Building Tomorrow's Society
Bâtir la Société de Demain

Fredericton, Canada
June 13 – June 16, 2018/ *Juin 13 – Juin 16, 2018*

FLEXURAL BEHAVIOUR OF REINFORCED CONCRETE BEAMS RETROFITTED USING EXTERNAL SHAPE MEMORY ALLOY BARS

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Abstract: Retrofitting Reinforced Concrete (RC) structures is required to upgrade their capacities and/or address deterioration happening overtime. Several retrofitting techniques and materials are available. More innovative and cost effective retrofitting techniques are continuously being developed. In this study, a new technique for retrofitting RC beams in flexure is introduced. The technique is based on using unbonded superleastic Shape Memory Alloy (SMA) bars. The technique is first assessed using the Finite Element (FE) method. A simplified sectional analysis approach is then presented, validated, and used to conduct an extensive parametric study. Results of the parametric study are used to develop equations to predict changes in the beam behaviour because of the suggested retrofitting technique.

1 INTRODUCTION

The civil infrastructure systems constitute a large portion of the national wealth. Because of ageing and exposure to the environment, they rapidly deteriorate and become more vulnerable to catastrophic failure. Therefore, these structures might need retrofitting to extend their service life. Retrofitting might also be needed to correct design and/or construction errors and to allow changing the structure function.

Examples of available retrofitting techniques for Reinforced Concrete (RC) sections are: (i) concrete jacketing; (ii) attaching steel plates; (iii) applying external post-tensioning; and (iv) using Fibre Reinforced Polymers (FRPs). Flexural retrofitting of RC beams using superelastic Shape Memory Alloy (SMA) bars is another potential technique. Main advantages of superelastic SMA bars are: (i) ability to undergo large deformations and return to their undeformed shape upon unloading (i.e. superelasticity); (ii) ability to dissipate large amounts of energy and release them upon unloading (i.e. flag shape stress-strain relationship) (iii) high resistance to corrosion; and (iv) high resistance to fatigue (Alam et al. 2007, Janke et al. 2005).

In this study, the possibility of using unbonded SMA bars to retrofit RC beams is analytically investigated. A Finite element (FE) model is first developed and validated using ABAQUS software (ABAQUS 2018). A simplified sectional method is then introduced. Results of the suggested method are validated using the FE model. A parametric study is then carried out using the simplified method. Results of the parametric study are used to develop design equations that can capture the change in the flexural behaviour of beams retrofitted using external unbonded SMA bars.

2 RESEARCH SIGNIFICANCE

RC structures are seismically designed to satisfy the strong column-weak beam concept, where the earthquake energy is dissipated in the form of steel yielding. Under moderate to strong earthquakes, severe permanent deformations are expected. These permanent deformations can make the structure irreparable. Thus, there is a need to minimize these permanent deformations. One way to do this is by retrofitting the structure using smart materials such as SMAs. Opposing to steel material, superelastic SMAs have great ability to recover deformations upon unloading. Thus, replacing the internal steel bars with external SMA bars will result in a significant reduction of permanent deformations of the beams.

There are no guidelines or design tools that can be used to determine the optimum amount and length of SMA bars. This study aims at developing simple design equations that can be used by practitioners to retrofit RC beams using external SMA bars.

3 FINITE ELEMENT SIMULATION

Three-dimensional FE models are developed in this study to investigate the behaviour of RC beams retrofitted using external SMA bars during the loading/unloading stages. Analysis is performed using the commercial FE program ABAQUS Version 6.9 (ABAQUS 2018).

Hexahedral (8-node) isoparametric linear solid elements with reduced integration (C3D8R) are used to model the RC beams. Same element type is used to model the internal and external reinforcement, external angles, hold down plates, and mid-span plate.

