Leadership in Sustainable Infrastructure

Leadership en Infrastructures Durables

Vancouver, Canada

May 31 – June 3, 2017/ *Mai 31 – Juin 3, 2017*

EXPERIENCES IN JET TESTING OF COHESIVE SOILS

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Abstract: This paper discusses the issues encountered over many years of soil erodibility testing using the jet erodibility test, which has been used in Canada and internationally. The test uses a submerged circular turbulent impinging jet, set at a large impingement height, to create scour in a soil sample and the time development of the depth of scour along the jet centerline is measured. The scouring rate and estimation of the equilibrium depth of scour then can be related respectively to the erodibility coefficient of the soil, which is a coefficient used in the relationship between erosion rate and excess shear stress on the soil, and the soil's critical shear stress. There have been issues with the diameter and depth of the test sample, fissures and rocks in the samples, vegetation, setting the appropriate jet velocity, and ultimately the underlying theory of the test. For the test theory, it has been seen that original analysis of the test significantly underestimates the equilibrium depth of scour and therefore under predicts the critical shear stress of the soil. Further, the analysis procedures assumed a linear relationship between the erosion rates of the soil and the excess shear stress on the soil, which is not the optimum model for this relationship. It also appears that the shape of the scour hole can impact results; however, the standard method of analysis of the test does not consider effects of the scour hole shape.

1 INTRODUCTION

The estimation of potential soil erosion by water flows is important to a number of problems: (i) design of hydraulic structures such as bridges, dams, weirs, drops, and culverts; (ii) determination of the river bank erosion in assessment of needs for bank protection; (iii) estimation of soil losses from agricultural fields and the associated pollutant transport into rivers; (iv) assessment of the stability of mine waste covers; (v) determination of potential shoreline erosion in reservoirs at dams; (vi) assessment of the potential for sediment mobilization in streams; and (vii) in the assessment of the stability of canals and irrigation channels. For "cohesionless soils" such as clean, coarse sands, there are reasonably reliable procedures for determining a soil's erosion resistance based on its characteristics, such as its mean particle size, gradation, density, shape, packing (Raudkivi 1998). However, there is currently no widely accepted method for determining the critical shear stress and erosion rates for clay-rich or "cohesive soils". Equations for estimating the erosion resistance of cohesive soils have been difficult to develop. This is partly because of the wide number of parameters that can affect the erosion resistance of a cohesive soil: soil density; clay content; clay mineralogy; pore and eroding fluid chemistry; temperature; fabric; water content; organic content; and matric suction (Paaswell 1973; Hanson and Cook 1998). In natural soils, there are also the additional complications of biological activity and vegetation (Black *et al.* 2002). As such, many have chosen to instead measure the erosion resistance of the material of interest in some way.

There exists a wide variety of apparatuses and testing methods for determining a cohesive soil's erodibility, yet none of them are currently accepted as a standard. Testing has been conducted in open flumes of various sizes (Enger 1963; Hanson 1990; Haralampides and Rodriguez 2006); in closed flumes such as the Erosion Function Apparatus (Briaud *et al.* 2001); within concentric rotating cylinders (Lim and Khalili 2009); by drilling a hole in the sample and treating the flow as a pipe flow as in the Hole Erosion and Slot Erosion tests (Wan and Fell 2002; Luthi *et al.* 2011); and by using a submerged circular turbulent impinging jet (Hollick 1976; Hanson 1991; Hanson and Cook 2004; Clark and Wynn 2007; Thoman and Niezgoda 2008; Mazurek 2010; Simon *et al.* 2010). The Jet Erodibility Test methodology is defined in ASTM Standard D5852 (ASTM Standard D5852, 2007) and Hanson and Cook (2004). The advantages of this test over other techniques has been that it can be used both in-situ and in the laboratory, with different scales of apparatuses, using both large and relatively smaller soil samples, and for a broad range of soil erosional strengths.

