



INTERFACE PLANES FOR DISCHARGE ESTIMATION IN NON-SYMMETRICAL RECTANGULAR COMPOUND CHANNEL

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Abstract: The complexity of flow area determination in compound channels may lead to errors in flow estimation. Although the uniform flow formulae, such as Manning's equation, are widely used to determine flows through open channels having simple cross-sections, this may lead to considerable errors in the case of compound channels. This paper evaluates alternatives to account for flood plain conveyance in non-symmetrical rectangular compound channel flows by determining the best interface plane arrangement leading to flow estimation for different flow depths. Evaluations are based on applying several traditional discharge estimation methods to a laboratory data set in a non-symmetrical rectangular compound channel cross-section. Five interface and two other standard methods for computing discharge in open channels were compared. Each interface method uses a particular arrangement of imaginary interface planes to artificially sub-divide the compound flow field into homogeneous zones. These methods are named for the particular interface plane arrangement adopted. The two standard methods do not involve interface planes. Computed discharges were compared and evaluated by applying various statistical measures to determine the degree of goodness-of-fit between computed and observed discharges. In terms of overall performance, the inward diagonal interface plane method produced the most accurate computations among the methods tested for the non-symmetrical compound channel cross-section under consideration.

1 Introduction

Discharge estimation is vitally important in the regulation, development, and management of river systems. Most laboratory studies involving channel flows have been performed in single regular-shaped channels with discharges estimated using either the Manning's or Chezy's formula. While this approach greatly simplifies matters, it has its limitations concerning accurate representation of river channel geometry. This is especially true in times of flooding when the bank-full stage of the river's main channel is frequently exceeded and adjacent flood plain zones are inundated. At such times, river cross-sections are compound shapes, comprising a deep main channel with one or two shallow flood plain zone(s). If the flood plain depth (d) to main channel depth (D) ratio is in the range $0.0 < d/D < 0.3$, lateral momentum transfer (LMT) is strong in the interface regions separating the deep and shallow zones and this feature can impact significantly on system conveyance, (Myers, 1978). Applying traditional discharge estimation methods in these circumstances generally results in overestimation of system discharge, (Wormleaton and Hadjipanos, 1982). Tingsanchali and Ackermann (1976) suggest that if the velocities on the flood plain zones are high, it is important to assess the dynamic effect of the flood plain flow when modelling system conveyance and computing discharges.

2 Compound Channel Flows

As stated above, a unique feature of compound flow fields is the LMT, or kinematic effect, associated with the interface regions separating the main channel and flood plain zones. Zheleznyakov (1965, 1971), who first

investigated this feature, demonstrated that at low flood plain depths ($d/D < 0.3$), flow velocities within the main channel portions of the mixing regions were significantly lowered by LMT. This, in turn, produced a corresponding reduction in system conveyance. Zheleznyakov also showed that, at large flood plain depths, ($d/D > 0.3$), channel velocities in the vicinity of the mixing regions approached bank-full values, which was an indication of weak LMT in these circumstances.

If a compound channel is treated as a single channel, i.e., if LMT effects are ignored entirely, system discharge is generally underestimated, (Wormleaton and Hadjipanos, 1982). On the other hand, if the cross-section is subdivided into hydraulically homogeneous main channel and flood plain zones, system discharge is generally overestimated, (ibid). A standard uniform flow equation (Manning or Chezy) is usually applied to calculate the discharge in each sub-section of the cross-section. Sub-section discharges are then summed to give total or system discharge. Standard methods employed to sub-divide the cross-section differ only in the assumptions made regarding the arrangement of imaginary interface planes used to effect the division. Yen and Overton (1973) produced an empirical relationship between the angle of inclination of inward diagonal interfaces and flood plain depth. When comparing the effectiveness of horizontal, vertical, and inward-diagonal interfaces Wormleaton and Hadjipanos (1982) noted that the most commonly used method of calculating discharge in rectangular-shaped compound channels is the vertical interface method. Chatila and Townsend (1996) applied several discharge estimation methods to a laboratory data set of steady uniform flows in a symmetrical trapezoidal compound-shaped channel. They reported that the outward-facing diagonal interface plane method and the vertical interface plane method produced the most accurate simulations among the methods tested.

Khatua et al. (2013) reported that a wrong selection of interface planes between the main channel and floodplain accounts for transfer of improper momentum, which introduces error in estimating the discharge for compound channel section. Their study indicates that for a straight compound channel, the horizontal division method provides better discharge results for low overbank flow depth and diagonal division method is good for higher overbank flow depths.

Huthoff et al. (2008) proposed the interacting divided channel method to calculate flow in compound channels. This new method is based on a new parametrization of the interface stress between adjacent flow compartments, typically between the main channel and floodplain of a two-stage channel. They reported good agreement between the analytical model results and laboratory data from the literature.

