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# ESTIMATING WATER AVAILABILITY AND UNDER ICE VOLUME OF ALBERTA LAKES USING MINIMAL DATA

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**Abstract:** Alberta's lakes support important environmental, social and economic values. The effect of cumulative allocations over time from lakes within a watershed may impact the health of the aquatic environment, which include fish and wildlife resources. Therefore, water withdrawals from lakes should be regulated in such a way so that ecosystems are preserved while balancing reliable, quality water supplies to sustain communities and economic and recreational opportunities. Accurate estimation of water availability (volume) in a lake requires a complete water balance study, which requires bathymetric information of the lake. However, only a fraction of Alberta lakes have surveyed bathymetry data. In support of provincial policy development and for quantifying the potential impacts of water withdrawal from lakes, approaches to estimating lake volume using limited available data were tested. In this study, we analysed available bathymetry data from 77 lakes and developed three different models to estimate maximum lake volume (a proxy of lake water availability) and 5% under ice volume (a proxy for winter allocation limit of lake water) assuming an ice thickness of 80 cm. These models have been developed in such a way that allows the user to apply the models based on data availability. These models can be used in absence of site specific data (e.g., bathymetry) to estimate volume, and subsequently water availability in lakes.

## 1 Introduction

Alberta's lakes support important environmental, social and economic values. The effect of cumulative allocations over time from lakes within a watershed may impact the health of the aquatic environment, which include fish and wildlife resources. Therefore, water withdrawals from lakes should be regulated in such a way so that ecosystems are preserved while balancing reliable, quality water supplies to sustain communities and economic and recreational opportunities.

In order to develop provincial policy (for example, setting limits on water withdrawal from lakes), information on water availability is essential. Accurate estimation of lake water availability requires a complete water balance study, which demands substantial data including lake bathymetry. However, only a fraction of Alberta lakes have surveyed bathymetry data. Moreover, acquiring bathymetric data is not always feasible for water availability assessments for small withdrawals. In absence of lake bathymetric data, lake shape models can be used to estimate lake volume and water availability.

Two distinct classes of lake shape model are found in literature, sinusoids and quadratic surfaces. Neumann (1959) presented an elliptic sinusoid model of lake shapes. The elliptic sinusoid is a geometric body whose base is an ellipse, and the planes perpendicular to the base passing through the center of the ellipse intersect the surface of the body along troughs of sine curves (Wetzel, 2001). Neumann's model was applied for 77 lakes in Alberta to investigate whether this model can be useful to idealize lake volume and surface area for Alberta lakes (Islam and Seneka, 2016). It was found that the model does not provide a satisfactory representation of Alberta lakes. Islam and Seneka (2016) developed dimensionless relationships between lake volume, lake surface area and lake depth, and then compared with analytical relationship between of five idealized quadratic lake shapes, viz. as cylindrical, pseudo-parabolic, parabolic, conic, and inverse-parabolic. They concluded that the volume-depth relationships of Alberta lakes are comparable with “idealized” lake shapes, and the majority of Alberta lakes examined are either “parabolic” (46% out of 77) or “conic” (32% out of 77) in shape. If a lake shape is comparable with an idealized lake shape, maximum depth and maximum lake surface area (at minimum) can be used to estimate lake volume (please see Section 2 for details). Where no lake shape is known, volume can be estimated assuming a parabolic-shaped lake (the most common category of the Alberta lakes examined).

During winter, lakes freeze over in Alberta. A representative ice thickness value for Alberta lakes was estimated to be 0.80 m (the 90th percentile of measured data from Alberta lakes, February and March, 1985 – 2016, n=686). Since potential water availability in lakes usually decreases in winter compared to the open water season, it is important to set separate winter allocation limits. Lower allocation limits during winter addresses the potential reduction in lake volume and surface area to minimize significant biological impacts to winter dissolved oxygen or littoral habitat. Assuming negligible inflow into lakes during the winter period, dissolved oxygen levels required for maintaining fish and other aquatic biota naturally degrades over time and can become limiting. Water withdrawals during this under ice period may accelerate this dissolved oxygen reduction. In a study on the effect of water withdrawals to dissolved oxygen concentrations in small lakes from the Northwest Territories, Cott et al. (2008) found that winter volumetric reductions ≤10% were not likely to adversely affect winter dissolved oxygen. Although similar studies in Alberta have not been completed, there is concern increased sediment oxygen demand may be more likely in Alberta lakes and may render them more sensitive to volumetric reductions (for instance, volumetric reductions ≤5%) than the less biologically productive lakes studied by Cott et al. (2008). Until further studies are conducted on Alberta lakes, a 5% change in volume will be adopted as a precautionary threshold to maintain dissolved oxygen levels.

