# DEVELOPMENT OF PROBABILISTIC COST FUNCTION FOR FLOOD DAMAGE TO RESIDENTIAL STRUCTURES 

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#### Abstract

Risk models, which describe the relationship between hazard intensity and a damage ratio are increasingly used in flood risk management. Direct tangible damage resulting from flooding is typically computed based on the internationally accepted method of depth-damage curves. Depth-damage curves relate absolute damage (in terms of currency) or relative loss (percentage of the estimated total replacement value of property) to a given flood depth. Many depth-damage functions in use today are computed from synthetic data, where data are collected from a representative sample of buildings with similar properties in a floodplain during field surveys. A primary problem when assessing risk at an objectbased spatial resolution using depth-damage curves is that these damage functions represent an average structure in the study location. There is great variability across any given structural class as well as variation within individual structures in a structural category (and across communities), for example, not all onestorey residences with basements are the same size, nor constructed of the same quality of materials and workmanship. The variability within a given class of buildings and the resulting depth-damage curve are often not transparent to the end user, thus damage estimates for individual buildings may be over/under estimated. In this paper, synthetic depth-damage curve data from communities in southern Ontario are used to develop probabilistic cost functions such that monetary damage estimates, as spent in Canadian dollars, and their likelihood of being exceeded at any given flood depth are more clearly expressed and communicated to end users.


## 1 INTRODUCTION

The most common and internationally accepted method for assessment of urban flood damage is through the use of depth-damage curves or stage-damage curves (Plazak1984, Prettenthaler et al. 2010). Depthdamage curves, as the name suggests, relate absolute cost damage (in terms of currency) or relative loss (percentage of the estimated total replacement value of property) to a given flood depth. While many factors can influence the damage incurred from flooding, e.g.: velocity, duration, etc., typically, only inundation depth is considered as a damage causing factor in the formulation of most depth-damage curves (Merz et al. 2004). These curves are developed separately for structural or load-bearing components; for contents (e.g.: interior furniture, art, appliances, etc.,); and for inventory in place (e.g.: commercial stock) (FEMA 2010).

Two common models are used to derive depth-damage curves: empirical and synthetic (Dottori et al. 2016). Empirical models use a data-driven approach, relying on actual damage datasets from past events in order to link building vulnerability and the flood event to damage data, whereas synthetic curves are generated based on a conceptual approach and expert knowledge, hypothesizing and making assumptions about the
damage, related to specific components in the building. This approach is thus less dependant on observed datasets for derivation, however data are necessary for validation and calibration (Dottori et al. 2016). Due to the relative scarcity of empirical data for a given community, synthetic depth-damage curves are common.

Depth-damage curves represent the generalized or "average" structure response for the community where they were derived. Within a single building class, e.g.: one-storey single family home with a basement, there is variability in building size, construction materials and quality of finishes. However, depth-damage curves give the impression of a one-to-one relationship, i.e.: for a single given water depth, " $x$ " percent damage or " $y$ " dollars of damage will incur. When considered over an entire community, the over and under estimations of damage typically average out (FEMA 2010) - but for a single building there is an unknown uncertainty in the damage estimation results, which is not explicitly stated.

In this paper, synthetic depth-damage curves derived from a 1985 report for the Ministry of Ontario are used as the basis to generate probabilistic cost functions which better represent the variability and variation in the estimated cost of building damages with respect to a range of flood levels. Depth-damage curve data are converted from 1984 dollars to 2016 dollars using indexes from Statistics Canada (https://www.statcan.gc.ca/): Consumer Price Index (CPI), Survey of Household Spending (SHS) and the New Housing Price Index. The resulting probabilistic cost functions are plotted to show the probability of exceeding a specific dollar amount (" $\$ \times X$ ") across a range of flood levels.

