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MESH-FREE TWO-PHASE MODELLING OF HIGHLY-DYNAMIC SEDIMENT TRANSPORT

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Abstract: Conventional numerical methods for the simulation of sediment transport solve single-phase hydrodynamics and advection-diffusion equations in combination with the empirical relations. These simplified methods are not capable of dealing with the highly-dynamic sediment movements which often induced by highly-erosive flows (e.g., floods) or powerful outer forces (e.g., landslides). Two-phase methods have recently proven their potential for such sediment flow problems. Among them the multiphase mesh-free particle methods provide a unique opportunity for the simulation of large interfacial deformations and fragmentations involved in two-phase sediment-water system. This article briefly overviews example applications of a multiphase mesh-free particle model for two-phase flow of water and sediments in different configurations. The model treats the sediment material as a continuum (a visco-plastic fluid) whose behaviour is predicted using a stress-dependent rheological model. The model is validated and applied for highly-dynamic movement of sediments in cases such as submarine landslide and sediment erosion. The results of this study, evaluate the capabilities of mesh-free particle methods for complex sediment transport problems and provide a more thorough understanding of their complex mechanism and processes.

1 INTRODUCTION

Sediment transport has a crucial role in many natural and engineering processes such as fluvial/coastal morpho-dynamics and ecosystem. The foundations of contemporary sediment research can be found in many experimental and field studies, while with the advancement in computing powers, the numerical simulations have recently gained much attention. Traditional approaches for simulation of sediment transport, have mostly been based on the single-phase modelling, where the flow equations are coupled with advection-diffusion equation and (semi-) empirical relations (such as Meyer-Peter and Müller, 1948; Van Rijn, 1984). These methods, which are the base of most of the existing models, have proved their effectiveness in some of past researches. However, they are not suitable for highly-dynamic movement of sediment material as they are not able to describe the underlying physics in detail. Such conditions are commonly initiated by highly-erosive flows or under powerful outer forces (e.g., gravity). Sediment transport during flood, dam-break, ice-break, river bank failure, and landslide are few examples of the highly-dynamic sediment movements.

Two-phase models, where the sediment and water act as separate phases, have proven to be capable of these sediment transport simulations. Two-phase models are either use continuum-based mesh-based Eulerian methods such as Finite Element or Finite Volume, or discrete-based Lagrangian methods such as Discrete Element Method, DEM. The first class of methods, has problems in dealing with flows with large interfacial deformations (Shakibaeinia and Jin, 2011). The second class of methods are computationally expensive for real-case large-scale problems as they simulate each individual sediment grain. The mesh-free particle (Lagrangian) methods developed for continuum mechanics such as Smoothed Particle Hydrodynamics, SPH (Gingold and Monaghan, 1977) and Moving Particle Semi-implicit method, MPS (Koshizuka and Oka, 1996) have the advantage of both classes of methods. Therefore, they provide an opportunity for dealing the complexities involved in highly-dynamic sediment transport problems.

The present paper will overview the development and implementing multiphase mesh-free model for the simulation sediment transport problems in authors' recent works. The method treats the multiphase system as a multi-viscosity and multi-density continuum. An exponentially-regularized visco-plastic rheological model with the Mohr-Coulomb yield criteria is applied to predict the non-Newtonian behavior of the sediment phase. The model is applied and evaluated for the simulation of one flow-driven and one gravity-driven sediment dynamic test cases.

2 NUMERICAL MODEL

1.1 Flow Governing equations

Considering the multiphase system of water and sediments as a multi-density multi-viscosity system, a single set of flow governing equations will describe both phases. The flow governing equations for a weakly-compressible flow system, including mass and momentum conservation and the motion in Lagrangian coordinates are as follows:

[1] $$\begin{cases} \dfrac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\nabla\cdot\mathbf{u} = 0 & \text{Mass conservation} \\ \dfrac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\dfrac{1}{\rho}\nabla p + \dfrac{1}{\rho}\nabla\cdot\boldsymbol{\tau} + \mathbf{f} = 0 & \text{Momentum conservation} \\ \dfrac{\mathrm{D}\mathbf{x}_p}{\mathrm{D}t} = \mathbf{u} & \text{Lagrangian motion} \end{cases}$$

where, **u:**(u,v) is velocity vector, t is time, ρ is density, p is pressure, **f** represents body forces, **x**:(x,y) is position vector and $\boldsymbol{\tau}$ is shear stress tensor.