In order to set winter allocation limits, determining lake under ice volume (UIV) is essential. UIV can be estimated using the lake bathymetry and ice thickness data. In absence of such site specific data, approximate methods can be used to estimate UIV with a known lake shape, maximum depth, and maximum lake surface area (please see Section 2 for details). Where no lake shape is known, volume can be estimated assuming a parabolic-shaped lake.

For most lakes, even the minimum data required for the approximate estimation of lake volume and UIV (i.e. lake shape and maximum depth) are unavailable. However, unlike lake shape and maximum depth, information on lake surface area is easily available through satellite imagery/Google Earth. In this study an attempt has been made to test the applicability of estimating lake volume and UIV from minimal and readily available data, such as lake surface area.

Based on the aforementioned background, the objectives of this study are:

i) Approximately estimate maximum lake volume ($V_{Max}$) and 5% Under Ice Volume for an ice thickness of 80 cm ($UIV5_{80}$) using a known lake shape, maximum depth, and maximum lake surface area ($A_{Max}$) and compare these with actual values (based on bathymetry).
ii) Re-estimate numbers from objective (i) assuming a parabolic lake shape.
iii) Test the applicability of estimating $V_{Max}$ and $UIV5_{80}$ only from the $A_{Max}$ and compare these with the estimated numbers from objective (i) & (ii), as well as with actual values.

## 2 Theoretical Derivation

### 2.1 Maximum Volume from Lake Shape, Maximum Surface Area and Maximum Depth

The general formulation of lake volume by schematizing the shape of a lake that consists of a volume of revolution of a quadratic surface bounded by $y = kx^n$ and y axis,

[1] $$V = \frac{\pi}{k^{2/n}} \frac{n}{n+2} D^{1+2/n}$$

where, *k* and *n* are constant for a specific lake shape

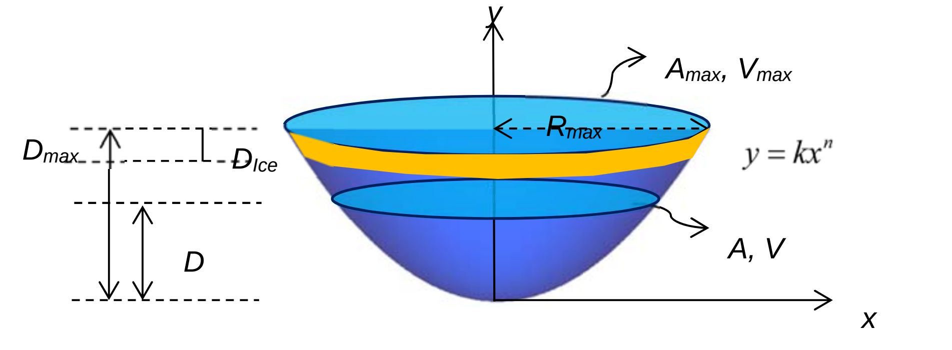


Figure 1. Schematic representation of a lake as a volume of revolution of a quadratic surface

The maximum volume ($V_{max}$) and surface area at a maximum depth ($D_{Max}$) are then defined as,

[2] $$V_{Max} = \frac{\pi}{k^{2/n}} \frac{n}{n+2} D_{Max}^{1+2/n}$$

[3] $$A_{Max} = \frac{\pi}{k^{2/n}} D_{Max}^{2/n}$$

The mean depth ($D_{Mean}$) is given by,

[4] $$D_{Mean} = V_{\max} / A_{\max} = \frac{n}{n+2} D_{\max} = \frac{1}{m} D_{\max}$$

where, $m$ is defined as the ratio of maximum depth to mean depth and is given by, $m = 1 + \frac{2}{n}$

So, the maximum volume can be estimated by,

[5] $$V_{Max} = \frac{1}{m} D_{Max} A_{Max}$$

Eq. 5 can be used to approximately estimate $V_{Max}$ from lake shape (see Table 1 for $m$), $D_{Max}$ and $A_{Max}$

Table 1. Calculated $m$ values (ratio of maximum depth to mean depth) for different lake shapes (Islam and Seneka, 2016).

| Shape | $m$ |
|---|---|
| Inverse Parabolic | 5 |
| Conic | 3 |
| Parabolic | 2 |
| Pseudo-Parabolic | 1.5 |
| Cylindrical | 1 |