Petersen-Overleir (2008) developed a statistical method based on a simple uniform flow depth-discharge model for a two-stage main channel-floodplain river section and tested for data from four hydrometric gauging stations. Applying this method to field data showed a good agreement between measured and estimated depth-discharge relationships and main channel-floodplain change-points. Some difficulties, caused by factors such as unequal measurement error variance, infinite parameter estimates and the presence of more than two depth-discharge change points, were apparent.

Lee et al. (2011) evaluated the accuracy of flow estimation for different materials of compound channels using combinations of traditional interface plane methods. They reported the complexity of flow in compound channel and the errors encountered by the interface plane methods in flow estimation in either smooth concrete or roughened channels. They recommended that more laboratory and field data need to be collected for further studies in understanding the flow estimation in either symmetrical or non-symmetrical compound channels to estimate flow accurately.

Seckin (2004) investigated the performance of four different methods for computing the discharge capacity of compound channels, when applied to a smooth compound channel and to a roughened flood plain. Seckin reported that the exchange discharge method and the Ackers Method are able to simulate the measured discharge values more accurately than those of the traditional methods, namely, the single-channel method, the divided-channel method.

Some of these discharge estimation methods, however, include the interface plane in the wetted perimeter computations (assuming the fluid shear stress at the junction is the same as the average boundary shear

stress) and others exclude it (assuming zero apparent shear stress), (Wright and Carstens, 1970; Yen and Overton, 1973; Wormleaton and Hadjipanos, 1982; Dracos and Hardegger 1987; Chatila and Townsend, 1996; among others). It has been established that, in the case of vertical interface planes, the apparent shear stress is large at low flood plain depths, (Wormleaton and Hadjipanos, 1982). Therefore, it would be useful to examine other possible division planes, where the apparent shear stress is lower, and hence can be ignored or equated to the local boundary shear stress.

Given that literature does not provide a unique method for discharge estimation in compound channels and it was noticed the lack of applications to non-symmetrical rectangular compound channels, it is crucial to evaluate the different discharge estimation methods to such data set.

This paper reports on the investigation of seven discharge estimation methods that were applied to a laboratory data set of unsteady flows in a non-symmetrical compound-shaped rectangular channel. Evaluation of the different methods was based on comparison between measured and the corresponding computed discharges along using statistical goodness-of-fit criteria.

3 Treske's Experimental Channel

Treske's experimental facility, which included: (1) a straight prismatic channel; (2) a lateral inflow channel to (1); and (3) a meandering channel, is shown schematically in figure 1, (Treske, 1980). In the present study, analysis was performed on Treske's straight prismatic channel only. Figure 2 shows the principal dimensions of the non-symmetrical rectangular compound channel. The main channel was 125 cm wide by 39 cm deep and had a left flood plain 300 cm wide by 30 cm deep, and a right side flood plain 150 cm wide by 37 cm deep. The working length of the channel was 210 m, the bed slope was 0.019 %, and Manning's roughness coefficient for the composite cross-section was estimated to be 0.012 (ibid). A head box was located at the entrance of Treske's three channels where the inflow hydrographs were controlled and measured. Two measurement stations were used. The upstream station was 14 meters downstream from the head box and the downstream station was located 210 meters from the upstream station. At each measuring station, both stage and discharge hydrographs were measured for each flood event. Discharge was measured in liters per second and stage in millimeters. To each value of depth, 819 m should be added to reference it to mean sea level. Treske's flood event 10 corresponds to the highest discharge and consequently highest depth over the flood plains ($d/D = 0.20$ at the highest value). It has 74 readings of stage and discharge over a time span of 219 minutes measured at 3-minute interval.

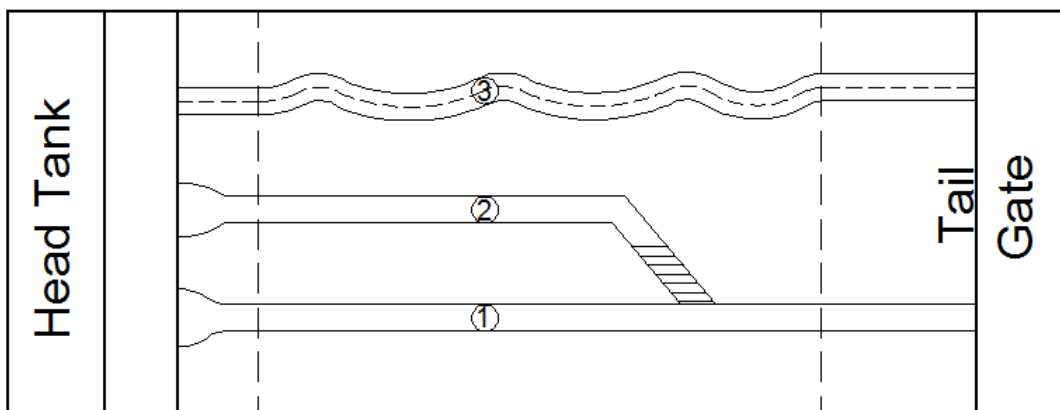


Figure 1 Treske's Experimental Facility, which included: (1) a Straight Prismatic Channel; (2) a Lateral Inflow Channel to (1); and (3) a Meandering Channel.