3.1 Concrete under Compression

The model developed by Scott et al. (1982), Figure 1(a), is used to model the concrete behaviour under compression loading. This model represents a good balance between accuracy and simplicity. During the unloading stage, behaviour of concrete in compression is assumed to follow the model proposed by Karsan and Jirsa (1969). When unloading starts, the material follows linear straight path that connects the strain at the unloading start, ε_r, to the unloading strain at zero-stress, ε_p. After reaching ε_p, the strains continue to reduce while keeping the stress value equal to zero. This continues till reaching the point of zero strain.

3.2 Concrete under Tension

Behaviour of concrete under tension loading is assumed to follow the model proposed by Stevens et al. (1987) and simplified by Youssef and Ghobarah (1999), Figure 1(b). In the pre-cracking zone, the concrete behaves in a linear fashion up to the cracking stress f_{cr}. This is followed by significant reduction in the stress value.

If unloading starts before reaching f_{cr}, the concrete behaves in a linear fashion similar to the loading stage. If unloading starts after reaching f_{cr}, the material follows a linear path with a slope equal to the modulus of elasticity of concrete. After reaching the zero-stress point, the strain continues to decrease while the stress is kept equal to zero. This continues until reaching the point of zero-strain.

3.3 Steel Bars

The behaviour of the steel material is assumed to follow a bilinear stress-strain relationship under both tension and compression loadings, Figure 1(c). The material behaves elastically until reaching its yielding strain, $\varepsilon_{y\text{-}s}$. Then, the modulus of elasticity is significantly reduced.

If unloading starts within the pre-yielding zone, the material behaves in an elastic manner similar to the loading stage with no residual deformations at complete unloading. If the unloading starts within the post-yielding zone, the material follows a linear unloading path until yielding on the other side (tension or compression).

3.4 Superelastic SMA Bars

The stress-strain model of SMA consists of four linear branches that are connected by smooth curves (Alam et al. 2007), Figure 1(d). To simplify the modelling process of the SMA material, the smooth curves are ignored and linear branches are assumed to directly intersect. The material behaves elastically until reaching the SMA critical stress $f_{cr\text{-}SMA}$ which represents the start of the martensite stress induced transformation. Exceeding this limit, the material stiffness significantly reduces to about 10% of its initial value. If loading continues until full transformation to martensite phase occurs, the material regains about 50% of its initial stiffness. If loading continues to the real yielding limit, another significant reduction in the material stiffness occurs.

The behaviour of SMAs during the unloading stage is illustrated in Figure 1(d). If unloading starts before reaching SMAs critical stress, the material behaves in an elastic manner similar to the loading stage (i.e. unloading path 1). If unloading starts when the stress in the material is in between the critical and yielding stresses, the material follows a flag shaped stress-strain relationship (i.e. unloading path 2). If unloading starts after the material reaches its yielding limit), the material follows a linear unloading path (i.e. unloading path 3).

4 EXPERIMENTAL VALIDATION

The experimental work performed by Abdulridha (2013) and Abdulridha et al. (2013) is used to validate the developed model. Six simply supported beams with dimensions of 2800 mm length, 2400 mm span, 125 mm cross-section width, 250 mm cross-section height, and reinforced with SMA and/or steel bars are experimentally tested. All beams are tested under two central point loads spaced at 125 mm around mid-span.