## 2.2 Maximum Volume from Maximum Surface Area

Eliminating $D_{Max}$ from Eq. 2 and Eq. 3,

[6] $$V_{Max} = \frac{n}{n+2} \frac{k}{\pi^{n/2}} A_{Max}^{1+n/2}$$

Define, $\alpha = \frac{n}{n+2} \frac{k}{\pi^{n/2}}$ and $\beta = 1 + \frac{n}{2}$ ,

[7] $V_{Max} = \alpha\, A_{Max}^{\beta}$

Note, $\alpha$ and $\beta$ are constant for a lake shape. Taking logarithm on both sides of Eq. 7,

[8] $\log V_{Max} = \beta \log A_{Max} + \log \alpha$

Eq. 8 represents a straight line with a slope of $\beta$ and an intercept of $\log \alpha$ . A linear regression model could be developed using log transformed $V_{Max}$ and $A_{Max}$ with known data, and then the model could further be used to estimate $V_{Max}$ for the lakes without a bathymetry data, where only $A_{Max}$ is available.

**2.3 5% Under Ice Volume from Lake Shape, Maximum Surface Area and Maximum Depth**

If the ice thickness is $D_{Ice}$, 'Under Ice Volume' of a lake is defined as (according to Eq. 2),

[9] $V_{UI} = \frac{\pi}{k^{2/n}} \frac{n}{n+2} \left(D_{Max} - D_{Ice}\right)^{1+2/n}$

From Eq. 2 and 9, ratio of the 'Under Ice Volume' to the 'Maximum Volume',

[10] $R_{UI} = \frac{V_{UI}}{V_{Max}} = \left(1 - \frac{D_{Ice}}{D_{Max}}\right)^{m}$

The 5% Under Ice Volume for a given Ice thickness of $D_{Ice}$, is then given by

[11] $UIV5_{D_{ice}} = \frac{5}{100} R_{UI} * V_{Max}$

Eq. 5, 10, and 11 can be used to estimate $UIV5_{D_{ice}}$ from a lake shape (see Table 1 for *m*), $D_{Max}$ and $A_{Max}$

**2.4 5% Under Ice Volume from Maximum Surface Area**

Combining Eq. 3, 7 and 9

[12] $V_{UI} = \alpha\, A_{Max}^{\beta} \left(1 - \frac{\pi^{n/2}}{k} D_{Ice}\, A_{Max}^{-n/2}\right)^{1+\frac{2}{n}}$

The 5% Under Ice Volume for a given Ice thickness of $D_{Ice}$, is then given by,

[13] $UIV5_{D_{ice}} = \frac{5\alpha}{100} A_{Max}^{\beta} \left(1 - \frac{\pi^{n/2}}{k} D_{Ice}\, A_{Max}^{-n/2}\right)^{1+\frac{2}{n}}$

Define, $\Phi\left(A_{Max}\right) = \left(1 - \frac{\pi^{n/2}}{k} D_{Ice}\, A_{Max}^{-n/2}\right)^{1+\frac{2}{n}}$

Although $\Phi\left(A_{Max}\right)$ varies with $A_{Max}$ , especially for smaller lake surface area, it eventually become constant ( ~1) for larger lake surface area. Assuming $\Phi\left(A_{Max}\right) \approx 1$, the 5% Under Ice Volume for a given Ice thickness of $D_{Ice}$ is approximately given by,

[14] $UIV5_{D_{ice}} \approx \alpha'\, A_{Max}^{\beta}$ , where $\alpha' = \frac{5\alpha}{100}$

Taking logarithm on both sides of Eq. 14,

[15] $\log UIV5_{D_{ice}} \approx \beta \log A_{Max} + \log \alpha'$

Eq. 15 represents a straight line with a slope of $\beta$ and an intercept of $\log \alpha'$. A linear regression model could be developed using log transformed $UIV5_{D_{ice}}$ and $A_{Max}$ with known data, and then the model could further be used to estimate $UIV5_{D_{ice}}$ for the lakes without a bathymetry data where only $A_{Max}$ is available.

