



RECONSTRUCTION OF AREA-CAPACITY CURVES FOR ALBERTA LAKES

Islam, Zahidul^{1, 3} and Seneka, Michael²

^{1, 2} Alberta Environment and Parks, Canada

³ Zahidul.Islam@gov.ab.ca

Abstract: Area-Capacity curves represent the relationship of surface area and volume of a lake or reservoir to elevation. Area-Capacity curves are essential to the development of water balance models for lakes; more specifically, they provide science-based information on surface water availability at any given time. This information can be vital to the execution of informed management decisions, such as the permitting of water withdrawals from lakes in such a way as to protect ecosystems while supporting reliable, quality water supplies that contribute to a sustainable economy. Currently, there are three major sources of area/capacity information in Alberta: the *Atlas of Alberta Lakes* (Mitchell 1990); Alberta Environment and Parks (AEP) archived files of historical bathymetric surveys and lake area/capacity calculations; and digital lake bathymetry from the Alberta Geological Survey (AGS). The *Atlas of Alberta Lakes* provides Area-Capacity curves for about 80 lakes in Alberta. However, these curves are only available on paper or in digital image format and are not usable unless digitized to retrieve the area/capacity information at various elevations. The AEP archived files contain manual calculations of area/capacity based on original bathymetry surveys for about 60 lakes and are not immediately accessible to the public. AGS data provide lakes bathymetry (GIS files) on their website, but not the actual information on area/capacity. In the current study, an attempt has been made to combine these sources and reconstruct Area-Capacity curves for numerous Alberta lakes. Digital images of Area-Capacity curves from the *Atlas of Alberta Lakes* were manually digitized, and higher order polynomial equations were fitted to develop Area-Capacity curves. Lakes for which area/capacity calculations are not available but digital bathymetry data could be acquired through the AGS, an ArcGIS based tool was developed to construct Area-Capacity curves. A mathematical extrapolation technique was applied to extend the Area-Capacity curves beyond the original surveyed water level.

1 INTRODUCTION

Area-Capacity curves represent the relationship of surface area and volume of a lake or reservoir to elevation (Figure 1). These curves are essential to the development of water balance models for lakes; more specifically, they provide science-based information on surface water availability at any given time. This information can be vital to the execution of informed management decisions, such as the permitting of water withdrawals from lakes in such a way as to protect ecosystems while supporting reliable, quality water supplies that contribute to a sustainable economy.

Currently, there are three major sources of area/capacity information in Alberta:

- I. *Atlas of Alberta Lakes* (the *Atlas*): The *Atlas of Alberta Lakes* (Mitchell 1990) is a collection of information on 100 of Alberta's most studied lakes. It provides information on location, drainage basin characteristics, water quantity, water quality, and biological characteristics of lakes. The *Atlas* provides Area-Capacity Curves for about 80 lakes in Alberta. However, these curves are only

available on paper or in digital image format and are not usable unless digitized to retrieve the area/capacity information at various elevations.

- II. Alberta Environment and Parks (AEP) archived files: AEP has a collection of historical information on lake bathymetric surveys. The AEP archived files contain manual calculations of area/capacity (Area-Capacity Table) based on original bathymetry surveys for about 60 lakes. However, this information is not immediately accessible to the public.
- III. Alberta Geological Survey (AGS) digital bathymetry: AGS has a collection of digital bathymetry, comprised of Geographic Information System (GIS) data of about 169 lakes (Alberta Geological Survey 2016). This dataset has been generated by capturing by digitizing the historic hydrographic survey maps and then digitally modelled to produce a continuous surface for each lakebed. However, AGS data provide lakes bathymetry, but not the actual information on area/capacity.

In the current study, an attempt has been made to combine these sources and reconstruct Area-Capacity curves for numerous Alberta lakes.