1.2 Sediment rheology

The sediment martial is treated as a visco-plastic continuum (Figure 1). Here the $\mu(I)$ rheological model proposed by GdR MiDi (2004) and Jop et al. (2006) is used to describe the non-Newtonian behaviour of sediment continuum. In this model the shear stress tensor is related to the inertial number, I, and the normal stress (pressure) between sediment grains as:

[2] $$\boldsymbol{\tau} = \frac{\mu(I)p}{\|\mathbf{E}\|}\mathbf{E} \quad \textit{where} \quad \mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1}; \quad I = \frac{\|\mathbf{E}\| d_s}{\sqrt{p/\rho_s}}; \quad \mathbf{E} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right) \;; \quad \|\mathbf{E}\| = \sqrt{II_E} = \sqrt{\frac{1}{2}\mathbf{E}:\mathbf{E}}$$

where, **E** is strain rate tensor, **||E||** and II_E are its magnitude and second invariant, respectively, and μ_s and μ_2 are first and second friction coefficients (i.e. tanϕ, ϕ being friction angle).

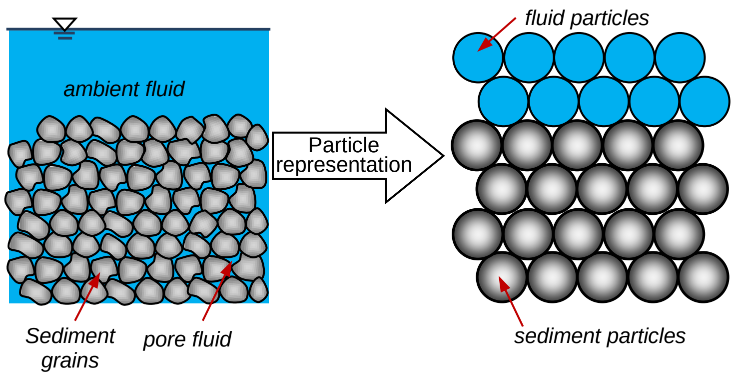

Figure 1:Particle representation of multiphase sediment continuum

1.3 MPS method

Here a weakly-compressible MPS method (Shakibaeinia and Jin 2010) is used for solving the governing equations. In MPS method, the continuum is represented by a set of mobile particles. Each particle possesses a set of field variables (e.g., velocity and pressure). MPS is based on a local weighted averaging of quantities, vectors and derivatives. The target particle *i* with position vector $\mathbf{x}_i$, interacts with particle *j* in its vicinity using a weight function, $W(x_{ij}, r_e)$, with $x_{ij}=|\mathbf{x}_j-\mathbf{x}_i|$ being the distance between particle *i* and *j*. Particles located within distance r_e from the target particle interact with that particle. A dimensionless parameter, the particle number density, *n,* represents the density of particles around a certain particle as:

[3] $$\langle n \rangle_i = \sum_{j \neq i} W\left(x_{ij}, r_e\right)$$

The pressure gradient term (Shakibaeinia and Jin 2010) and the divergence-free viscous term for multiphase flow (Shakibaeinia and Jin, 2011a, 2012) are given by:

[4] $$\langle \nabla p \rangle_i = \frac{d}{n_0} \sum_{j \neq i} \left(\frac{p_j - \hat{p}_i}{x_{ij}} \mathbf{e}_{ij} W\left(x_{ij}, r_e\right) \right); \quad \hat{p}_i = \min_{j \in J}(p_i, p_j) \; ; \; J = \left\{ j : W\left(x_{ij}, r_e\right) > 0 \right\}$$

[5] $$\langle \nabla \left(\mu \nabla \cdot \mathbf{u} \right) \rangle_i = \frac{4d}{\lambda n_0} \sum_{j \neq i} \left(\frac{\mu_i \mu_j}{\mu_i + \mu_j} \left(\mathbf{u}_j - \mathbf{u}_i \right) W\left(r_{ij}, r_e\right) \right)$$

The MPS approximation of the strain rate tensor, **E**, can be written as:

[6] $$\mathbf{E}_i = \begin{bmatrix} \dot{e}_{xx} & \dot{e}_{xy} \\ \dot{e}_{yx} & \dot{e}_{yy} \end{bmatrix}_i = \frac{1}{n_0} \begin{bmatrix} 2\sum_{j \neq i} \left(\frac{u_{ij} x_{ij}}{r_{ij}^2} W_{ij} \right) & \sum_{j \neq i} \left(\frac{u_{ij} y_{ij} + v_{ij} x_{ij}}{r_{ij}^2} W_{ij} \right) \\ \sum_{j \neq i} \left(\frac{u_{ij} y_{ij} + v_{ij} x_{ij}}{r_{ij}^2} W_{ij} \right) & 2\sum_{j \neq i} \left(\frac{v_{ij} y_{ij}}{r_{ij}^2} W_{ij} \right) \end{bmatrix}$$

where n_0 is the initial value of particle number density, *d* is the number of space dimensions and $\mathbf{e}_{ij} = \mathbf{x}_{ij}/x_{ij}$ is the unit direction vector. The viscous term, considers the multiphase force due to the discontinuity in the viscosity field. A fractional-step time integration methods is employed, where the velocity in a new time step is the sum of the predicted velocity and corrected velocity.

[7] $$\mathbf{u}_i^{k+1} = \mathbf{u}_i^{*} + \mathbf{u}_i' \quad \textit{where} \quad \mathbf{u}_i^{*} = \mathbf{u}_i^{k} + \frac{\Delta t}{\rho_i}\left(\mathbf{f}_i + \mathbf{f}_{si} + \mu_{ij} \nabla^2 \mathbf{u}_i^{k} - (1-\alpha) \nabla p_i^{k} \right) \; ; \; \mathbf{u}' = -\frac{\alpha \Delta t}{\rho_i} \nabla p_i^{k+1}$$

where α is a relaxation factor varying between 0 and 1. Particles are moved based on their predicted velocity and therefore a new particle number density, n^*, is calculated. The n^* used to calculate the pressure value. The pressure gradient term is implicitly deployed to calculate the velocity correction. As in the WC-MPS (Shakibaeinia and Jin, 2010) the fluids are assumed to be weakly compressible and the Tait's equation of state is used for prediction of the pressure, as:

[8] $$p_i^{n+1} = \frac{\rho_i c_0^2}{\gamma}\left(\left(\langle n^* \rangle_i / n_0\right)^{\gamma} - 1\right)$$

where c_0 is the numerical sound speed. The typical value of γ=7 is used in this relation. Since an explicit algorithm is used, the time intervals in each time step are selected in the way to satisfy the stability condition. To avoid the sudden pressure, jump across the interfaces created by the large density discontinuity, Shakibaeinia and Jin (2012) used the smoothed value of density for the pressure calculation. To deal with the large Reynolds number flow, the model uses the Sub-Particle-Scale (SPS) LES model. The inflow/outflow and solid boundaries are treated by the method proposed by Shakibaeinia and Jin (2010, 2011b).

3 TEST CASES

Model of this study can be applied to any multiphase sediment transport problem. Here we consider two benchmarks. The first benchmark mimics the submarine landslide or river bank failure. The second benchmark is study of sediment erosion resulted from a water jet. Figure 2 shows the schematic of the first benchmark. It is based on the experiments done by Rzadkiewicz et al (1997) on slide of a wedge of sand on a 45 degrees inclined surface under water. The computational domain is represented by particle with size of 0.015 m. Figure 3 show the snapshots of the numerical results in different times. In this figure two color sediment particles have been used to study the internal deformation of sediments. At t=0 s, the sediments that are bounded by a gate are released and start to slide down the surface. This creates a water wave that propagates downstream. Going down the slope, the wage of sediments stretches along the slope under friction with the solid surface. Eventually the sediments form a pile down the slope. The numerical results show the ability of the model in representing sediment and water surface deformations. Figure 4 compares (for an example time) the surface profile and sediment profile resulted from the model of this paper with those from experimental and numerical work of Rzadkiewicz et al (1997). It shows better compatibility of present model results with the experiments (bed profile Err.=0.041m) comparing the model of Rzadkiewicz et al (1997) (Err.=0.053m).