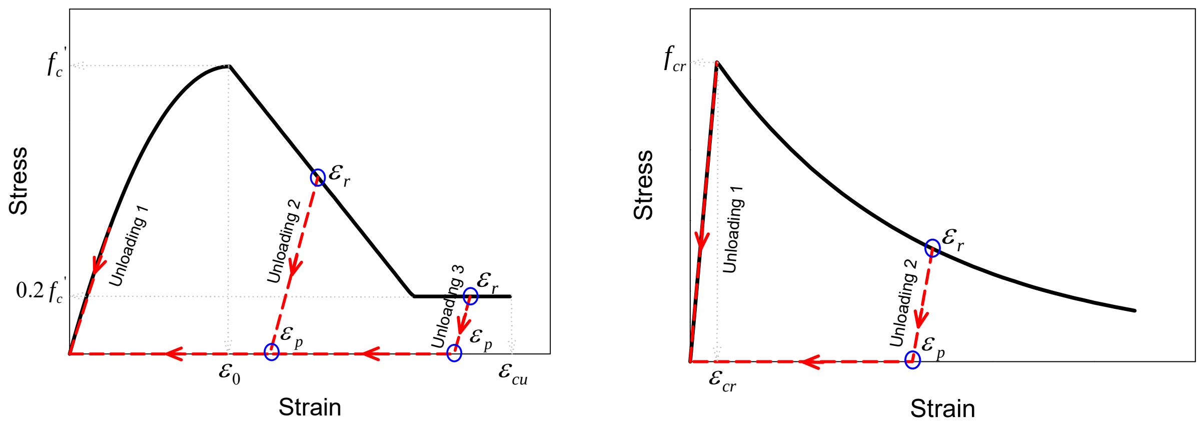
The clear concrete cover of the beams is 20 mm. The average concrete compressive strength is 32.7 MPa for the SMA RC beams and 34.6 MPa for the steel RC beam. The beams are transversely reinforced with 6.35 mm wires spaced at 100 mm.

The length of the SMA bars is 600 mm centred at the mid-span of the beams. The diameter of the middle 300 mm of the SMA bars is reduced to 9.50 mm. M15 steel bars connected the SMA bar using mechanical couplers are used to reinforce the remaining length of the beam. Reinforcement details of the tested beams are summarized in **Table 1**.

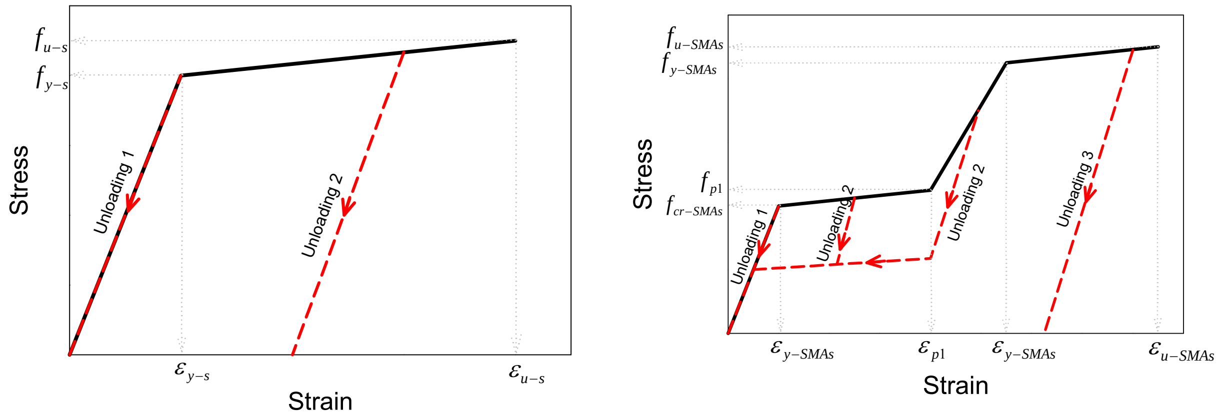
Obtained load-displacement relationships are compared with the experimental results in case of monotonic loading, and with experimental envelopes in case of cyclic and reversed-cyclic loading. Experimental load-displacement results are plotted versus the numerically obtained results in Figure 2 for SMA RC beams. As shown in the figure, very good agreement between experimental and analytical results is obtained for all beams.

Table 1: Details of the tested beams by Abdulridha et al. (2013).