## 2 DATA

The data utilized in this research are from a 1985 report for the Ministry of Ontario (Paragon Engineering Limited et al. 1985), in which synthetic depth-damage curves were created from a sample of seven communities in southern Ontario. The synthetic depth-damage curves were computed via an on-site survey of households of representative structures in each of these communities. The survey results were processed against numerous water depths to generate depth-damage curves. A total of 34 curves were derived for residential structures, including curves for: one-storey, two-storey, split-level residences as well as townhouses and mobile homes. Eleven of these curves are devoted to one-story residences, including variants such as building quality, presence or absence of basement, and construction style (wood frame or masonry foundation). A recommendation in the Paragon Engineering Limited et al. (1985) report suggests mean damage curves for 7 basic types of structures are sufficient. Figure 1 illustrates the aggregation of (a) one-storey structures with basements and (b) two -storey structures with basements with damage cost converted to 2016 dollars and highlights the variability within the Ontario sample datasets over a range of water depths. Figure 1a represents summarized data from a total of 76 structures, while Figure 1b is the result of 99 field surveys on two-storey structures with basements. In both Figures 1a and 1b, the variability in the damage estimate to the structure and contents increases with increasing depth, from approximately $\$ 1,000$ at -2.4 m , to $\$ 10,000$ at 0 m , up to $\sim \$ 25,000$ difference between different buildings within the same class at 2.4 m flood level.

## 3 METHOD

The original depth-damage curves were developed in 1984 and reported in 1984 dollars. To convert to present day dollars, a combination of tables and data from the CPI and SHS tables via the procedure outlined by Natural Resources Canada and Public Safety Canada (2017) in the Canadian Guidelines and database of Flood Vulnerability Functions were followed. The method to convert contents depth-damage data is via the CPI for weights and classes from the SHS of household spending data tables, Eq. 1 and Eq. 2. While the SHS is not a direct measure of changing prices, spending amounts in each of the categories account for changes in quality and quantity over time (Natural Resources Canada and Public Safety Canada 2017). The CPI is a widely used measure of inflation, published by Statistics Canada (http://www.statcan.gc.ca/). Structure damage is updated to 2016 dollars using the New Housing Price Index (http://www23.statcan.gc.ca/imdb-bmdi/pub/indexN-eng.htm), Eq. 3.


Figure 1: Variability in the total estimated damages for (a) one-storey residences with basement and (b) two-storey residences with basement. The depth-damage curves are based on data from the 1985 Ministry of Ontario, report converted to 2016 dollars
[1] Contents Flood Damage Index $=\Sigma\left(\right.$ Component Index $_{i} *$ Weight $\left._{i}\right)$
[2] Current \$ (Contents) = Base Year \$ $x$ (Current Weighted SHS / Base Year Weighted SHS)
[3] Current $\$($ Structure $)=\frac{\text { Current New Housing Index }}{\text { Base Year Housing Index }} *$ Base Year Damages
The probabilistic cost functions (as shown in Figure 2) are the result of applying a series of treatments to the depth-damage curve data. The mean ( $\mu$ ) and standard deviation ( $\sigma$ ) for each of the 7 structure classes were used to compute the Coefficient of Variation ( $\delta$ ), Eq. 4. Next the lognormal standard deviation (弓) is calculated using the square of $\delta$, Eq. 5 . The median of the lognormal distribution $(\lambda)$ at each water depth is computed with Eq. 6. Finally, the probabilistic cost functions are derived based on the computed probability of exceedance of a specific dollar loss using the threshold values from Table 1 and Eq. 7. These damage class values ( $a_{i}$ ) are based on the Hazus damage classification scheme (FEMA 2010), converted to Canadian dollars and rounded to the nearest hundred for all values other than the minimum.
[4] $\delta=\frac{\sigma}{\mu}$
[5] $\zeta=\sqrt{\log \left(1+\delta^{2}\right)}$
[6] $\quad \lambda=\log (\mu)-\left(\frac{1}{2} \zeta^{2}\right)$
[7] PCF $=1-\operatorname{NormDist}\left(\log \left(a_{i}\right), \lambda, \zeta\right.$, true $)$

Table 1: Damage Class Values

| Damage Class Value (a) | Class |
| :---: | :---: |
| $\$ 1$ | Slight |
| $\$ 3,500$ | Moderate |
| $\$ 6,000$ | Minor |
| $\$ 10,000$ | Major |
| $\$ 20,000$ | Extensive |

## 4 RESULTS

The results for 6 of the 7 structure classes (mobile homes are omitted) are presented in Figure 2. For structures with basements, flood depths ranging from -2.4 m to +2.4 m are considered, to include basement/below grade level flooding, those without basements consider flood depth from -0.6 m to +2.4 m . Zero metres flooding represents the upper plane of the height of the first floor. In all scenarios, there is damage expected at this water level.