In the second benchmark the sediment erosion resulted from an un-submerged jet is studied. The experimental data are taken from Pagliara et al. (2006). Figure 5 shows the geometry of the problem and snapshots of the numerical results. The jets' momentum (and the resulting circulations) induces the erosion of the sediments and deposits them further downstream. This creats an scour hole, which developes over time. Figure 6 compares the numerical and exerimental bed profile and the evolution of the scour hole depth. The comparosons show a good compatibility.

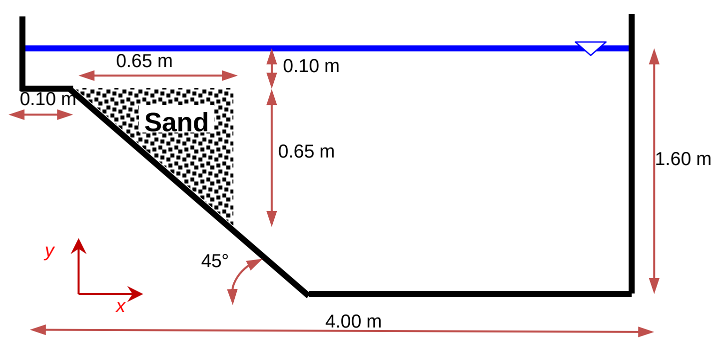

Figure 2: schematic of submarine landslide test case

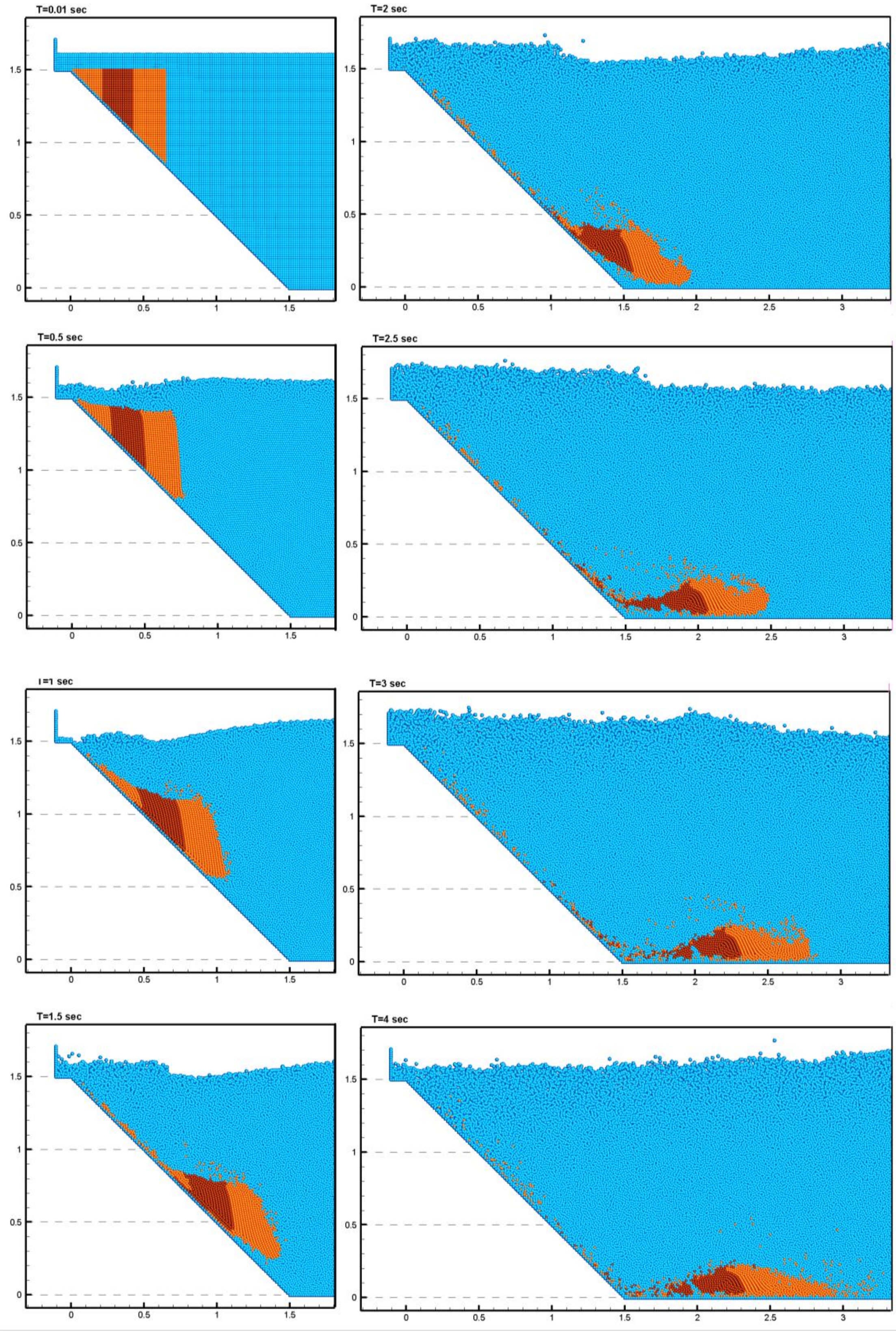

Figure 3:Time evolution of numerical submarine landslide

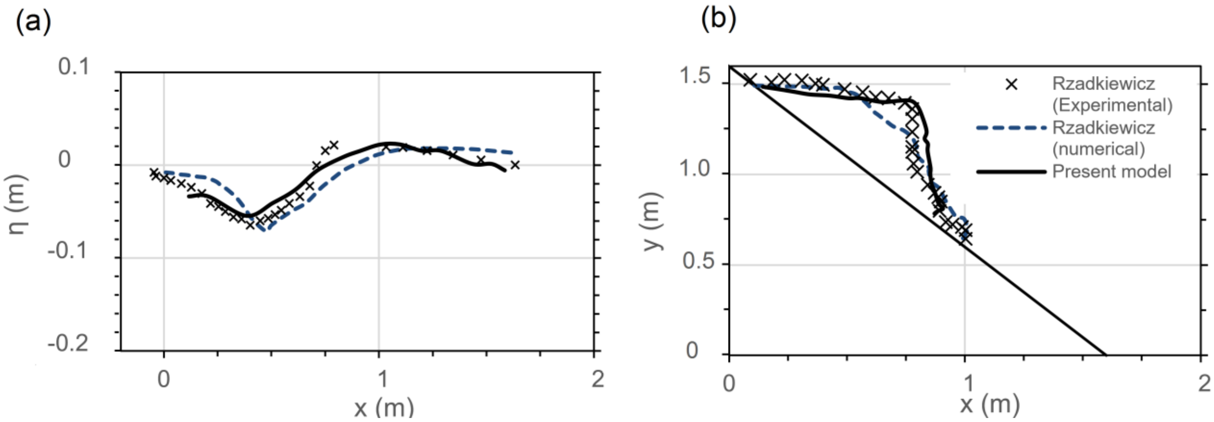

Figure 4: comparison of experimental and numerical (a) surface wave profile and (b) sediment profile at t=0.4 s.