Specimen	Loading type	Reinforcement type at mid-span	Longitudinal reinforcement at mid-span	
			Bottom	Top
B4-NM	Monotonic	SMAs	2 bars, ϕ = 9.5 mm	2 bars, ϕ = 6.35 mm
B6-NR	Cyclic	SMAs	2 bars, ϕ = 9.5 mm	2 bars, ϕ = 9.5 mm
B7-NCM	Cyclic	SMAs	2 bars, ϕ = 9.5 mm	2 bars, ϕ = 9.5 mm



(a) Concrete in compression (b) Concrete in tension

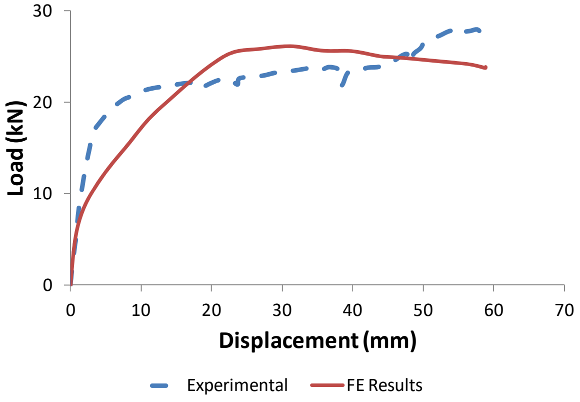


(c) Steel in tension/compression (d) SMA in tension/compression

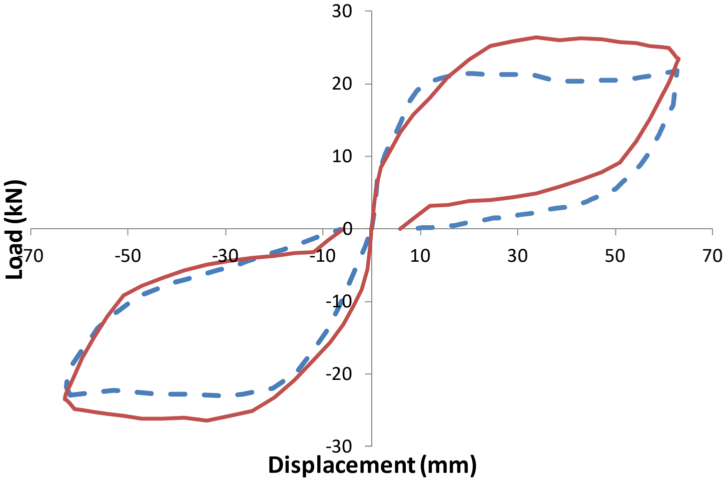
Figure 1: Stress-strain models during loading and unloading stages

5 SUGGESTED RETROFITTING TECHNIQUE

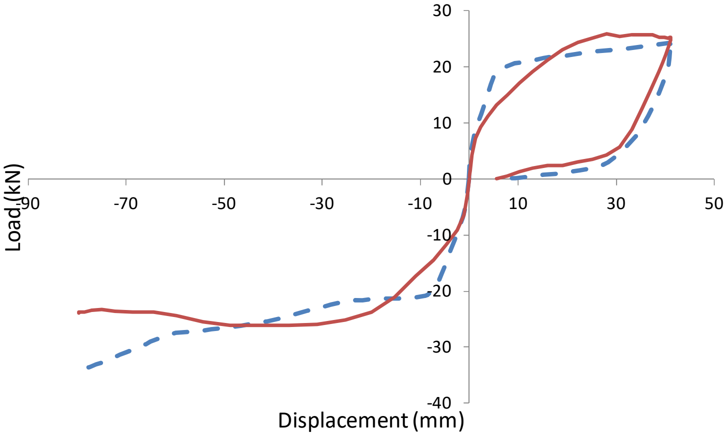
Figure 3 shows the suggested retrofitting technique. Two angles are first attached to the ends of the beam using steel bolts. They are attached to the ends of the beam to avoid adding tensile and shear stresses to the middle area of the beam due to the angles fixation. The angles are then connected with SMA-steel bars. Each bar is made of a middle SMA bar connected using two mechanical couplers to two steel bars. The use of steel bars is to minimize the length of the SMA bars and thus their cost. Hold down plates are used along the length of the beam to enforce the external bars to bend with the beam.



(a) B4-NM results



(b) B6-NR results



(c) B7-NCM results

Figure 2: Experimental vs. numerical load-displacement results of SMA RC beams tested by Abdulridha (2013); (a) B4-NM results; (b) B6-NR results; (c) B7-NCM results.