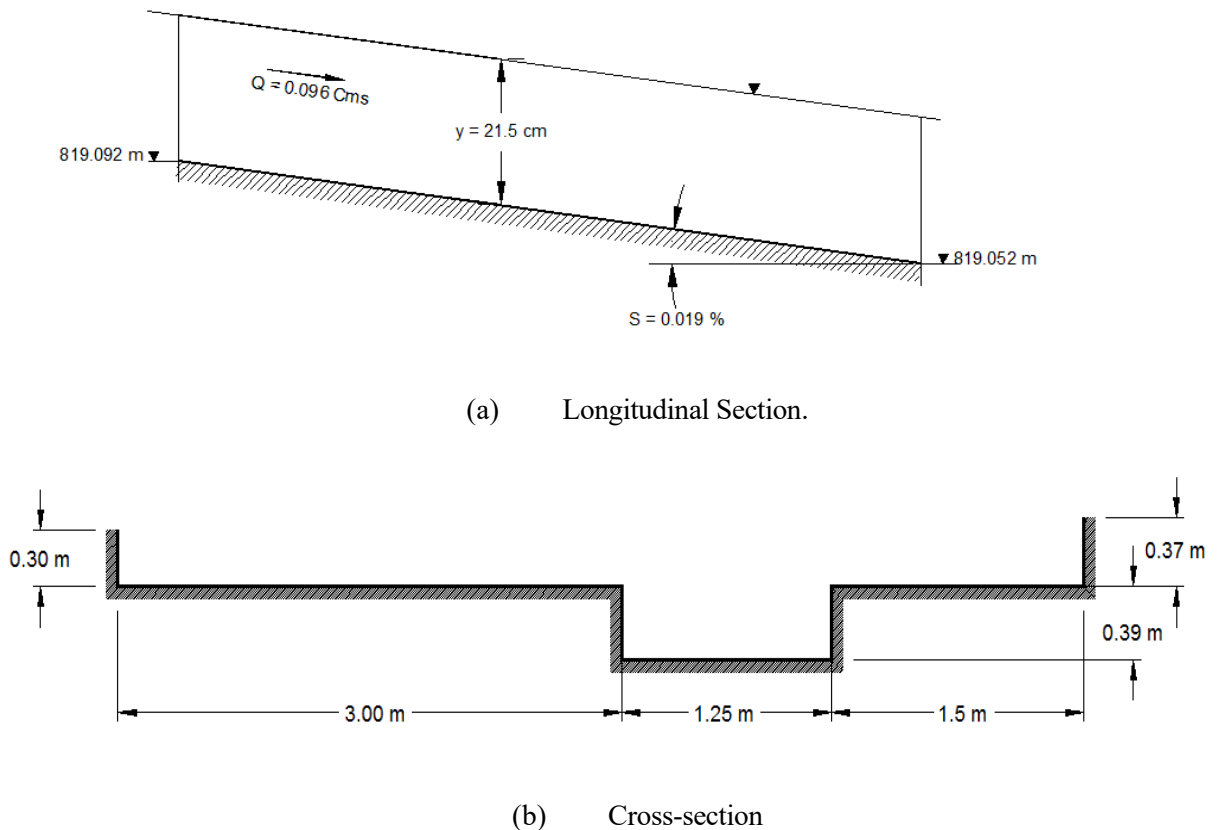


Figure 2 Treske's Straight Prismatic Non-Symmetrical Rectangular Compound Channel.