## 3 Data and Methodology

Three types of data are required for this study:

i) Area/Capacity data (relation of lake surface area and lake volume with elevations as derived from the bathymetry data)
ii) Lake shape information
iii) Maximum recorded lake level (in order to estimate the maximum lake volume and maximum lake surface area)

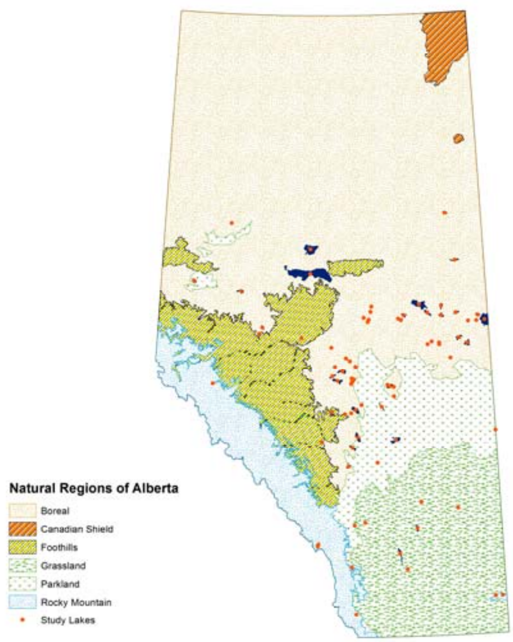


Figure 2. Location of 77 Alberta lakes analyzed in the current study

Figure 2 shows the location of 77 lakes selected for this study. These lakes are selected such a way that the bathymetric information is available; lakes are distributed over major river basins and over different natural regions/sub-regions; and also cover a wide spectrum of depth (mean depth ranges from 1.3 meter to 50 meter; maximum depth ranges from 3.01 meter to 111.9 meter) and lake surface area (lake surface area ranges from 0.17 km$^2$ to 1200 km$^2$). Out of 77 lakes, 16 are from the Beaver River basin; 15 are from the Athabasca River basin; 22 are from the North Saskatchewan River basin; 6 are from the Peace/Slave River basin; 17 are from the South Saskatchewan River basin; and one is from the Milk River basin. Most of the selected lakes are from the Boreal natural zone (73%) with the remainder distributed in the Parkland, Rocky Mountain, Grassland, and Foothills natural zones.

The historical mean daily lake level data has been collected from two different sources: Water Survey Canada (WSC) Historical Hydrometric Data of Environment Canada, and the Miscellaneous Streams and Lake Levels (MSLL) database of Alberta Environment and Parks (Islam and Seneka, 2015). Lake Area/Capacity curves of 77 lakes are collected from the reconstructed Area/Capacity curves of Alberta Environment and Parks (Islam and Seneka, 2017). These curves have been used to calculate maximum depth, maximum surface area and maximum volume of the study lakes. Note, assumptions have been made that the maximum depth, maximum surface area and maximum volume of the study lakes are corresponding to the maximum recorded water level of these lakes. Lake shape information of these 77 lakes are collected from Islam and Seneka (2016).

### 3.1 Estimation of $V_{Max}$ and $UIV5_{80}$ from lake shape, $D_{Max}$ and $A_{Max}$

First, pick the value of $m$ from Table 1 based on the known shape of a lake. Where no lake shape is known, $m$ value can be approximated assuming parabolic-shaped lake ($m$=2).

Second, for a known $m$, $A_{Max}$, and $D_{Max}$, estimate the $V_{Max}$ from Eq. 5

Third, estimate the ratio of the Under Ice Volume to the Maximum Volume ($R_{UI}$) from Eq. 10 for a known $D_{Max}$ and ice thickness of $D_{ice}$.

Finally, for an estimated $V_{Max}$, and $R_{UI}$, estimate the $UIV5_{80}$ from Eq. 11

### 3.2 Development of Linear Models: '$V_{Max}$ vs $A_{Max}$' and '$UIV5_{80}$ vs $A_{Max}$'

First, for each lake, maximum surface area ($A_{Max}$), maximum volume ($V_{Max}$), and the 5% Under Ice Volume for an ice thickness or 80 cm ($UIV5_{80}$) are estimated from the Area/Capacity curve. Note, assumption has been made that lake surface area and volume at the maximum recorded water level represents the maximum surface area and maximum volume, respectively.

Second, $V_{Max}$ and $A_{Max}$ for all lakes were log-transformed so that they could be fitted according to Eq. 8.

Third, $UIV5_{80}$ and $A_{Max}$ for all lakes were log-transformed so that they could be fitted according to Eq. 15.