In Figure 2a, the one-storey building with basement structure class, the probability of exceeding $\$ 1$ in damages is lowest at lower water levels, with the inverse being true for the major class ( $>\$ 20,000$ ) of damage costs. At 0 m water depth, the probability of exceeding $\$ 20,000$ is negligible, $0.3 \%$. There is a $43 \%$ chance of exceeding $\$ 10,000$ in damages, and $41 \%$ of exceeding minor ( $\$ 6,000$ ) damage at 0 m water level for a one-storey building with basement. At this 0 m water depth there is a $10 \%$ probability of minimal damage, less than $\$ 3,500$. In both one-storey probability cost functions (with and without basements), the probability of exceeding $\$ 20,000$ in damage rises steeply beyond 0 m water depth, reaching $85 \%$ at 0.6 m for structures with and without a basement respectively.

Two-storey structures display a different damage response function to flooding than the one-storey structures. In the two-storey structures, the probability of exceeding $\$ 20,000$ is lower than the one-storey buildings, with greater than $80 \%$ probability at 1.2 m and 1.8 m for basement and no basement respectively. This is, in part, due to the consideration of the second storey. In a two-storey home, the total value of contents (furnishings, furniture, art, etc.) is distributed among multiple levels, whereas a single storey will have its contents on the first floor, thus, the expected total damages incurred, are typically less in a twostorey than a one-storey. It can be observed in Figures $2 a$ and 2 c , that the shape of the probabilistic cost functions are quite similar, but shifted toward lower probabilities of exceedance in the two-story building, indicating expected lower costs at the same water levels. At the maximum water depth of 2.4 m the twostorey with no basement shows there is, an $88 \%$ probability of exceeding $\$ 20,000$, which differs from the other one- and two-storey curves which are between $97 \%$ and $100 \%$ at this flood level.

For a split-level home, which typically has a finished lower level, which is partially below grade, the probability of exceeding $\$ 20,000$ is the greatest at lower water levels, with a $16 \%$ probability of exceeding $\$ 20,000$ at 0 m water depth and $100 \%$ exceedance at 1.2 m water depth.

The computed probability cost function for the townhouse shows a difference damage response than the other classes. In this class, the probability of exceedance of $\$ 20,000$ in damages is very low, with only a $27 \%$ probability at the maximum water depth of 2.4 m . At 0 m water depth, there is a $50 \%$ probability of damages being less than $\$ 3,500$. Thus the cost of expected damages overall is lower; however, as is seen in the other classes, cost of damage shows a steep increase between 0 and 0.6 m of flood water.


Figure 2: Probabilistic functions of exceeding a given monetary damage level (in 2016 CAD) for 6 structure classes (a-f) based on the 1984 Ontario depth-damage curves.