4 Discharge Estimation Methods

Five interface and two standard methods for computing discharge in compound open channels were compared by applying them to Treske's data set flood event case No. 10 comprised of non-symmetrical rectangular-shaped compound channel. Each of the five interface plane methods uses a different imaginary interface planes to artificially sub-divide the compound flow field into homogeneous zones. The zones include a main channel and some flood plain areas depending on the method. These discharge estimation methods are named for the particular interface plane arrangement adopted, as follows, (figure 3):

- 4.1 Horizontal (H): applies a horizontal interface at the bank-full depth to separate the component flows. Flows above and below the interface are considered flood-plain and main-channel flows; respectively. This is represented by interface a-a in figure 3.
- 4.2 Inward Diagonal (I); (Yen and Overton, 1973): applies interfaces that extend from the junctions (between the main channel and flood plains), to the center-line of the main channel at the water surface. This is represented by interface a-c in figure 3.
- 4.3 Vertical (V): vertical interfaces separate the main channel from the flood plain flows. This method is most widely used for compound channels comprised of rectangular-shaped sub-sections. This is represented by interface a-b in figure 3.
- 4.4 Modified Vertical (MV): similar to the V method above, vertical interfaces separate the main channel from the flood plain flows. However, the interface planes were not considered in the computation of the wetted perimeter.
- 4.5 Weighted Hydraulic Radius (W); (French, 1985): this method has the same interface plane arrangement as the I method. However, areas and wetted perimeters for the sub-sections are computed and a weighted hydraulic radius for the composite section is used based on the following:

$$R_w = \frac{\frac{A_{lf}^2}{P_{lf}} + \frac{A_{mc}^2}{P_{mc}} + \frac{A_{rf}^2}{P_{rf}}}{A_{lf} + A_{mc} + A_{rf}}$$

Where: A_{lf} , A_{mc} , A_{rf} , P_{lf} , P_{mc} , and P_{rf} are the areas and wetted perimeters of the left floodplain, main channel, and right floodplain, respectively.

- 4.6 Weighted Extended Side Slope (E); (Chow, 1959): interfaces are straight-line extensions to the water surface of the sloping sides of the main channel. Since the main channel is rectangular in nature and not trapezoidal, then this method once applied coincides with the V method.

The two standard methods, which do not involve imaginary interface planes, are:

- 4.7 Single-Channel Method (S): the entire cross-section is assumed to convey the flow as a single unit, i.e. flow area and wetted perimeter are calculated as for a single channel.
- 4.8 Single-Channel Empirical Method (SE); (Dracos and Hardegger, 1987): is a more sophisticated version of S method involving several more parameters. The ratio of the hydraulic radius to the total depth R/D can be computed using:

$$\frac{R}{D} = \frac{K_1 D^2 + K_2 D^2 + K_3}{K_4 D^2 + K_5 D}$$

Where:

$$K_1 = 0.5*(Z_3 + Z_4),$$

$$K_2 = P_m + P_{lf} + P_{rf} + (Z_1 + Z_2)*d,$$

$$K_3 = 0.5*(Z_3 + Z_4 - Z_1 - Z_2)*d^2 - (P_{lf} + P_{rf})*d,$$

$$K_4 = (1 + Z_3)^2 + (1 + Z_4)^2,$$

$$K_5 = P_m + P_{lf} + P_{rf} + [(1 + Z_1^2)^{1/2} + (1 + Z_2^2)^{1/2}]*d,$$

Z_1 and Z_2 = main channel side slopes,

Z_3 and Z_4 = floodplain side slopes

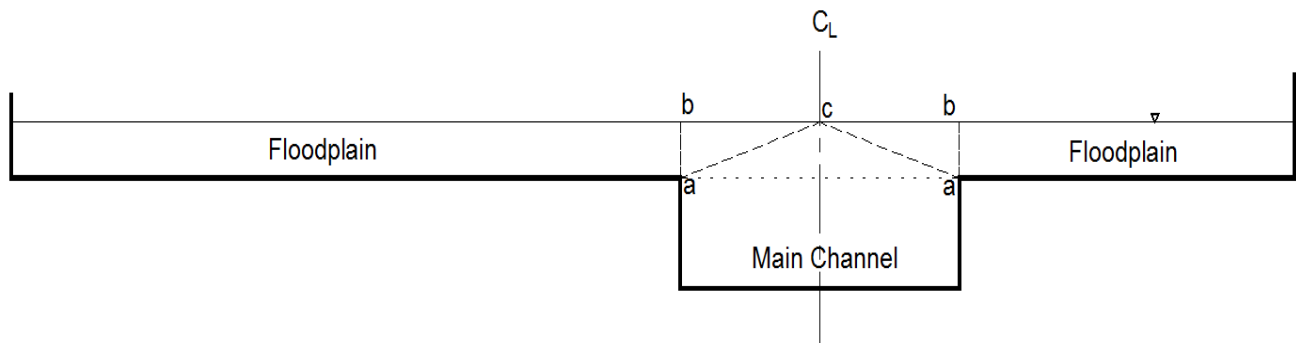


Figure 3 Different Interface Options of Dividing the Compound Cross-Section (a - a: Horizontal, a - b: Vertical, a - c: Inward diagonal).