Finally, Linear Model tools of "R Commander" package were used to develop linear regression models: $\log V_{Max}$ vs $\log A_{Max}$ and $\log UIV5_{80}$ vs $\log A_{Max}$

## 4 Results

### 4.1 Estimation of $V_{Max}$ and $UIV5_{80}$ from lake shape, $D_{Max}$ and $A_{Max}$

$V_{Max}$ and $UIV5_{80}$ has been estimated using the Eq. 5 and Eq. 11, respectively for all 77 study lakes for which the lake shape is known through the Islam and Seneka (2016). $V_{Max}$ and $UIV5_{80}$ for all those 77 lakes were then re-estimated assuming a parabolic lake shape. Figure 3 shows a comparison of these estimated numbers with the actual values. In general, the estimated $V_{Max}$ and $UIV5_{80}$ are in a good agreement with the calculated values. As expected, applying a parabolic geometry slightly overestimated lake volume (and 5% under ice volume) when comparing with the estimated numbers from the actual shape.

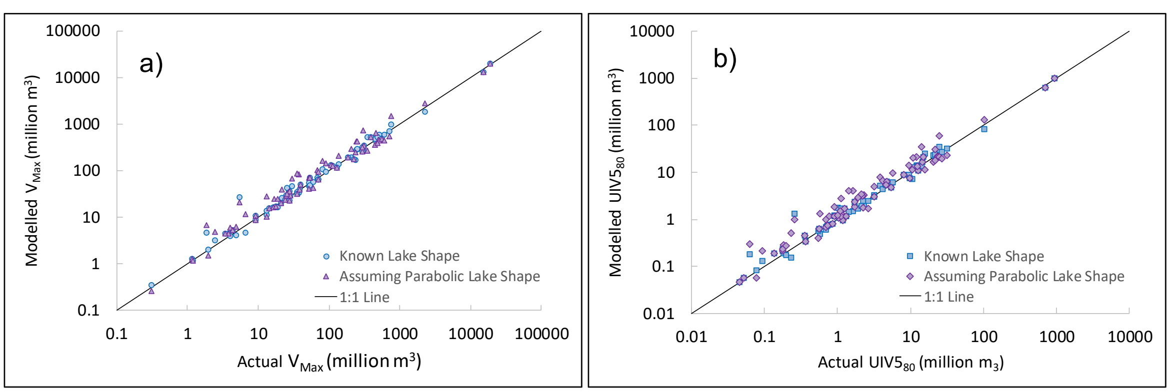


Figure 3. Comparison of a) modelled and actual $V_{Max}$ and b) modelled and actual $UIV5_{80}$ for known lake shape as well as assuming a parabolic lake shape.

### *4.2* Linear Model: $V_{Max}$ vs $A_{Max}$

The fitted linear regression model was given by,

[16] $$\log_e V_{Max} = 1.0504 * \log_e A_{Max} + 1.595;\ R^2 = 0.904$$

[17] $$V_{Max} = 4.9284 * (A_{Max})^{1.0504}$$

where, $V_{Max}$ is in millions of cubic meters and $A_{Max}$ is in millions of square meters (or km$^2$)

Note, the fitted equation presented in Equation 16 and 17 is based on 76 sample points (Cold Lake excluded). Initially a linear model was developed based on all 77 sample points including Cold Lake. However, the data point of Cold Lake acts as a noticeable outlier when comparing the modelled volume with the actual volume. Cold Lake is atypical of most lakes in Alberta as it has a very high mean depth compared to the maximum volume. Since the inclusion of Cold Lake results in an overall significant increase in the per cent bias of the linear model, it was decided to exclude the lake from the linear model sample data.

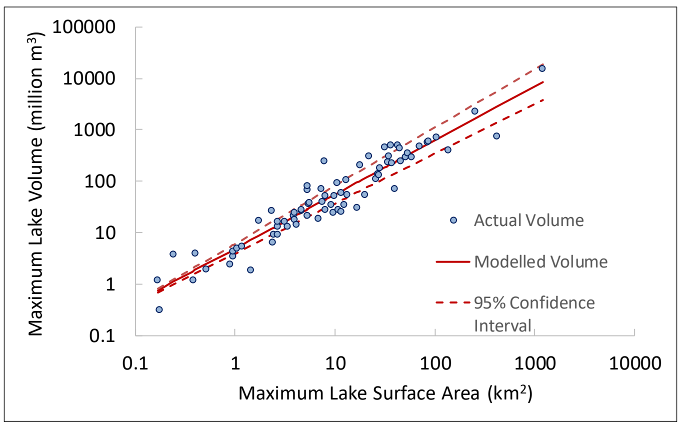


Figure 4. Comparison of modelled and actual maximum volume of Alberta lakes.