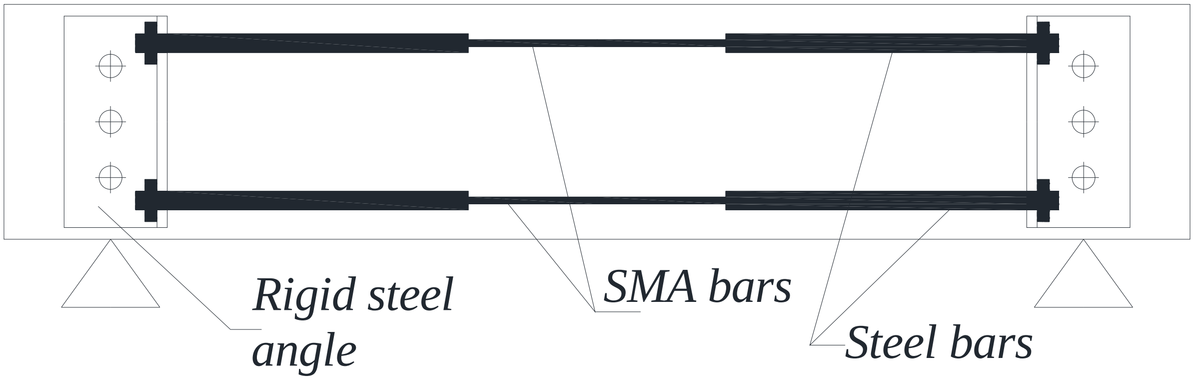


Figure 3: Suggested strengthening technique.

The modulus of elasticity of steel is much higher (3 to 5) than the modulus of elasticity of SMAs. Thus, attaching a small or moderate ratio of SMA will improve the beam strength and stiffness, but is not expected to reduce the residual deformations. Thus it is proposed to cut the internal tensile steel at the mid-span section and replace it with external SMA bars.

A RC beam is assumed to have a cross-section of 125x250 mm and a span of 1200 mm, Figure 4. An external angle with dimensions of 320x210x40 mm is connected to the RC beam using 8 bolts. The three hold down plates have dimensions of 110x210x10 mm and each of them is attached to the beam using 3 bolts. The main hold down plate at mid-span has dimensions of 110x210x40 mm and is attached to the beam using 9 bolts. The bolts are assumed to be 91 mm in length and 12.7 mm in diameter. The external bars are attached to the external angles using end couplers. The end couplers are modeled as nuts perfectly tied to the bars.

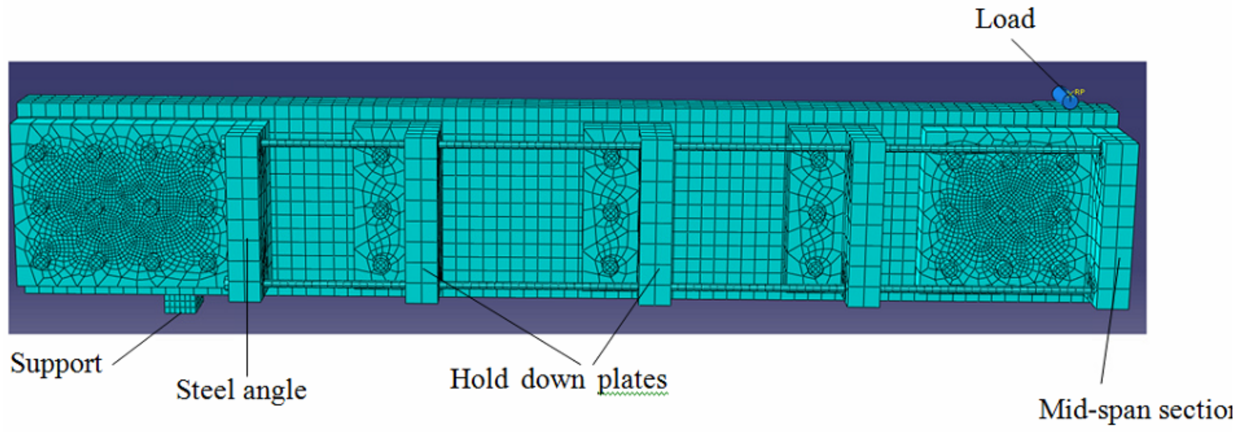


Figure 4: FE model of the strengthened beam.