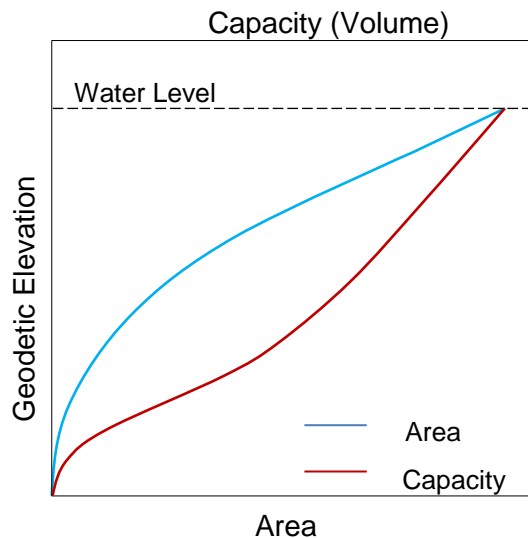


Figure 1: Schematic Diagram of Area-Capacity Curves

2 DATA AND CHALLENGES

The Atlas is publicly accessible digitally through the [University of Alberta website](#) (Mitchell 1990). First, the digital images (JPEG Format) of Area-Capacity curves of about 80 lakes are downloaded from the website. Then, an open source screen digitizing software [Plot Digitizer](#) (SOURCEFORGE.NET 2015) is used to manually digitize those digital images of Area-Capacity curves. Plot Digitizer is a Java program used to digitize functional X-Y type scatter or line plots from a scanned image of a plot (SOURCEFORGE.NET 2015). The advantage of this screen digitizing is that enough points can be collected from the digital images to capture the curvature of Area-Capacity curves. However, this process has human errors which can lead to irregular shapes, especially when they are zoomed in.

The AEP archived files contains Area-Capacity calculations based on original bathymetry contours. Lake surface area and lake volume are then calculated at regular elevation intervals (from the lake bottom to the surveyed water level) from those bathymetry contours. However, often these calculations have fewer points compared to the depth of a lake.

The [AGS digital lake bathymetry data](#) is a combination of multiple data sources (Alberta Geological Survey 2016). All available bathymetry and related information for lakes were collected and hard copy maps digitized where necessary. The data were validated against more recent data (Shuttle Radar Topography Mission 'SRTM' imagery and Indian Remote Sensing 'IRS' imagery) and corrected where necessary. ESRI ArcGIS 3D Analyst could be used to calculate lake surface area and volume (capacity) at various elevations to construct Area-Capacity curves from this digital lake bathymetry dataset. However, often Area-Capacity curves constructed through this process do not match with existing Area-Capacity curves collected from other sources, whenever available (e.g., the Atlas, AEP archived files).

Moreover, most of the Area-Capacity curves available through historical sources (e.g., the Atlas or AEP Archived files) are based on bathymetry surveys conducted in 1960s/1970s/1980s. As the calculation of lake surface area and volume (capacity) are limited by water level information available at the time of the survey, sometimes these curves are unable to provide information on area/capacity at a water level above the surveyed water level. Hence, a mathematical extrapolation is often required to estimate area/capacity above the surveyed water level.

