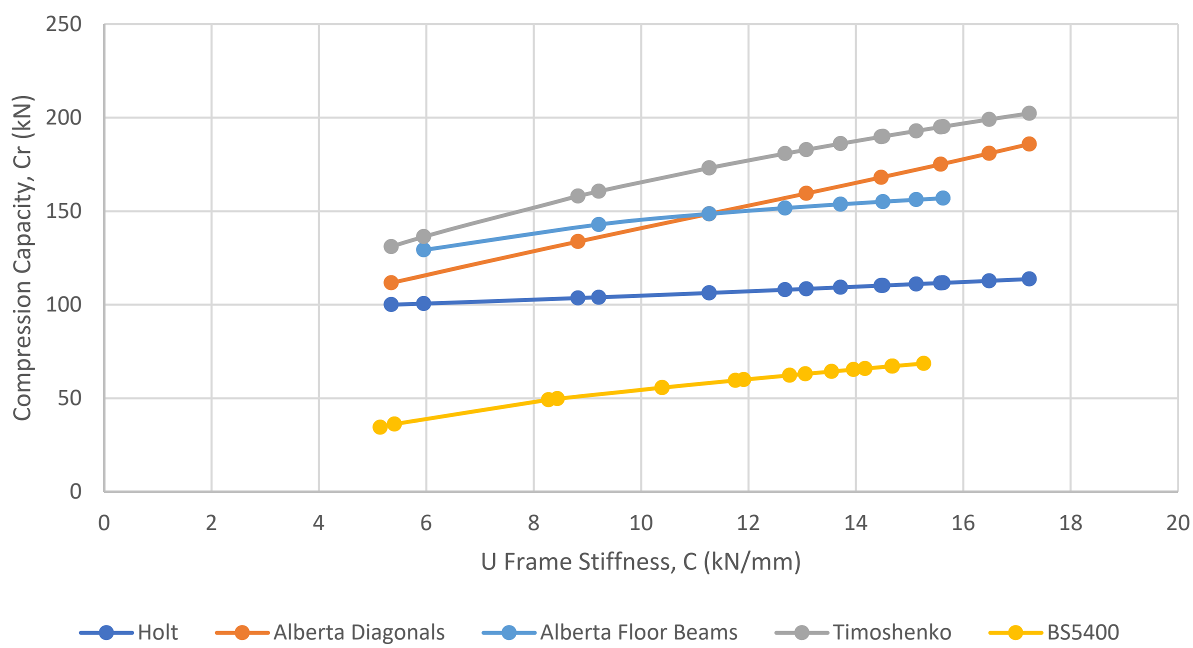

Figure 5: Compression capacity of top chord vs. U-Frame stiffness.

Figure 6 and Figure 7 were created to further investigate the effects of the floor beams and diagonals on the capacity. The Timoshenko and Gere, Alberta Transportation, and BS 5400 methods all demonstrate similar trends for the changes in both the floor beams and diagonals, with the Timoshenko and Gere method typically producing the largest capacities, the Alberta Transportation method producing a slightly lower capacity, and BS 5400 producing the lowest capacity. The Holt method in both cases shows little change as the floor beam or diagonal moments of inertia are varied.