Table 2. Summary of statistics for the fitted linear models

| Statistical Test | | Linear Model (Log Transformed) | |
|---|---|---|---|
| | | **$V_{Max}$ vs $A_{Max}$**<br>Intercept=+1.5950<br>Exponent=1.0504 | **$UIV5_{80}$ vs $A_{Max}$**<br>Intercept=-1.5856<br>Exponent=1.0588 |
| $R^2$ | | 0.904 | 0.879 |
| t-Statistics | Standard Error (Intercept) | 0.1107 | 0.1273 |
| | Standard Error (Exponent) | 0.0397 | 0.0457 |
| | t-Value (Intercept) | 14.41 | -12.45 |
| | t-Value (Exponent) | 26.45 | 23.18 |
| | P-Value (t-statistic) | $<2 \times 10^{-16}$ | $<2 \times 10^{-16}$ |
| 95% Confidence Interval | Intercept | (1.3744, 1.8156) | (-1.8392, -1.3319) |
| | Exponent | (0.9713, 1.1296) | (0.9678, 1.1498) |
| F-Statistics | F-Value | 699.4 | 537.4 |
| | p-Value (F-Statistic) | $<2.2 \times 10^{-16}$ | $<2.2 \times 10^{-16}$ |

Figure 4 shows the modelled relationship based on Eq. 17. The solid maroon line shows the modelled maximum lake volume as estimated from the maximum lake surface area using Eq. 17, and the blue dots show the actual data. The 95% confidence limit was also calculated for the fitted exponent and intercept, represented by the dotted maroon line. It was found that about 51% of the actual data falls within the confidence interval. Statistical significance tests (t-Statistics and F-Value) were performed to assess the performance of the regression model (See Table 2). Note, t-value is a measure of the likelihood that the actual value of the exponent and intercept is not zero. The larger the absolute value of t, the less likely that the actual value of the parameter could be zero. Moreover, the probability (p-value) of the t-Statistics was also estimated for both of the parameter and found to be less than 2 x $10^{-16}$. The p-value is the probability of obtaining the estimated value of the parameters if the actual parameters value are zero. The smaller the p-value, the more significant the parameter and the less likely that the actual parameter value is zero. The F-value and associated p-value for the regression model was found to be 699.4 and less than 2.2 x $10^{-16}$, respectively. Low p-value associated with the F-value would imply that the regression equation does have some validity in fitting the data. Various statistics of residuals (e.g., normality test of residuals, variance of residuals, Cook's distance) of the linear model presented in Eq. 16 were also checked in order to ensure that the equation represent an ideal linear regression.

### 4.3 Linear Model: *UIV5* (Ice Thickness=80 cm) vs $A_{Max}$

The fitted linear regression model is given by,

[18] $$\log_e UIV5_{80} = 1.0588 * \log_e A_{Max} - 1.5856;\ R^2 = 0.879$$

[19] $$UIV5_{80} = 0.2048 * (A_{Max})^{1.0588}$$

where, $UIV5_{80}$ is in millions of cubic meters and $A_{Max}$ is in millions of square meters (or km$^2$). Note, like Equation 16 & 17, the fitted equation presented in Equation 18 and 19 is based on 76 sample points (excluding Cold Lake). Figure 5 shows the modelled relationship based on Eq. 19. The solid maroon line shows the modelled 5% Under Ice Volume (for an ice thickness of 80 cm) as estimated from the maximum lake surface area using Eq. 19, and the blue dots show the actual data. The 95% confidence limit was also calculated for the fitted exponent and intercept, represented by the dotted maroon line. It was found that about 51% of the actual data falls within the confidence interval. Statistical significance tests (t-Statistic and F Value) were performed to asses the performance of the regression model (Table 2).The probability (p-value) of the t-Statistic was also estimated for both of the intercept and exponent and found to be less than 2 x $10^{-16}$. The F-value and associated p-value for the regression model was found to be 537.4 and less than 2.2 x $10^{-16}$, respectively. Low p-value associated with the F-value would imply that the regression equation does have some validity in fitting the data. Various statistics of residuals of the linear model presented in Eq. 18 were also checked in order to ensure that the equation represent an ideal linear regression.