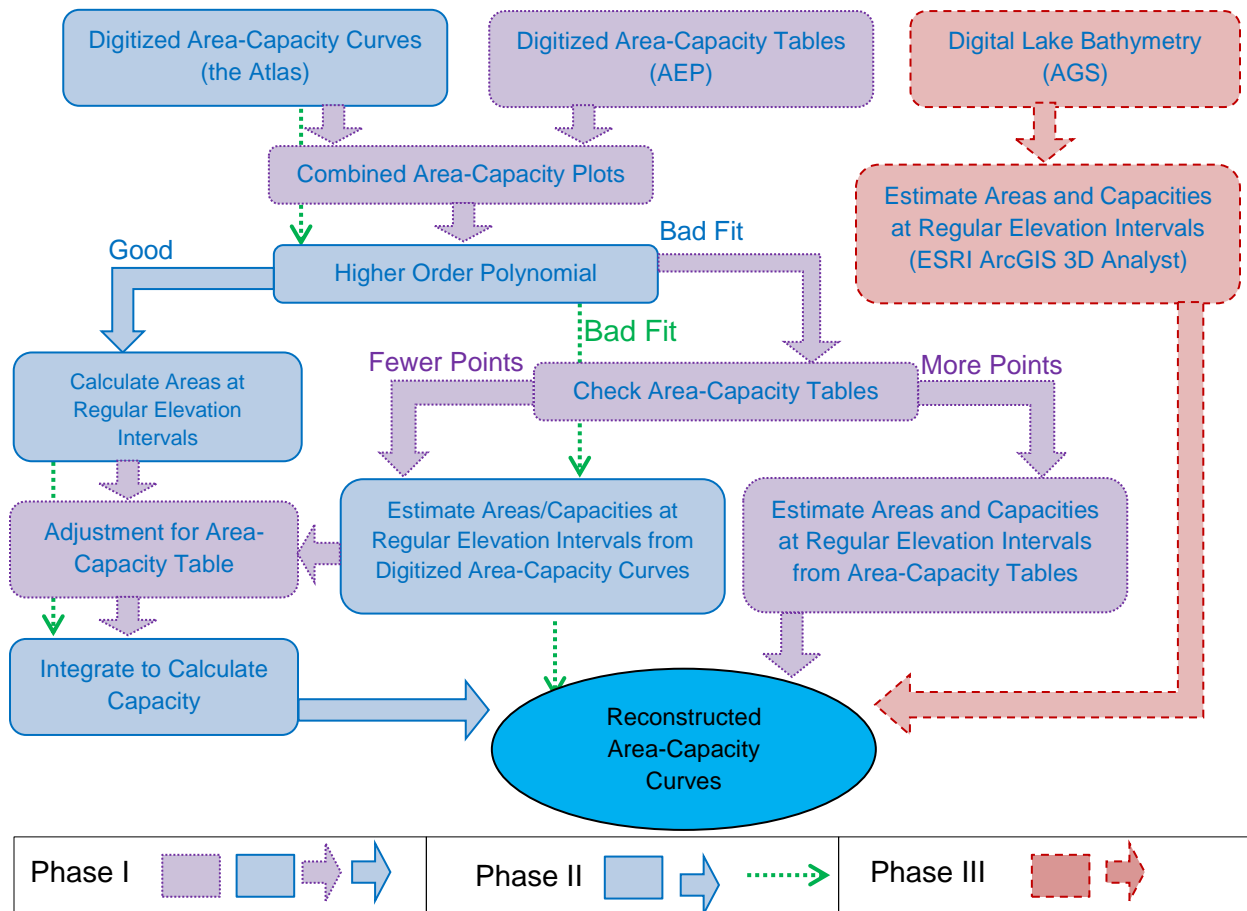


Figure 2: Reconstruction Flowchart for Phase I, II, and III

3 METHODOLOGY

Based on data availabilities and challenges discussed in the aforementioned sections, three different phases have been planned to reconstruct Area-Capacity curves. The following subsections will demonstrate the reconstruction methodologies of these three phases:

Phase I: Lakes have “Area-Capacity Curves” (through the Atlas) and “Area-Capacity Tables” (through AEP Archived files)

Phase II: Lakes have “Area-Capacity Curves” (through the Atlas) but no “Area-Capacity Tables” (through AEP Archived files)

Phase III: Lakes have no Area-Capacity curves/tables; have AGS lake bathymetry data

3.1 Phase I Methodology

Figure 2 shows a flowchart of Area-Capacity curves reconstruction methodology for all Phases. The purple & blue rectangular boxes and purple & blue wide arrows show steps for the Phase I reconstruction process (i.e. for the lakes which has “Area-capacity Curves” available through the Atlas as well as “Area-Capacity Tables” available through AEP archived files). The step by step methodology is also given below with the demonstration of the steps for the Marie Lake and Crimson Lake of Alberta in Figure 3:

Step 1: The digital images of Area-Capacity Curves are manually digitized using the Plot Digitizer.

Step 2: The Area-Capacity Tables are collected (if available) from AEP archived files, digitized and plotted.

Step 3: Area-Capacity plots from both sources (Step 1 & Step 2) are combined (Figure 3a) for each lakes.

Step 4: Based on the combined points, a higher order polynomial (usually 6th order) is fitted to represent the analytical relationship between lake surface area and lake depth (with respect of the surveyed water level) (Figure 3b).

Step 5a: If the polynomial shows a good fit between lake surface area and depth (Figure 3b), then the polynomial equation is used to reconstruct lake surface area at regular elevation intervals from the lake bottom to the lake surface (surveyed water level).

It was observed that, often the reconstructed lake surface areas calculated from the polynomial equation do not match with the values from the Area-Capacity Tables. The Area-Capacity Tables from AEP archived files are based on original bathymetric contours and ideally the reconstructed curves should follow these points. Hence, an adjustment of reconstructed curves was necessary. Following steps were applied to adjust the reconstructed curves so that they follow the data points based on original bathymetry:

- I. The polynomial equation developed in **Step 4** is used to estimate lake surface area at the elevations where original bathymetric contours are available (points of the Area-Capacity Tables based from AEP archived files).
- II. Area Adjustment Factors are calculated by dividing the lake surface areas found from the Area-Capacity Table by the lake surface areas found in **Step 5a (I)** (for the same elevation or lake depth). Note, near the lake bottom, these factors could provide unrealistic higher values (dividing a small number with another small number). In order to avoid this to happen, these factors were capped at 1.5.
- III. The Area Adjustment Factors were then interpolated at regular elevation intervals (as mentioned in **Step 5a**).
- IV. The reconstructed lake surface areas were then adjusted by multiplying the lake surface areas from **Step 5a** (i.e., using the polynomial equation) by the Area Adjustment Factors.