FE analysis is performed for the retrofitted beam. Four element different sizes (61.25, 43.75, 26.25, and 17.5 mm) are first considered to determine the appropriate mesh size. It is found that element size of 17.5 mm gives good results and further refinement of the mesh will not noticeably change the behaviour.

Figure 5 shows the load-displacement relationship of the retrofitted beam vs. the original beam. The maximum moment capacity of the beam increased from 20 kN.m to 24 kN.m. The pre-yielding stiffness of the beam reduced significantly due to cutting the internal steel bars. Suggested retrofitting technique reduced the amount of residual displacement from 32mm to 5 mm (84%). Thus, it is clear that SMA can be used to reduce seismic residual deformations. However, such use affects the stiffness and strength of the retrofitted element. The following sections present a simplified method that can be used to assess such changes, a parametric study, and simplified equations to calculate them.

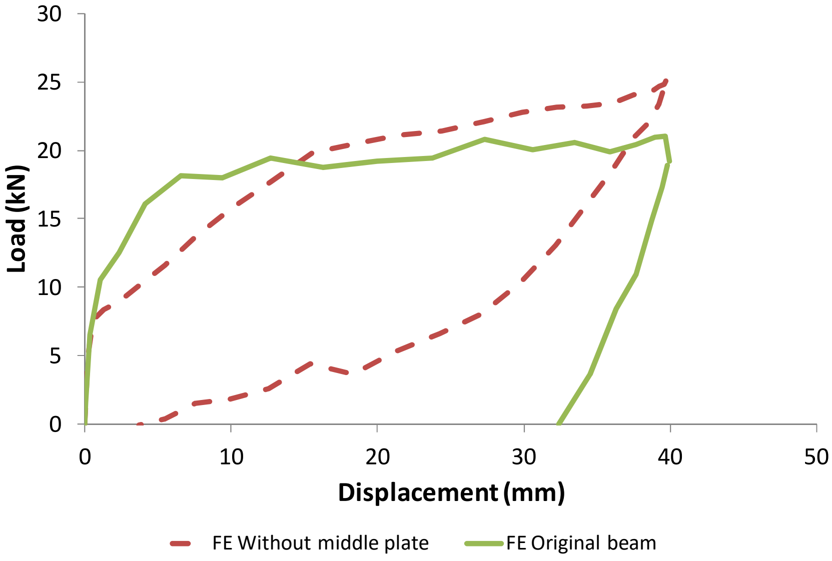


Figure 5: Load displacement results of the retrofitted beam vs. the original beam.

6 SIMPLIFIED ANALYSIS METHOD

A simplified method of analysis is introduced in this section. A computer program is first developed using JAVA programming to predict the flexural behaviour of RC beams retrofitted using unbonded superelastic SMAs bars. The program is based on the sectional analysis methodology, where the cross-section of the retrofitted beam is divided into a discrete number of horizontal layers. Using the predefined stress-strain relationship of each layer, and considering the cross-section equilibrium and kinematics, the flexural behaviour of the retrofitted beam can be predicted (Youssef and Rahman 2007; Elbahy et al. 2009). Two main assumptions are proposed in the suggested analysis procedure: (i) plane sections remain plane (i.e. linear strain distribution); and (ii) perfect bond exists between concrete and internal reinforcement layers.

7 PROGRAM VALIDATION

The FE model is used in this section to validate the results obtained using the simplified sectional analysis method. The load-displacement relationship of the ABAQUS model is plotted versus the load-displacement relationship obtained using the simplified sectional analysis method in Figure 6. The FE model showed good agreement with the simplified method. Therefore, the simplified method is used in the analysis of the following sections of the paper.