5 Results and Analysis

For each case considered, computed discharge was based on the measured flow depth and the selected method of defining flood plain hydraulic boundaries. Manning's equation was used to calculate the discharge for each subsection and total discharge was the sum of the subsection discharges. The flows were measured

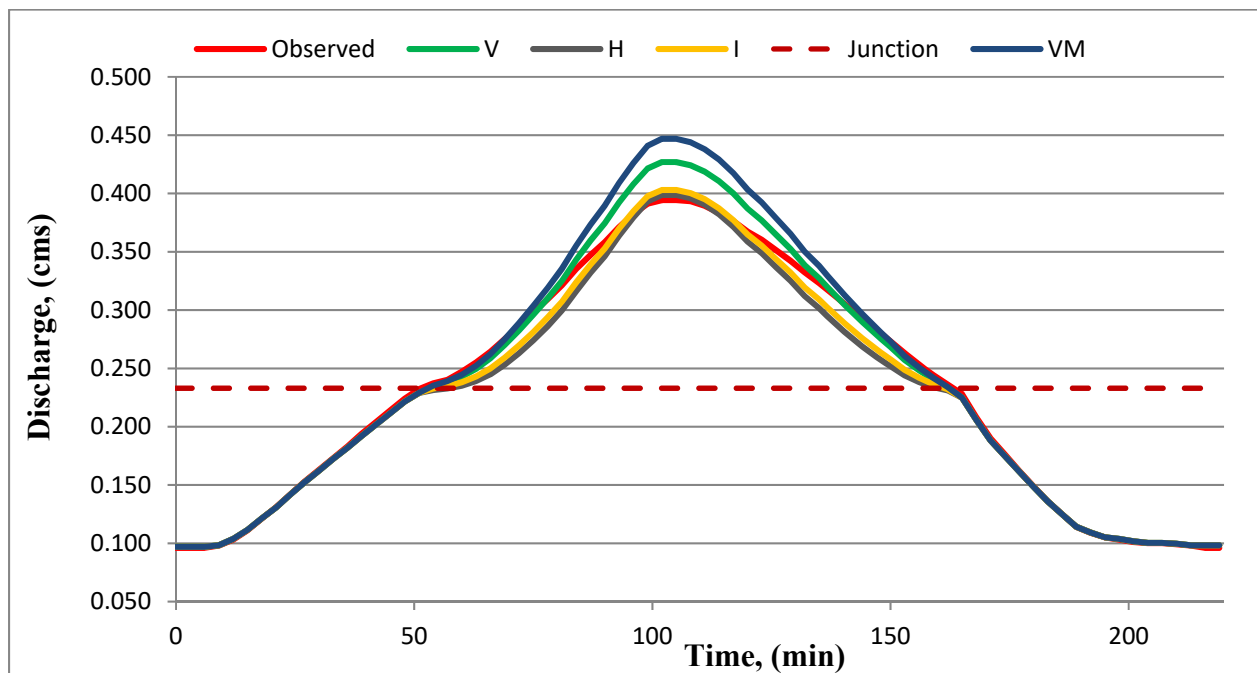
up to $d/D = 0.20$, as stated earlier. Table 1 and figures 4 (a) and 4 (b) show the observed and computed discharges (m^3/s) resulting from the corresponding discharge estimation method application.

Table 1 Observed and Computed Discharges Using Corresponding Discharge Estimation Method.