Finally, the capacities (volumes) at regular elevation intervals are then calculated by numerically integrating the adjusted reconstructed lake surface areas using trapezoidal rule.

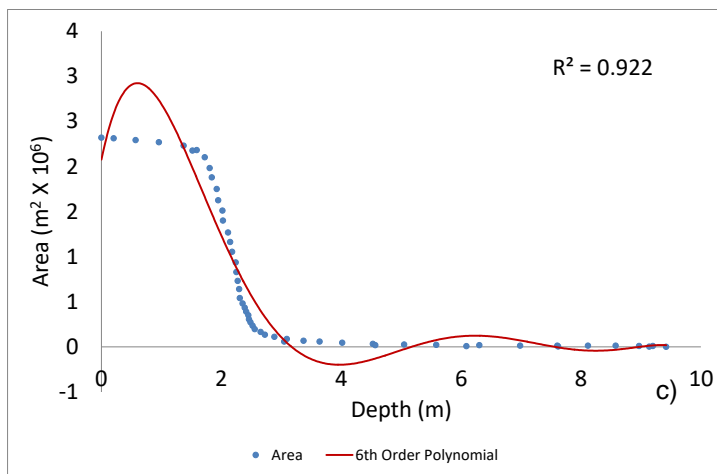
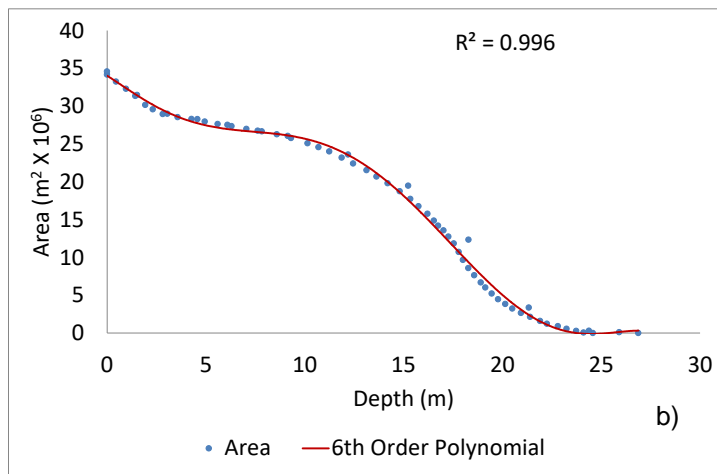
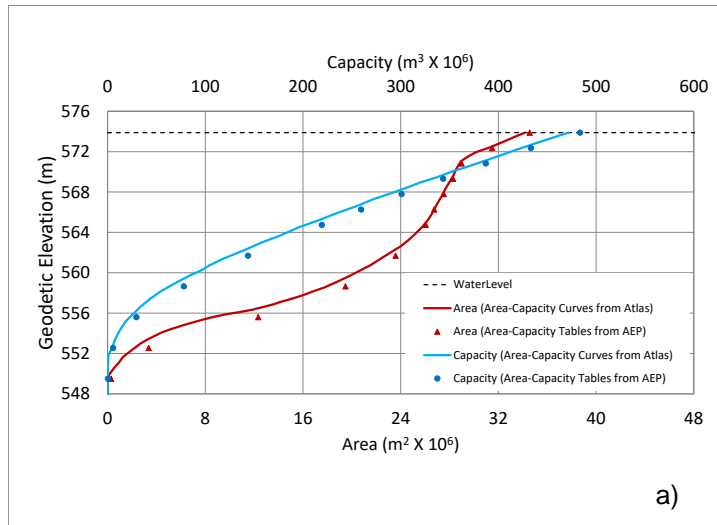


Figure 3: a) The combined (from the Atlas and from the AEP archive) Area-Capacity plots for the Marie Lake b) 6th Order Area-Depth Polynomial for the Marie Lake (example of good fit), c) 6th Order Area-Depth Polynomial for the Crimson Lake (example of bad fit)

Step 5b: If the polynomial does not show a good fit between lake surface area and depth (Figure 3c), then the Area-Capacity curves are reconstructed either from the Area-Capacity Tables (AEP) or from the digitized Area-Capacity Curves (the Atlas).