## 5 VALIDATION

To test the probabilistic cost functions, a flood risk simulation of residential structures was run. Using data from the city of Fredericton, 116 single family residences, ( 36 single storey and 80 two-storey buildings) the total estimated flood damages (structural and contents) were computed using ER2 calculator (http://hmcgrat1.ext.unb.ca/ER2 Online/Index.html). Water levels of 0.1 m and 0.6 m were applied as flood hazard input input along with the necessary building details and the original depth-damage curve data (in present day dollars, normalized with total loss value) to estimate damages. The result are shown in Figures 3 and 4 . The 36 single story residences with basements have an average building value of $\$ 185,500$. At
0.6 m water depth the average damage among this sample is $\$ 29,558$, with a standard deviation of $\$ 7,621$, Figure 3b. From this sample, the probability of exceedance, using the 5 exceedance values, Table 1, was computed and is plotted in Figure 3a. In this sample, at 0.6 m water depth, $100 \%$ of the structures had damage estimates greater than $\$ 1, \$ 3,500$ and $\$ 6,000.94 .44 \%$ of the buildings compute damage exceeding $\$ 10,000$ and $88.89 \%$ of these 36 residences exceed $\$ 20,000$ in losses. These damage estimates line up well with the derived probabilistic cost functions. Damages at 0.1 m depth level, in the single story do not align as well with the cost function. At this water level, as much as $97.22 \%$ of the structures have damage greater than $\$ 1$, with $3 \%$ reporting no damage at all. Only $22 \%$ of the buildings have loss estimates greater than $\$ 3,500$ and no structures having damage greater than $\$ 6,000$, Figure $3 a, b$.


Figure 3 Estimated damage level from a sample of single storey buildings with basements, with simulated flooding of 0.1 m and 0.6 m . (a) sample distribution overlaid on probabilistic cost functions, (b) estimated damage for each buildling in the sample.

These two water levels were also applied to the dataset of two storey buildings, Figure 4. At 0.6 m water depth, it was found that $100 \%$ of the properties had total damage estimates greater than $\$ 10,000.58 .75 \%$ of the structures estimate monetary damage greater than $\$ 20,000$, which lines up well with the threshold of the probabilistic cost function. At the 0.1 m flood depth level, the estimated damages fit well within the probabilistic cost function for all thresholds except the $\$ 6,000$ threshold. In the sample, $86.25 \%$ of the two storey residences has damages exceeding $\$ 6,000$, however the cost function exceedance range predicts $20-70 \%$ probability within this damage range.


Figure 4. Estimated damage level from a sample of two storey buildings with basements, with simulated flooding of 0.1 m and 0.6 m . (a) sample distribution overlaid on probabilistic cost functions, (b) estimated damage for each buildling in the sample.

## 6 CONCLUSIONS

Depth-damage curves represent a standard method to estimate building damage resulting from flood events. These curves are created either synthetically or empirically and represent the average response for a broad class of similar structures in the region for which they were created. Often, the data required to generate and validate these curves are unavailable in all regions, thus, it is common for curves from one community, country or province to be used in other geographic areas. Given that these curves are a representative average, the one-to-one relation of water depth to damage (either in dollars or as a percent of the structure/contents) conveys to the user a fixed damage estimate, with no indication of the uncertainty or variability within the class.

In this research, synthetically derived depth-damage curves from a 1985 report were used as the basis to generate probabilistic cost distribution functions, in an attempt to provide a better idea of the probability of exceeding damages of a specific dollar amount over a range of water depths.

Starting with depth-damage curves from 34 structural classes, the data was converted from 1984 dollars to 2016 dollars. Using the present day currency and the Ministry of Ontario report, which suggests damage
curves for 7 basic types of structures are sufficient, the depth-damage curve data were processed and probabilistic cost distribution functions for exceedance of " $\$ \times X$ " were developed. The exceedance classes were based on the Hazus classification for minor and major damage and converted to Canadian dollars.

While the underlying data for this research are based on field surveys conducted in 1984, the described methodology is general and can be applied to any depth-damage data. In addition, the class value ranges as well as the number of classes used can be easily modified.

The probabilistic cost functions were validated against single and two storey residences with basements, using a sample building dataset from Fredericton, NB. Preliminary results indicate that the developed probabilistic cost functions represent the sample datasets well. Further testing of the remaining residential occupancy classes and additional datasets, in other locales, is in progress. Additionally, work is ongoing to access actual claims data to verify the probabilistic cost functions - as the current validation method simply tests one method against another.

The result of this research are probabilistic cost functions which better represent the variability in estimated building damages with respect to a range of flood levels for 7 types of structural classes. The important benefit of these proposed probabilistic cost curves is a more transparent assessment of variability in the expected cost and damages over a range of flood depths.

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