8 PARAMETRIC STUDY

A parametric study is carried out in this section to investigate the behaviour of RC beams retrofitted using unbonded SMA bars. Analysis is performed for the loading and unloading stages. Three parameters are investigated: (i) the ratio between the added external SMA reinforcement to the amount of internal steel reinforcement in the beam (A_{SMA}/A_s); (ii) applied load level (ratio between the maximum applied displacement to the displacement at which yielding of the external reinforcement occurs δ_{max}/δ_y); and (iii) ratio between the length of the used SMA bars to the span of the beam (L_{SMA}/L).

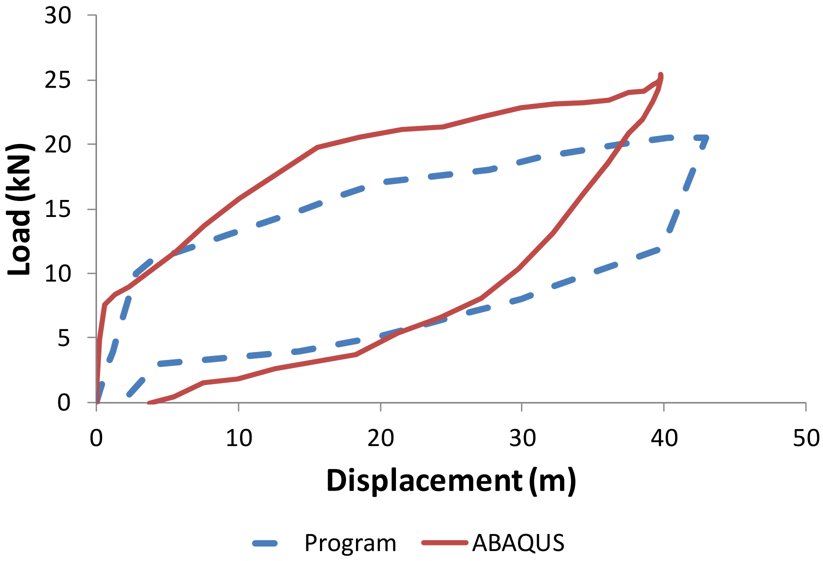


Figure 6: Load-displacement relationship of the FE method vs. the developed program.

The parametric study is performed on simply supported beams with cross-sectional dimensions of 300 mm by 700 mm and span of 7,000 mm. They beams loaded/unloaded under either one or two point loads. For each of the studied parameters, the parameter under investigation is varied within the desired range while keeping all other parameters constant during the analysis. While varying A_{SMA}/A_s and load level, the length of the SMA bars is assumed equal to the full length of the beam. For the third parameter (L_{SMA}/L), nine different lengths of the SMA bars are investigated (0.05 *L*, 0.10 *L*, 0.125 *L*, 0.167 *L*, 0.25 *L*, 0.33 *L*, 0.50 *L*, 0.67 *L*, and 0.75 *L;* where *L* is the span of the studied beams).

9 CHOICE OF SMA BARS

Multiple linear regression is used to model the relationship between the outputs and the inputs obtained from the parametric study. After trying numerous number of models that utilize different transformations (i.e. linear transformations, quadratic power transformation, and logarithmic transformation), the best five models for the five outputs are noted and presented in this section.

The data used in this analysis are the data obtained from the parametric study. A total of 350 data sets are used in the models. All parameters (i.e. inputs and outputs) are non-dimensional parameters. The inputs are: L_{SMA}/L, A_{SMA}/A_s, and load level. The outputs of the parametric study are: δ_r/δ_{max}, M_{rt}/M_{org}, ST_{rt}/ST_{org}, $\delta_{y\text{-}rt}/\delta_{y\text{-}org}$, and $\delta_{max\text{-}rt}/\delta_{max\text{-}org}$. Analysis of the data starts with investigating the correlation between each pair of the variables and noting the highly correlated pairs and their signs. Correlation matrix is determined using the STATA software V.12 (STATA 2018) and is given in Table 2.