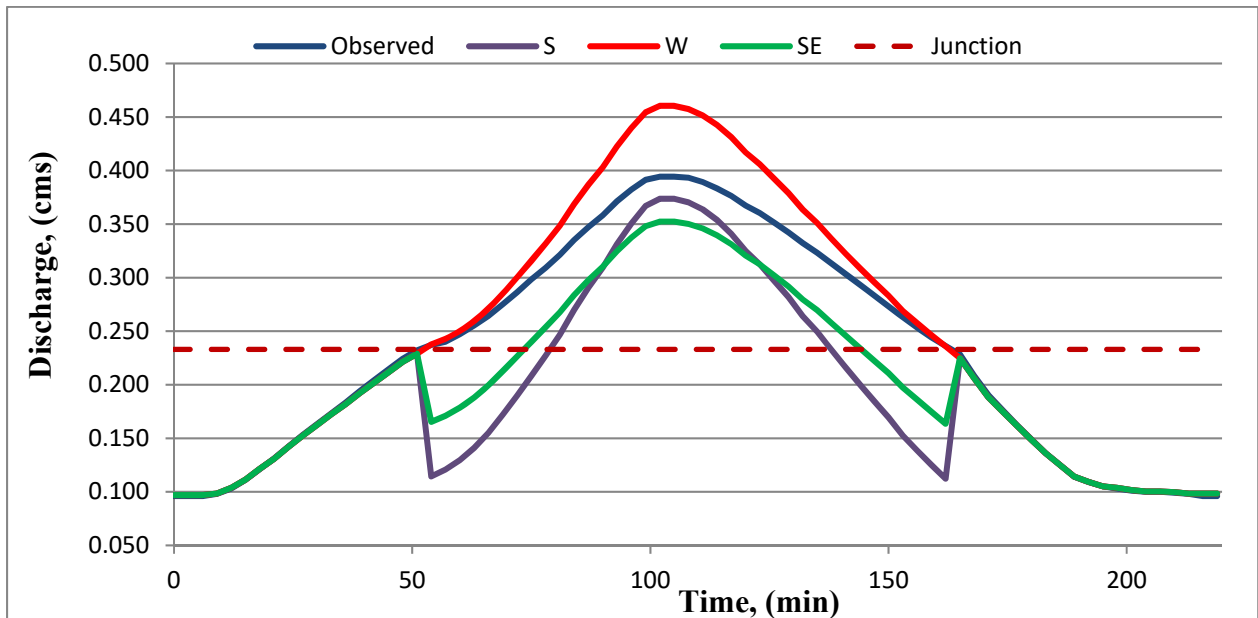
Time (min)	Observed (m^3/s)	y (m)	y/D	Computed Discharges (m^3/s)						
				H	I	V	VM	W	S	SE
54	0.237	0.394	0.01	0.231	0.233	0.235	0.235	0.238	0.114	0.165
57	0.240	0.397	0.02	0.233	0.235	0.238	0.239	0.243	0.121	0.171
60	0.247	0.401	0.03	0.235	0.238	0.243	0.245	0.250	0.130	0.179
63	0.255	0.406	0.04	0.239	0.243	0.250	0.252	0.259	0.141	0.188
66	0.264	0.412	0.05	0.245	0.250	0.259	0.262	0.271	0.155	0.199
69	0.275	0.419	0.07	0.253	0.259	0.270	0.275	0.285	0.172	0.213
72	0.286	0.426	0.08	0.263	0.270	0.283	0.289	0.300	0.190	0.226
75	0.298	0.433	0.10	0.274	0.281	0.296	0.303	0.316	0.208	0.240
78	0.309	0.440	0.11	0.286	0.294	0.310	0.319	0.332	0.227	0.254
81	0.321	0.447	0.13	0.300	0.307	0.325	0.335	0.349	0.247	0.268
84	0.335	0.455	0.14	0.316	0.324	0.343	0.355	0.369	0.270	0.284
87	0.347	0.462	0.16	0.332	0.339	0.360	0.373	0.387	0.291	0.298
90	0.358	0.468	0.17	0.346	0.353	0.374	0.389	0.403	0.309	0.310
93	0.371	0.475	0.18	0.364	0.370	0.392	0.409	0.423	0.331	0.325
96	0.382	0.481	0.19	0.380	0.385	0.408	0.426	0.440	0.351	0.337
99	0.391	0.486	0.20	0.393	0.398	0.421	0.441	0.454	0.367	0.348
102	0.394	0.488	0.20	0.399	0.403	0.427	0.447	0.460	0.374	0.352
105	0.394	0.488	0.20	0.399	0.403	0.427	0.447	0.460	0.374	0.352
108	0.393	0.487	0.20	0.396	0.400	0.424	0.444	0.457	0.370	0.350
111	0.389	0.485	0.20	0.390	0.395	0.419	0.438	0.451	0.364	0.346
114	0.383	0.482	0.19	0.382	0.387	0.411	0.429	0.443	0.354	0.340
117	0.376	0.478	0.18	0.372	0.377	0.400	0.417	0.431	0.341	0.331
120	0.367	0.473	0.18	0.359	0.365	0.387	0.403	0.417	0.325	0.321
123	0.360	0.469	0.17	0.349	0.355	0.377	0.392	0.406	0.312	0.312
126	0.351	0.464	0.16	0.337	0.344	0.364	0.379	0.392	0.297	0.302
129	0.342	0.459	0.15	0.325	0.332	0.352	0.365	0.379	0.282	0.292
132	0.332	0.453	0.14	0.312	0.319	0.338	0.350	0.364	0.264	0.280
135	0.323	0.448	0.13	0.302	0.309	0.327	0.338	0.351	0.250	0.270
138	0.313	0.442	0.12	0.290	0.297	0.314	0.324	0.336	0.233	0.258
141	0.303	0.436	0.11	0.279	0.286	0.302	0.310	0.322	0.216	0.246