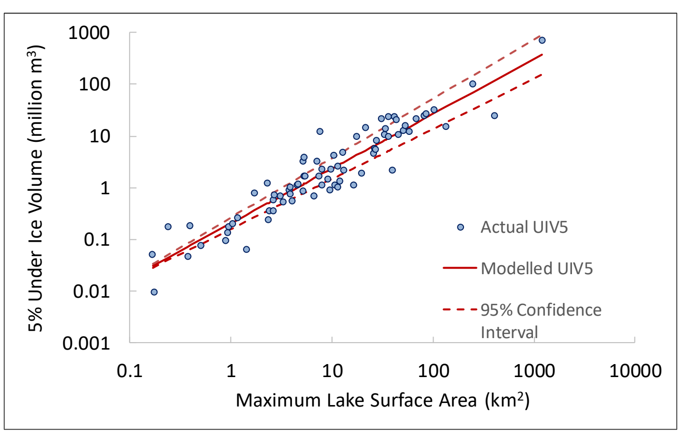


Figure 5. Comparison of modelled and actual 5% Under Ice Volume (Ice Thickness=80 cm)

### 4.4 Comparison between Models

Figure 6 shows a comparison of modelled and actual $V_{Max}$ and $UIV5_{80}$ for three different models: $V_{Max}$ from known lake shape, $A_{Max}$ and $D_{max}$; $V_{Max}$ from assumed parabolic lake shape, $A_{Max}$ and $D_{max}$; and $V_{Max}$ from linear regression. Various measures of the Goodness of Fit (GOF) have been calculated using the R package "hydroGOF" and presented in Table 3. In general, the estimated $V_{Max}$ from a known lake shape, $A_{Max}$ and $D_{max}$ has the better GOF compared to the other models.

As expected, applying a parabolic lake shape slightly overestimated lake volume (and 5% under ice volume) when comparing with the estimated numbers from the actual shape. Since the linear regression model uses only one parameter, the lake surface area, to estimate maximum volume and 5% under ice volume, their GOF is relatively poor compared to the other models that used the additional physical data/parameters.

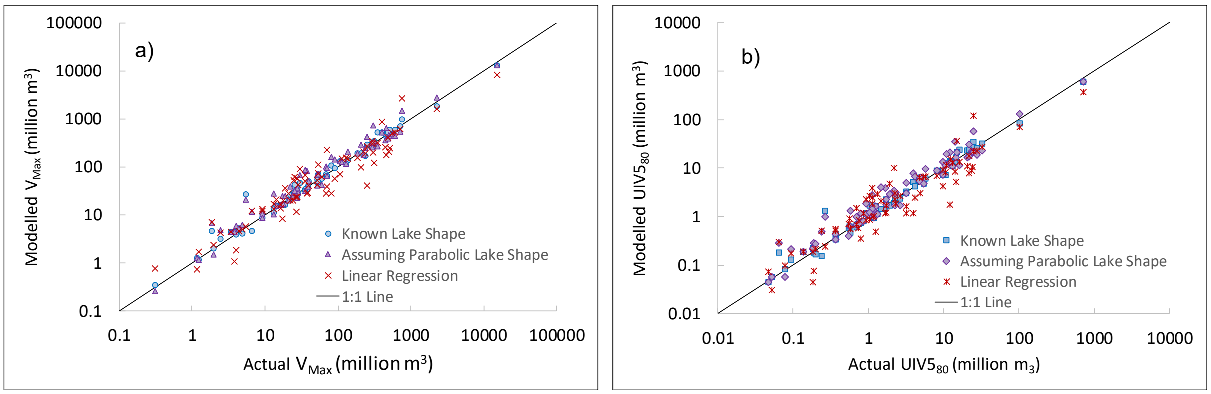


Figure 6. Comparison of a) modelled and actual $V_{Max}$ and b) modelled and actual $UIV5_{80}$ for three different models.

Table 3. Statistical Goodness of Fit (GOF) for different models to estimate $V_{Max}$ and $UIV5_{80}$

| Statistical Goodness of Fit (GOF) | $V_{Max}$ Models | | |
|---|---|---|---|
| | $V_{Max}$ → Known Lake Shape, $A_{Max}$ and $D_{max}$ | $V_{Max}$ → Assumed Parabolic Lake Shape, $A_{Max}$ and $D_{max}$ | $V_{Max}$ → Linear Regression |
| Mean Error | -4.19 | 21.40 | -84.53 |
| Normalized Root Mean Square Error | 9.2 | 10 | 46.40 |
| Percent Bias | -0.7 | 3.5 | -23.30 |
| Nash-Sutcliffe efficiency | 0.99 | 0.99 | 0.78 |
| Coefficient of Determination | 0.99 | 0.99 | 0.93 |
| | $UIV5_{80}$ Models | | |
| | $UIV5_{80}$ → Known Lake Shape, $A_{Max}$ and $D_{max}$ | $UIV5_{80}$ → Assumed Parabolic Lake Shape, $A_{Max}$ and $D_{max}$ | $UIV5_{80}$ → Linear Regression |
| Mean Error | -0.25 | 1.01 | -4.41 |
| Normalized Root Mean Square Error | 9.3 | 10.20 | 49.50 |
| Percent Bias | -0.9 | 3.6 | -26.70 |
| Nash-Sutcliffe efficiency | 0.99 | 0.99 | 0.75 |
| Coefficient of Determination | 0.99 | 0.99 | 0.92 |