If the Area-Capacity Table has sufficient points (to maintain the curvature), the Table is used to linearly interpolate areas/capacities at regular elevation intervals (from lake bottom to lake water level).

If the Area-Capacity Table does not have sufficient points, the digitized Area-Capacity Curves are used to linearly interpolate areas/capacities at regular elevation intervals. However, in order to make sure that the reconstructed curves developed through this interpolation represent calculated areas/capacity from the original bathymetric contours (points from the Area-Capacity Tables), adjustments are necessary. Following steps were applied for the adjustments:

- I. From the digitized Area-Capacity Curves, linear interpolation is used to estimate lake surface area and capacity at the elevations where original bathymetric contours are available (points of the Area-Capacity Tables).
- II. Area Adjustment Factors and Capacity Adjustment Factors were calculated by dividing the lake surface areas and capacities found from the Area-Capacity Table by the lake surface areas and capacities found in **Step 5b (I)**, respectively. Note, near the lake bottom these factors could provide unrealistic higher values (dividing a small number with another small number). In order to avoid this to happen, they (factors) were capped at 1.5.
- III. The Area Adjustment Factors and Capacity Adjustment Factors were then interpolated at regular elevation intervals.
- IV. The reconstructed lake surface areas and capacities were then adjusted by multiplying the lake surface areas and capacities from **Step 5b** by the Area Adjustment Factors and Capacity Adjustment Factors, respectively.

3.2 Phase II Methodology

The blue rectangular boxes, blue wide arrows, and green arrows in Figure 2 show steps for the Phase II reconstruction process (i.e. for the lakes which has "Area-capacity Curves" available through the Atlas **but no** "Area-Capacity Tables" are available through AEP archived files). The step by step methodology is also given below:

Step 1: The digital images of Area-Capacity Curves are manually digitized using the Plot Digitizer.

Step 2: Based on the digitized points, a higher order polynomial is fitted to represent the analytical relationship between lake surface areas and lake depths (with respect of the surveyed water level)

Step 3a: If the polynomial shows a good fit between lake surface areas and depths, then the polynomial equation is used to reconstruct lake surface areas at regular elevation intervals from the (lake bottom to the lake surface, or the surveyed water level). The lake capacities (volumes) at those elevations are then calculated by numerically integrating the reconstructed lake surface areas using trapezoidal rule.

Step 3b: If the polynomial does not show a good fit between lake surface area and depth, then the digitized Area-Capacity Curves are used to linearly interpolate areas/capacities at regular elevation intervals.

3.3 Phase III Methodology

For the lakes where no information of area/capacity is found, however digital bathymetry data is available through AGS, the following methodology is applied to construct Area-Capacity curves:

Step 1: ESRI ArcGIS© is used to collect information on lake bottom elevation from lake bathymetric DEM. It was assumed that the lowest elevation of the bathymetry grid represent the lake bottom.

Step 2: Lake surface areas and volumes at regular elevation intervals (from lake bottom to lake water level) are calculated using the “Surface Volume” tool under 3D Analyst toolset of ESRI ArcGIS. Surface Volume is a Geoprocessing tool that calculates the area and volume of a surface above or below a given reference plane (ESRI 2017). An ArcGIS tool was developed to make the process automated by providing lake bottom elevation, water level, and the elevation increments.

Step 3: Lake surface areas and volumes are plotted against the geodetic elevations to construct Area-Capacity curves.

3.4 Mathematical Extrapolation of Reconstructed Area-Capacity curves

Lake area/capacity calculations are limited by water level information available during the hydrographic survey. Anytime lake level increases from the surveyed lake level, these curves are unable to provide information on area/capacity at a water level above the surveyed water level. Figure 4 shows an example (Baptiste Lake) of such case where the lake level during the hydrographic survey (conducted in January 1979) is 578.52 meter. However, the lake levels fluctuated within 577.84 to 579.20 meter over a period of 38 years (1972-2009). The historical Area-Capacity Curves/Tables is unable to provide a full range of information on lake area and volume for this case.