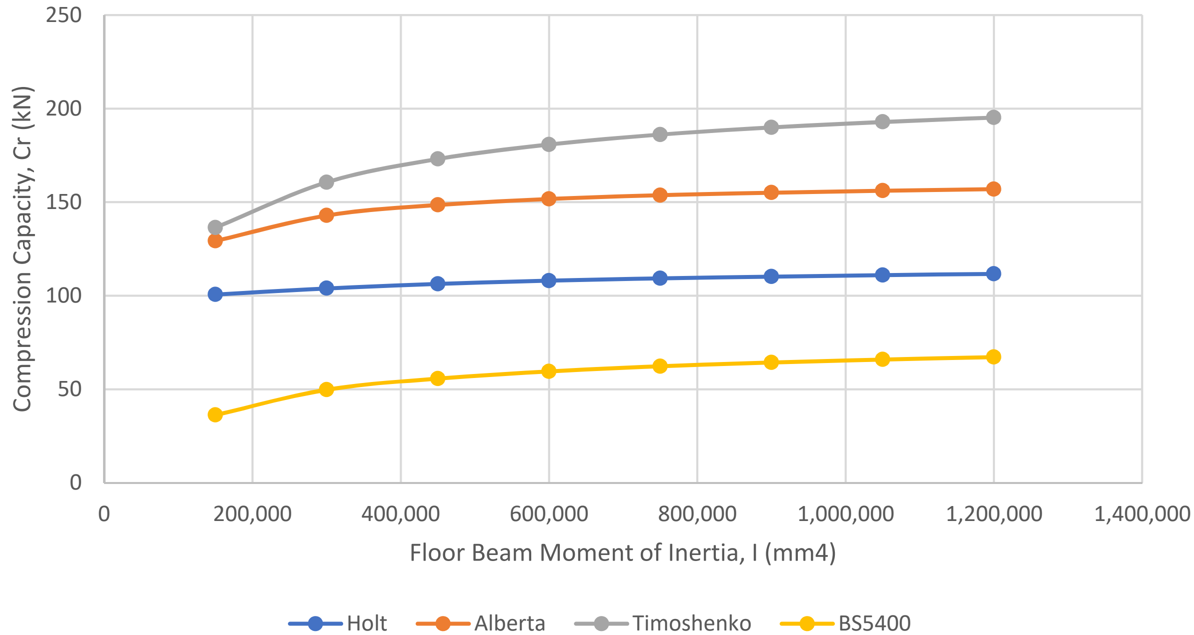

Figure 6: Compression capacity of top chord vs. floor beam moment of inertia.

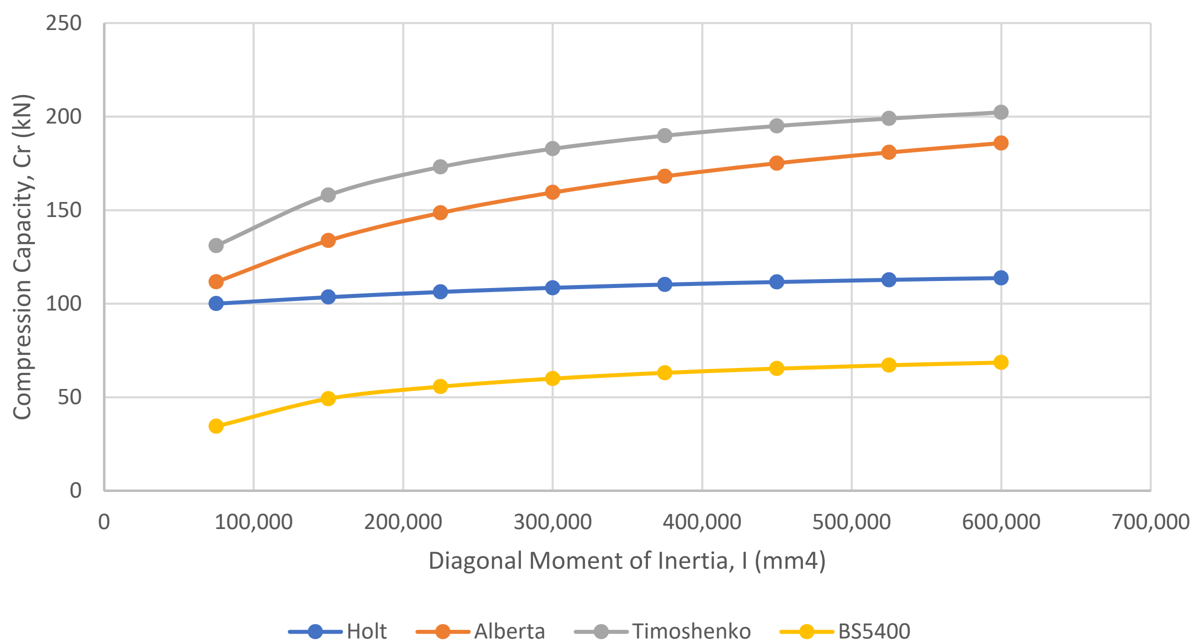

Figure 7: Compression capacity of top chord vs. diagonal moment of inertia.