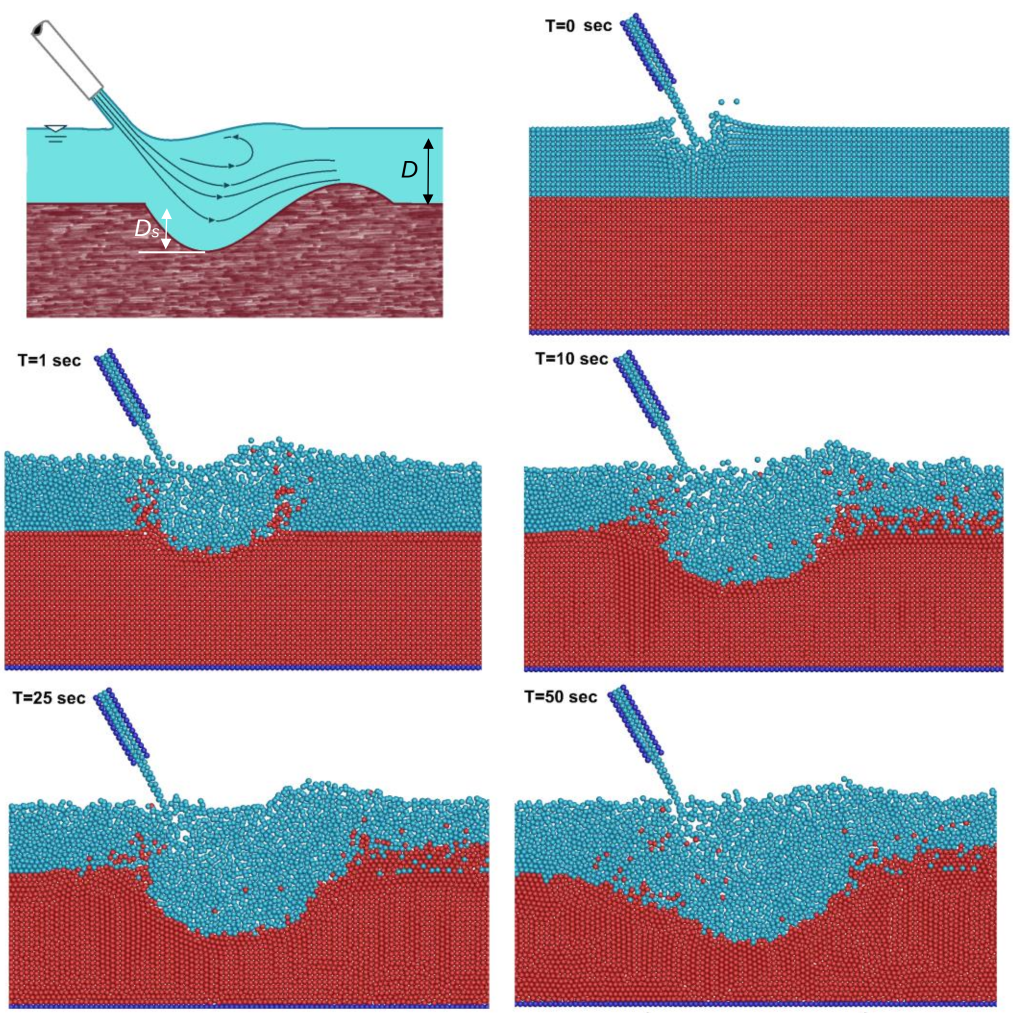

Figure 5:Geometry and time sequence of numerical results for the jet-erosion test case.

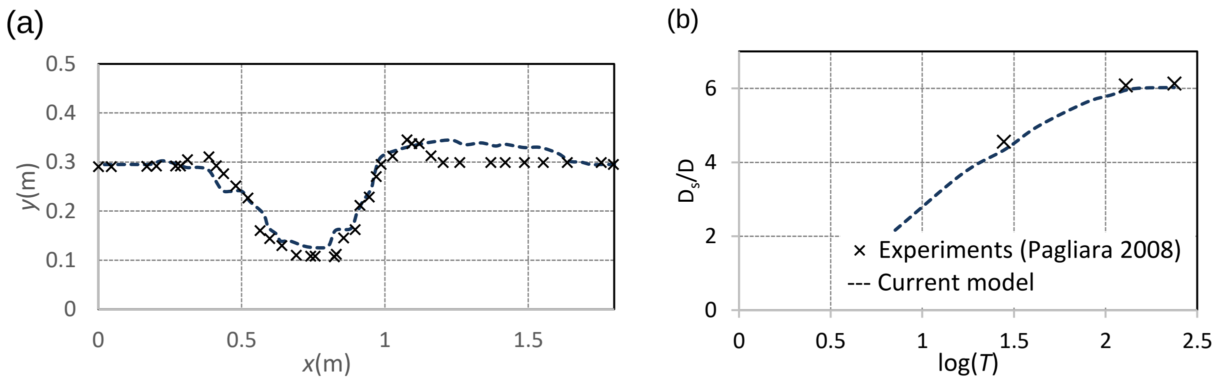

Figure 6: (a) final bed profile, and (b) time evolution of the maximum scour depth (dimensionless)

4 CONCLUSION

A multiphase mesh-free particle model base on the MPS mesh-free particle method is evaluated for two-phase flow of water and sediments in two example test cases. The sediment material is treated as a continuum (a visco-plastic fluid) whose behaviour is predicted using $\mu(I)$ rheological model. The model is validated and applied for highly dynamic movement submerged sediments in various regimes in cases such under-water landslide and erosion (resulted from a water jet). The results of this study, proved the capabilities of mesh-free particle methods for accurate simulation of the complex sediment transport problems for the test cases of this study. The model of this study can be applied to the other complex sediment dynamics problems such as sediment slump, mobile-bed dam-break, and sediment scouring.

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