144	0.293	0.430	0.09	0.269	0.276	0.290	0.297	0.309	0.200	0.234
147	0.283	0.424	0.08	0.260	0.267	0.279	0.285	0.296	0.185	0.222
150	0.273	0.418	0.07	0.252	0.258	0.268	0.273	0.283	0.170	0.211
153	0.263	0.411	0.05	0.244	0.249	0.257	0.261	0.269	0.153	0.198
156	0.253	0.405	0.04	0.238	0.242	0.248	0.251	0.258	0.139	0.186
159	0.244	0.399	0.02	0.234	0.236	0.240	0.242	0.246	0.125	0.175
162	0.236	0.393	0.01	0.231	0.232	0.234	0.234	0.236	0.112	0.164

Figures 4 (a) and (b) compare computed with observed discharges in the range $d/D < 0.2$. While figure 5 presents the variation of discharge with d/D for the different discharge estimation methods applied in the current study.



(a) V, I, H and VM Methods



(b) S, W, SE Methods

Figure 4 Observed and Computed Discharges Using Various Methods.

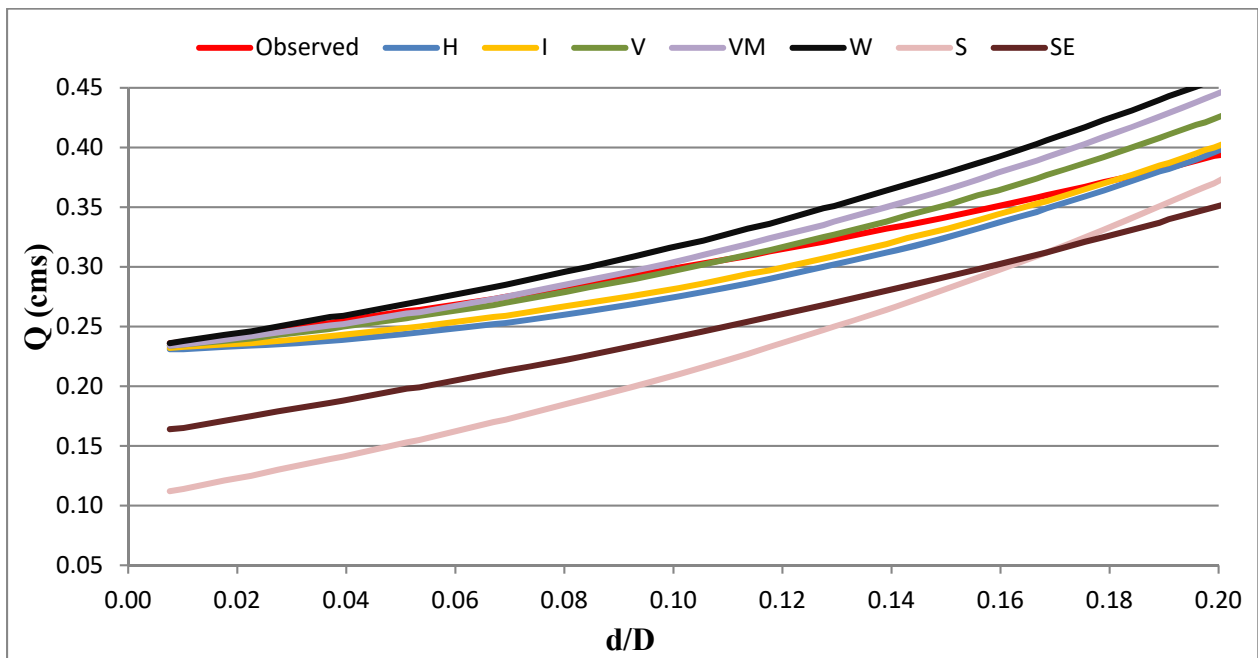


Figure 5 Observed and Computed Discharges Variation with d/D.

Based on the computations, the peak discharge (Q_p) was determined and values are summarized in table 2 below. The peak discharge is mainly over-estimated for the interface plane methods considered and under-estimated for the two single-channel methods.

Table 2 Computed Peak Discharges and Percent Error Relative to Observed Peak

Method	Q_p (m ³ /s)	Error	Comment
Observed	0.394	N/A	N/A
H	0.399	1.3%	Over-estimated
I	0.403	2.3%	Over-estimated
V	0.427	8.4%	Over-estimated
VM	0.447	13.5%	Over-estimated
W	0.460	16.8%	Over-estimated
S	0.374	-5.1%	Under-estimated
SE	0.352	-10.7%	Under-estimated

Furthermore, linear regression analysis was performed on the data in table 1 and 1 (b), with the computed discharge (Q_c) as the dependent variable and the observed discharge (Q_o) as the independent variable, (i.e., $Q_c = a Q_o + b$). Ideally a and b should be 1.0 and 0.0 respectively, however, as indicated in table 3, method S and SE produced quite large discrepancies between Q_c and Q_o . Methods W and VM, whose estimates were comparable, also produced discrepancies; however, these were generally smaller than those associated with methods S and SE. As evidenced in table 1 (b), methods S and SE consistently underestimated system discharge, whereas the interface plane methods overestimated system discharge. Methods H and V produced generally acceptable results, the latter method being somewhat superior to the former, (table 3). Method I produced the best results for the case considered and the methods evaluated.