3.4 Discussion

The differences between the methods reflect the uncertainty in calculating the capacity of pony truss structures. The Timoshenko and Gere method consistently returns the highest compression capacity, except for when the top chord has a relatively small lateral stiffness. BS 5400 consistently returns the most conservative compression capacity. As the top chord moment of inertia increases, BS 5400 becomes limited by Equation [6, which is due to L and r both increasing with the square root of I, resulting in a constant slenderness. The Holt method is affected much more by increases in the top chord lateral stiffness as opposed to increases in the U-Frame stiffness. This is due to the moment of inertia influencing the radius of gyration, and therefore the slenderness of the section, directly, while the U-Frame stiffness influences the capacity more subtly through the use of the $C \cdot L / P_{cr}$ parameter and design tables.

Only one of the methods differentiated between increased stiffness of the floor beams and the diagonals. The Timoshenko and Gere, Holt, and BS 5400 methods are all calculated using the stiffness of the U-Frame, and do not otherwise differentiate between the diagonal and floor beam stiffnesses. The Alberta Transportation method, on the other hand, relies on a 3D model to calculate the effective buckling length, and this model is affected by the stiffness of the floor beams and diagonals differently. The governing buckling mode found using the 3D SAP 2000 model was the lateral-twisting mode shown in Figure 8 rather than the symmetric U-Frame buckling mode assumed by the other methods. This mode corresponds to the smallest lateral stiffness in the model. It would appear that for this buckling mode, the influences of the floor beam and diagonal member moments of inertia on the compression chord effective length are not the same, which means that variations in the frame lateral stiffness (i.e. calculated assuming the deflected shape in Figure 2) have a different effect on the compression chord capacity, depending on whether they are achieved by varying the floor beam stiffness or the diagonal stiffness.

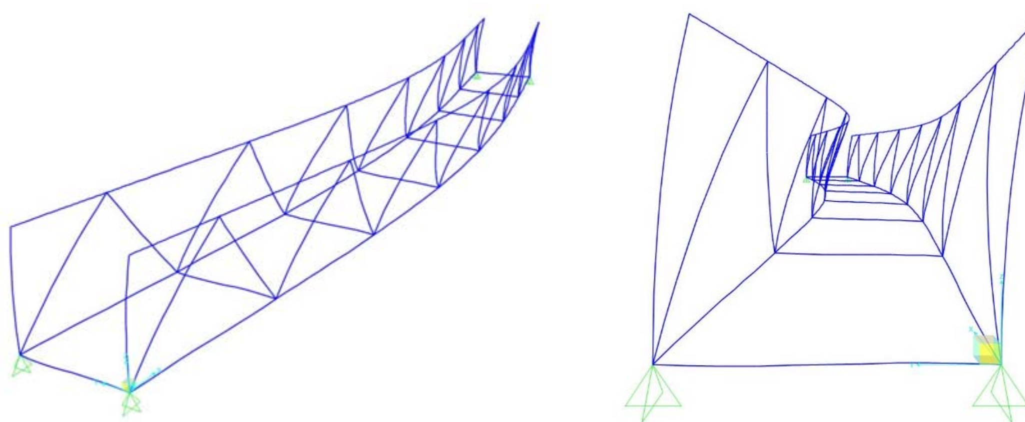

Figure 8: Lateral-twisting buckling mode.

4 CONCLUSIONS

This study has examined several methods for calculating the compression chord capacity of an aluminum pony truss structure. Four different methods were examined: the Holt method, proposed in the 1950s; the Timoshenko and Gere method, proposed in the 1960s; the Alberta Transportation method, created by the Alberta Ministry of Transportation; and the BS 5400 method. These methods were found to vary significantly as the section properties changed. In general, the Timoshenko and Gere method was found to predict the largest capacities except for relatively small top chord lateral stiffnesses, while the BS 5400 method was found to predict the most conservative compression capacities. The Holt and Alberta Transportation methods calculated capacities between the other two methods. The Alberta Transportation method typically predicted higher strengths than the Holt method for the same U-Frame stiffness, but neither method consistently predicted higher strengths for the same top chord stiffness. The Alberta Transportation method also varied depending on whether the floor beams or the diagonals varied their stiffness, while the other methods did not, due to the different buckling mode found using the 3D model.

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