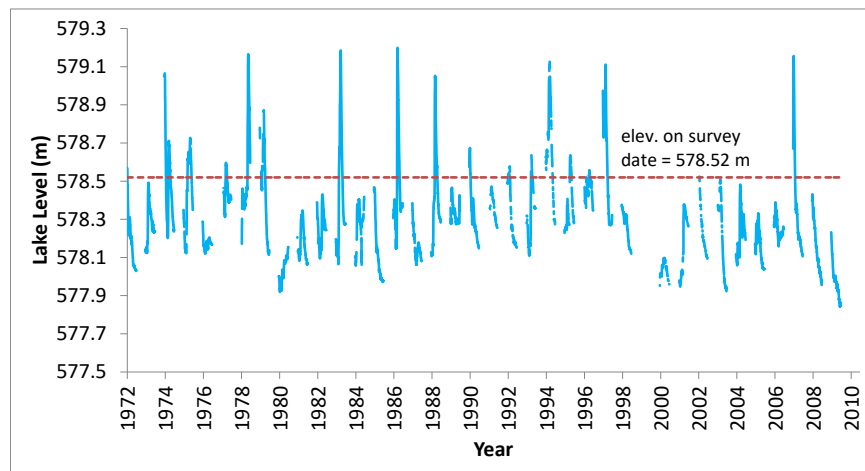


Figure 4: Historical water levels of Baptiste Lake, Alberta. The red horizontal dashed line shows lake level (578.52 meter) during the hydrographic survey conducted in January 1979

Based on the observation of lake levels of about 80 lakes, following assumptions are considered to mathematically extrapolate the reconstructed Area-Capacity curves above the surveyed water level:

- I. Volume-Depth relationship of a lake at the higher elevations (nearer the surface water level portion of a lake) is more realistic to extrapolate for the elevations higher than the surveyed lake level.
- II. The top 1/4th depth of a lake is defined as the “near water level portion” and is chosen the development of mathematical extrapolation.
- III. Area/Capacity curves should not be extended beyond 1.5 meter above the surveyed water level.

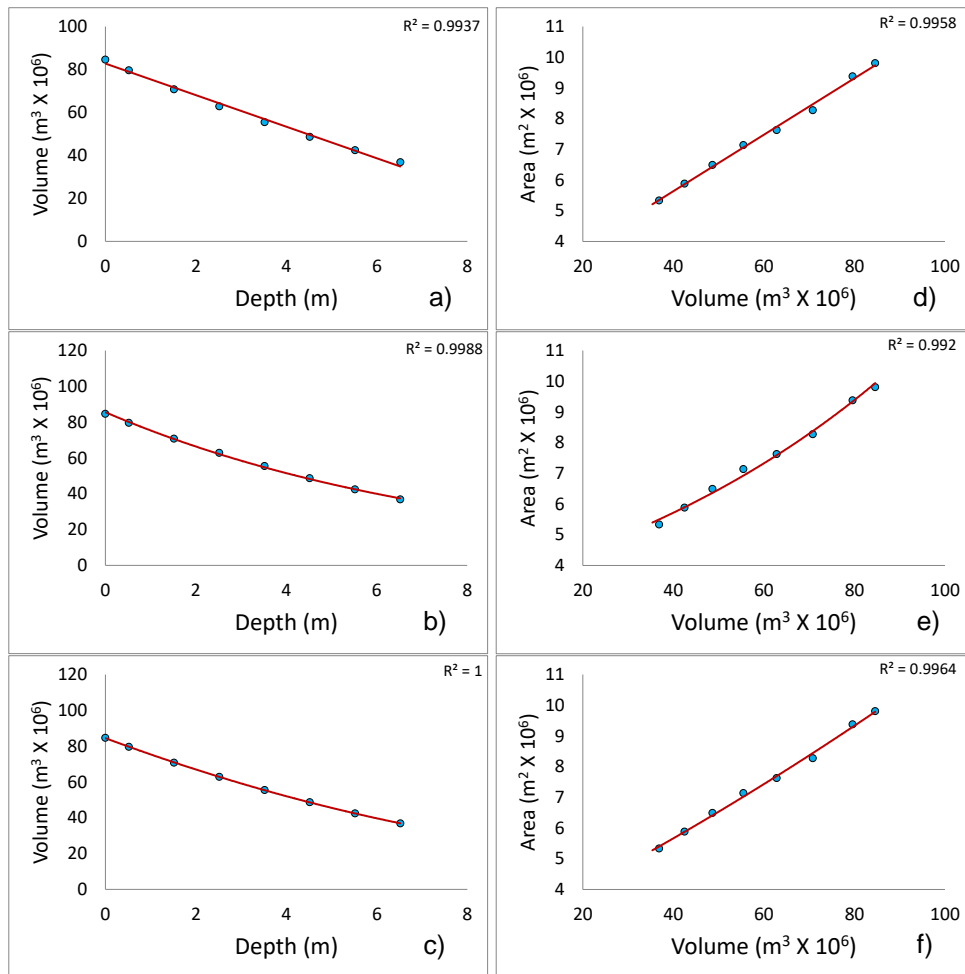


Figure 5: Analytical models for near-surface water level portion Volume-Depth and Area-Volume relationships of Baptiste Lake: Linear (a & d), Exponential (b & e), and 2nd Order Polynomials (c & f)