Table 3 Linear Regression Analysis on Observed and Computed Discharges.

Method	Slope	Intercept	r^2	Comments
H	0.9548	0.0037	0.9934	Better
I	0.9803	0.0006	0.9956	Best
V	1.0663	0.0114	0.9947	Better
VM	1.1275	0.0203	0.9913	Good
W	1.1831	0.0275	0.9912	Good
S	0.7678	0.0163	0.8143	Poor
SE	0.7843	0.0218	0.9516	Poor

6 Statistical Criteria

In general, no single statistical goodness-of-fit criterion is sufficient to adequately assess the measure of fit between simulated and observed data points. Different goodness-of-fit criteria are weighted in favour of different values (volumes, peak flows, maximum depths ...etc). Thus, there is no general criterion, and the one ultimately selected should depend on the objective of the modelling exercise. The various statistical criteria applied in this study included: graphical techniques, sum of squares (SS), Nash and Sutcliffe (N & S), root mean square error (RMSE), standard error of estimate (SEE), reduced error of estimate (REE), proportional error of estimate (PEE), and total absolute relative error (TARE). Except for N & S, which should approach 1.0 for better simulations, the main goal is to minimize all of the above objective functions. Tables 4 shows the statistical goodness-of-fit criteria applied to Treske's data set for compound channel over-bank flows.

Table 4 Statistical Goodness-of-Fit Criteria Applied to Compound Over-Bank Flows.

Statistical Criteria	Discharge Estimation Method						
	H	I	V	VM	W	S	SE
SS	0.0094	0.0046	0.0090	0.0268	0.0506	0.2445	0.1163
N&S	0.9083	0.9552	0.9119	0.7384	0.5067	-1.3860	-0.1351
RMSE	0.0159	0.0111	0.0156	0.0269	0.0370	0.0813	0.0561
SEE	0.0164	0.0115	0.0161	0.0277	0.0380	0.0836	0.0577
REE	0.3027	0.2118	0.2968	0.5115	0.7023	1.5447	1.0654
PEE	0.0082	0.0058	0.0081	0.0139	0.0191	0.0419	0.0289
TARE	0.5117	0.3676	0.4276	0.7256	1.0955	2.7286	2.0449

Clearly table 4 (b) shows that method I produced the best results with the minimum value of the statistical test.

7. Summary and Conclusions

This paper evaluates various alternatives to account for flood plain conveyance in non-symmetrical rectangular compound-shaped channel. Evaluations are based on applying several traditional discharge estimation methods. Many of the techniques applied overestimate compound channel discharge. Five interface and two other standard methods for computing discharge in open channels were compared and results are reported below:

- Except for the S and SE methods, applying traditional discharge-estimation methods to data in a non-symmetrical prismatic rectangular compound channel flows generally results in over-estimation of system discharge. This is in line with the conclusion reached at by Chatila and Townsend (1996) for trapezoidal symmetrical compound channel.
- Method V slightly over-estimated system discharge and the difference between estimated and observed values increased with increasing floodplain depth.
- Method H slightly underestimated the discharge values up to $y/D < 0.19$, and overestimated discharge for $y/D > 0.19$ while Chatila and Townsend (1996) reported that H grossly overestimated system discharge for a trapezoidal symmetrical compound channel.
- Method I slightly underestimated the discharge values up to $y/D < 0.18$, and slightly overestimated discharge for values of $y/D > 0.18$.
- Method V slightly underestimated the discharge values up to $y/D < 0.10$, and overestimated discharge for values of $y/D > 0.10$.
- Method VM underestimated the discharge values up to $y/D < 0.07$, and overestimated discharge otherwise.
- Method W slightly overestimated the discharge values up to $y/D < 0.10$, and overestimated discharge for values of $y/D > 0.10$.
- Method S grossly underestimated system discharge.
- Method SE grossly underestimated system discharge.

It can be concluded that no single method can be applied safely for compound channels discharge determination. The channel geometry plays an important role in the applicability of the method. For the range of hydraulic conditions examined in a non-symmetrical rectangular compound channel cross-section, the inward-facing diagonal interface plane method proved to be the most accurate, especially for $d/D < 0.20$ under consideration.

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