Equations [1-5] represent the summary of the final regression models for the five outputs.

[1] δ_r/δ_{max} = -0.14318 x A_{SMAs}/A_s + 0.061737 x $(A_{SMAs}/A_s)^2$ + 0.013083 x $(\text{Load level})^2$ + 0.751673

[2] M_{rt}/M_{org} = 82.72809 x (A_{SMAs}/A_s) + 5.773622 x (Load level) -23.0628

[3] $\ln(ST_{rt}/ST_{org}) = -0.71767 \times \ln(L_{SMAs}/L) + 0.88563 \times \ln(A_{SMAs}/A_s) + 3.210665$

[4] $\ln(\delta_{y\text{-}rt}/\delta_{y\text{-}org}) = 8.336842 \times (L_{SMAs}/L) – 11.2 \times (L_{SMAs}/L)^2 + 0.408727 \times (A_{SMAs}/A_s) – 0.06615 \times (A_{SMAs}/A_s)^2 + 2.603485$

[5] $\ln(\delta_{max\text{-}rt}/\delta_{max\text{-}org}) = 14.02687 \times (L_{SMAs}/L) - 17.2568 \times (L_{SMAs}/L)^2 - 0.14746 \times (A_{SMAs}/A_s) + 4.255832$

Table 4: Correlation coefficients between all variables.

	L_{SMA}/L	A_{SMA}/A_s	**Load level**	δ_r/δ_{max}	M_{rt}/M_{org}	ST_{rt}/ST_{org}	$\delta_{y\text{-}rt}/\delta_{y\text{-}org}$	$\delta_{max\text{-}rt}/\delta_{max\text{-}org}$
L_{SMA}/L	1.00							
A_{SMA}/A_s	0.00	1.00						
Load level	0.00	0.00	1.00					
δ_r/δ_{max}	0.08	-0.43	0.13	1.00				
M_{rt}/M_{org}	0.00	0.96	0.11	-0.22	1.00			
ST_{rt}/ST_{org}	-0.63	0.55	0.00	-0.08	0.61	1.00		
$\delta_{y\text{-}rt}/\delta_{y\text{-}org}$	-0.20	-0.16	0.00	0.08	-0.15	-0.24	1.00	
$\delta_{max\text{-}rt}/\delta_{max\text{-}org}$	-0.05	-0.29	0.01	0.08	-0.30	-0.50	0.74	1.00

10 CONCLUSIONS

The use of external unbonded SMA bars to retrofit RC beams is investigated in this study. A FE model is first developed to simulate the behaviour of the retrofitted beams and validated using available experimental results. Experimental results included RC beams that are internally and externally reinforced using SMA and steel bars. Good agreement between experimental and analytical results is observed. A simplified analysis method is then developed to capture the flexure behaviour of the retrofitted beams. The analysis method is based on the sectional analysis technique for unbonded bars. Results obtained from the developed program/method are then validated using FE results.

An extensive parametric study is then carried out to investigate the flexural behaviour of RC beams retrofitted using SMA bars. Effect of varying three different parameters is studied. These parameters are: A_{SMA}/A_s, load level (δ_{max}/δ_y), and L_{SMA}/L. For each of the studied parameters, load-displacement relationships are constructed using the moment-area method. Out of the different load-displacement relationships, δ_r/δ_{max}, M_{rt}/M_{org}, ST_{rt}/ST_{org}, $\delta_{y\text{-}rt}/\delta_{y\text{-}org}$, and $\delta_{max\text{-}rt}/\delta_{max\text{-}org}$ are used to capture the change happening in the behaviour due to varying one of the parameters.

Results of the parametric study are then used in multiple linear regression analysis. A numerous number of models are first developed for the five outputs. Different transformations of the inputs are used. Best five models for the five outputs are then reported in this study. The five models are summarized in the form of simple equations to help the designers to decide the optimum amount and length of the used SMA bars.

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