## 5 Conclusions

In this study, we developed three different models to estimate maximum lake volume (a proxy of lake water availability) and 5% under ice volume (a proxy for winter allocation limit of lake water) assuming an ice thickness of 80 cm. These models have been developed in such a way that allows the user to apply the models based on availability of data (e.g. analytical model when more data is available, and regression model when minimal information is available). The summary of the models, data requirements and relevant caveats are listed in Table 4. Note, no models are recommended when the complete bathymetry of a lake is available. These models are recommended only when partial physical lake data is available.

Table 4. Recommended models for maximum volume ($V_{Max}$) and 5% under ice volume for an ice thickness of 0.8 m ($UIV5_{80}$) based on availability of data. Note, equations presented in second and third row are unit independent; however, equations presented in the last row are unit dependent ($V_{Max}$ and $UIV5_{80}$, is in million m$^3$, $A_{Max}$ is in km$^2$)

| Data Availability | Recommended Model | Remarks & Unit |
|---|---|---|
| Complete Bathymetry | No models recommended, $V_{Max}$ and $UIV5_{80}$ should be calculated based on the area/capacity curves | Only a fraction of Alberta lakes have bathymetric data. |
| Shape, Maximum Surface Area, Maximum Depth | $V_{Max} = \frac{1}{m} D_{Max} A_{Max}$ ; *m* Values from Table 1<br>$UIV5_{D_{ice}} = \frac{5}{100} R_{UI} * V_{Max}$ ; $R_{UI} = \left(1 - \frac{D_{Ice}}{D_{Max}}\right)^m$ | *Unit Independent.* Information on lake shapes is available for 77 lakes in Alberta (Islam and Seneka, 2016). Lake shape can be estimated from Table 1 for a known *m* |
| Maximum Surface Area, Maximum Depth | $V_{Max} = \frac{1}{2} D_{Max} A_{Max}$<br>$UIV5_{D_{ice}} = \frac{5}{100} R_{UI} * V_{Max}$ ; $R_{UI} = \left(1 - \frac{D_{Ice}}{D_{Max}}\right)^2$ | *Unit Independent.* Assuming a Parabolic shaped lake (the most common category for the 77 Alberta lakes with available data) |
| Only Maximum Surface Area | $V_{Max} = 4.9284 * (A_{Max})^{1.0504}$<br>$UIV5_{80} = 0.2048 * (A_{Max})^{1.0588}$ | *Unit dependent:* $V_{Max}$ and $UIV5_{80}$, is in million m$^3$, $A_{Max}$ is in km$^2$ Modelled values represent medians as data has been log-transformed |

## References


Cott, P.A., P.K. Sibley, A.M. Gordon, R.A. Bodaly, K.H. Mills, W.M. Somers, and G.A. Fillatre. 2008. Effects of water withdrawal from ice-covered lakes on oxygen, temperature and fish. *Journal of the American Water Resources Association,* 44: 328-342.

Islam, Z. and Seneka, M. 2015. Development of Alberta Lake Level Index: A Simple Approach. *Proceedings of the Annual General Conference*, Canadian Society for Civil Engineering, Halifax.

Islam, Z. and Seneka, M. 2016. Development of Generic Shapes of Alberta Lakes to Support Water Policy Development. *Proceedings of the Annual General Conference*, Canadian Society for Civil Engineering, London.

Islam, Z. and Seneka, M. 2017. Reconstruction of Area-Capacity Curves for Alberta Lakes. *Proceedings of 23rd Hydrotechnical Conference*, Canadian Society for Civil Engineering, Vancouver.

Neumann, J. 1959. Maximum Depth and Average Depth of Lakes. *Journal of the Fisheries Research Board of Canada*, **16**(6): 923-927.

R Core Team. 2017. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/.

Wetzel, R. G. 2001. *Limnology: lake and River Ecosystems*, Academic Press.