The step by step methodology is given below:

Step 1: For the near-surface water level portion of a lake (top 1/4th depth) the “Volume-Depth” and the “Area-Volume” analytical relationship are developed. Three types of analytical models are used for each of those two relationships: linear, exponential and 2nd order polynomial. Figure 5 shows different analytical models for these two relationships of the Baptiste Lake. Note, in some cases these analytical models developed for the top 1/4th portion of the lake do not produce a good relationship. Manual adjustment of the “near water level portion” depth is necessary for those cases to develop a representative analytical relationship (See Table 1 for details).

Step 2: The “Volume-Depth” analytical model, which provides the best Coefficient of Determination (R^2), is then chosen to extrapolate lake volumes at regular elevation intervals above the surveyed water level (not exceeding 1.5 meter above the surveyed water level).

Step 3: A Volume Adjustment Factor is calculated by dividing the lake volume at surveyed water level found from the reconstructed Area-Capacity curves by the lake volume at surveyed water level found from the chosen analytical model (**Step 2**). The lake volumes above the surveyed water levels estimated

by the analytical model are then adjusted by multiplying with the Volume Adjustment Factor to avoid any discontinuity.

Step 4: The “Area-Volume” analytical model, which provides the best Coefficient of Determination (R^2), is then chosen to extrapolate the lake surface areas for the extrapolated and adjusted lake volumes from **Step 3**.

Step 5: An Area Adjustment Factor is calculated by dividing the lake surface area at surveyed water level found from the reconstructed Area-Capacity curves by the lake surface area at surveyed water level found from the chosen analytical model (**Step 4**). The lake surface areas above the surveyed water levels estimated by the analytical model are then adjusted by multiplying with the Area Adjustment Factor to avoid any discontinuity.

Note, as most of the reservoirs are surveyed up to their full supply level, no mathematical extrapolation is considered /necessary for reservoirs.

4 RESULTS AND DISCUSSIONS

Appendix A shows a list of Alberta lakes for which the reconstruction process is completed. The table also includes information on the reconstruction method and which portion of the lake considered in extrapolation of Area-Capacity curves above the surveyed water level. This is an ongoing process and preliminary data generated from this project need to be reviewed to ensure the quality of information. Once the Quality Assurance and Quality Control (QA/QC) is performed, the data will potentially be shared with public. Reconstructed Area-Capacity curves for numerous lakes have already been applied for the development of generic shapes of Alberta lakes to support water policy development (Islam and Seneka 2016).

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Appendix A: List of Alberta lakes for which reconstructions of Area-Capacity curves are completed

Name	Reconstruction Phase	Reconstruction Method	Extrapolation Depth ¹
Amisk Lake	Phase I	Polynomial ²	25%
Baptiste Lake	Phase I	Interpolation (Tables) ³	25%
Battle Lake	Phase I	Polynomial	25%
Beauvais Lake	Phase I	Interpolation (Curves) ⁴	19%
Bonnie Lake	Phase I	Interpolation (Curves)	29%
Buffalo Lake	Phase I	Polynomial	31%
Chestermere Lake	Phase I	Polynomial	30%
Coal Lake	Phase I	Interpolation (Curves)	4%
Cold Lake	Phase I	Interpolation (Tables)	25%
Crimson Lake	Phase I	Interpolation (Curves)	5%
Dillberry Lake	Phase I	Polynomial	28%
Ethel Lake	Phase I	Polynomial	20%
Figure Eight Lake	Phase I	Polynomial	6%
Garner Lake	Phase I	Polynomial	25%
Gregoire Lake	Phase I	Polynomial	25%
Halfmoon Lake	Phase I	Polynomial	25%
Hasse Lake	Phase I	Polynomial	25%
Hastings Lake	Phase I	Polynomial	25%
Hubbles Lake	Phase I	Polynomial	25%
Iosegun Lake	Phase I	Polynomial	25%
Island Lake	Phase I	Interpolation (Tables)	25%
Isle Lake	Phase I	Interpolation (Curves)	25%
Kananaskis Lakes	Phase I	Polynomial	25%
Lac La Nonne	Phase I	Polynomial	25%
Lac St. Cyr	Phase I	Interpolation (Tables)	25%
Lac Ste. Anne	Phase I	Polynomial	25%
Lake Eden	Phase I	Polynomial	25%
Lessard Lake	Phase I	Interpolation (Tables)	25%
Lesser Slave Lake	Phase I	Polynomial	25%
Little Fish Lake	Phase I	Interpolation (Curves)	25%
Long Lake (near Athabasca)	Phase I	Polynomial	25%
Mann Lakes	Phase I	Polynomial	25%
Marie Lake	Phase I	Polynomial	25%
McLeod Lake	Phase I	Interpolation (Tables)	18%
Miquelon Lake	Phase I	Polynomial	25%
Moose Lake	Phase I	Interpolation (Tables)	25%
Muriel Lake	Phase I	Polynomial	25%
Nakamun Lake	Phase I	Polynomial	25%
Narrow Lake	Phase I	Interpolation (Curves)	25%
North Buck Lake	Phase I	Interpolation (Curves)	11%
Pine Lake	Phase I	Interpolation (Tables)	25%
Pinehurst Lake	Phase I	Polynomial	25%
Rock Lake	Phase I	Polynomial	25%
Sandy Lake	Phase I	Polynomial	15%
Skeleton Lake	Phase I	Interpolation (Tables)	25%

Name	Reconstruction Phase	Reconstruction Method	Extrapolation Depth ¹
Smoke Lake	Phase I	Interpolation (Tables)	25%
Spring Lake	Phase I	Polynomial	25%
Steele Lake	Phase I	Polynomial	25%
Sylvan Lake	Phase I	Polynomial	25%
Thunder Lake	Phase I	Interpolation (Curves)	25%
Touchwood Lake	Phase I	Polynomial	25%
Twin Lake	Phase I	Polynomial	25%
Utikuma Lake	Phase I	Polynomial	25%
Wizard Lake	Phase I	Polynomial	25%
Wolf Lake	Phase I	Interpolation (Tables)	25%
Beaver Lake	Phase II	Interpolation (Curves)	25%
Blood Indian Creek Reservoir	Phase II	Interpolation (Curves)	N/A
Buck Lake	Phase II	Polynomial	25%
Chain Lakes Reservoir	Phase II	Interpolation (Curves)	N/A
Cooking Lake	Phase II	Polynomial	6%
Crawling Valley Reservoir	Phase II	Interpolation (Curves)	N/A
Elkwater Lake	Phase II	Interpolation (Curves)	7%
Gleniffer Lake	Phase II	Polynomial	N/A
Glenmore Reservoir	Phase II	Interpolation (Curves)	N/A
Gull Lake	Phase II	Interpolation (Curves)	25%
Lac La Biche	Phase II	Polynomial	25%
Little Bow Lake Reservoir	Phase II	Interpolation (Curves)	N/A
McGregor Lake	Phase II	Interpolation (Curves)	N/A
Pigeon Lake	Phase II	Polynomial	25%
Reesor Lake	Phase II	Polynomial	N/A
Sturgeon Lake	Phase II	Interpolation (Curves)	16%
Tucker Lake	Phase II	Polynomial	25%
Wabamun Lake	Phase II	Polynomial	25%
Christina Lake	Phase III	Surface Volume ⁵	N/A
Bear Lake	Phase III	Surface Volume	N/A
Red Deer Lake	Phase III	Surface Volume	N/A
Seibert Lake	Phase III	Surface Volume	N/A
Jackfish Lake	Phase III	Surface Volume	N/A

¹ Top portion of the depth considered in mathematical extrapolation (As most of the reservoirs are surveyed up to their full supply level, no mathematical extrapolation is considered for reservoirs)

² 6th Order Polynomials were fitted

³ Area-Capacity Tables (AEP archived files) is used to linearly interpolate areas/capacities at regular elevation intervals

⁴ Area-Capacity Curves (the Atlas) is used to linearly interpolate areas/capacities at regular elevation intervals

⁵ Surface Volume tool under 3D Analyst toolset of ESRI ArcGIS is used to calculate area/capacity from AGS digital bathymetry