The Jet Erodibility Test was originally developed by Greg Hanson and his colleagues at the United States Agricultural Research Station in Stillwater, Oklahoma, based on early work with jet testing discussed in Hanson (1991). The earlier analysis procedures of the test given in Hanson (1991) were later modified to adapt the work of Stein *et al.* (1993), who studied the time development of scour in soils by obliquely impinging plane jets, to scour by circular impinging jets (Hanson *et al.* 1998; Hanson and Cook 2004). The resulting analysis then produced two important parameters for describing soil erodibility: the critical shear stress of the soil and its erodibility coefficient. The critical shear stress is the shear stress created by flow on the soil's surface at which the soil is first observed to erode. The erodibility coefficient is the ratio of the erosion rate of the soil to the excess shear stress on the bed.

Over many years of experience with Jet Erodibility Test (JET) and in analyzing the scour produced by vertical, circular turbulent jets, the authors have encountered issues with both conducting the test and with its underlying theory. These issues are discussed herein.

2 CONDUCTING A JET ERODIBILITY TEST

2.1 Standard Jet Erodibility Test Procedures

In the Jet Erodibility Test, the soil sample and jet are submerged in water. The jet is created by flow through a plenum and then through a nozzle. The jet of diameter, d, is set in such a way that it impinges at 90˚ to the soil surface at a height, H, above the sample. Figure 1 gives a definition sketch. In the original standard, which is summarized in ASTM Standard D5852(2007), a 13 mm diameter nozzle is suspended 0.22 m above the sample surface. Water is pumped into a constant head tank to feed the nozzle, which produces the jet. The recommended head on the jet is 0.91 m though other heads could be used depending on the soil conditions. A test timing sequence of 10, 20, 30, and 60 minutes is used, with a total test duration of 2 h. At each time, the jet flow is stopped and the profile of the scoured soil bed is taken using a pin profiler. Alternatively, a point gauge may be lowered through the nozzle to facilitate measuring the centreline scour depth, ε_{cl}, instead of measuring the profile of the bed.

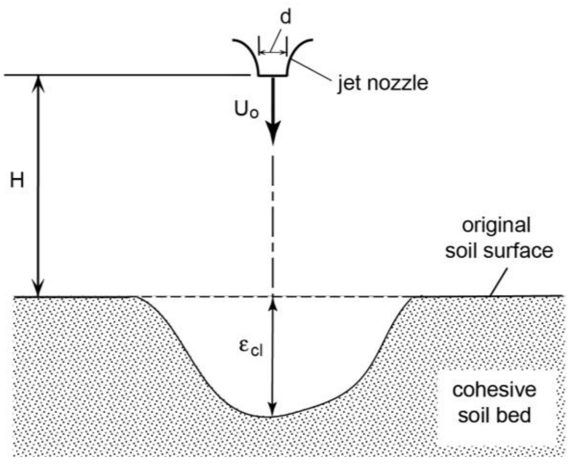

Figure 1: Definition of jet test variables

In further developing the jet erodibility test procedures, Hanson and Cook (2004) noted that the jet could be set at a height above the sample from 6 to 35 nozzle diameters, with a recommended initial height equal to 12 diameters, and a jet diameter of 6.4 mm. The head of water above the nozzle could be adjusted depending on the soil conditions. Measurements of the scour depth at the jet centreline should be taken at intervals of 5 to 10 minutes, with a recommendation of taking at least 10 to 12 readings.

2.2 Experimental Setup and Experiments

The results from two sets of experiments in the scour of cohesive soils by submerged vertical, circular, turbulent impinging jets are discussed herein. In the first set of tests, the maximum and centreline scour depths were measured with time until scour hole was deemed to have reached equilibrium. The entire scour hole was then measured. In the second set of experiments, the entire scour hole was additionally measured at intermediate times of scouring until equilibrium was reached. For all these tests, the samples were set on a table within a 1.2 m deep, 1.1 m wide octagonal tank that was filled with water. A circular jet was created by flow through a 0.95 m long plenum, with internal flow straighteners, then a nozzle, which was designed so that that the flow contraction occurred smoothly without any significant head loss and the velocity profile across the nozzle was uniform; both the plenum and nozzle were designed following the standards laid out in American Society of Mechanical Engineers (1990). To create the jet, City of Saskatoon tap water was pumped from a constant tank through a magnetic flow meter and then finally to the top of the jet plenum. The plenum was suspended from a steel frame on hinges so that it could be moved to the side of the jet tank during scour measurements. The water in the tank overspilled the top of the tank around all its edges so that a constant submergence of the jet was maintained. The apparatus is shown in Figure 2.

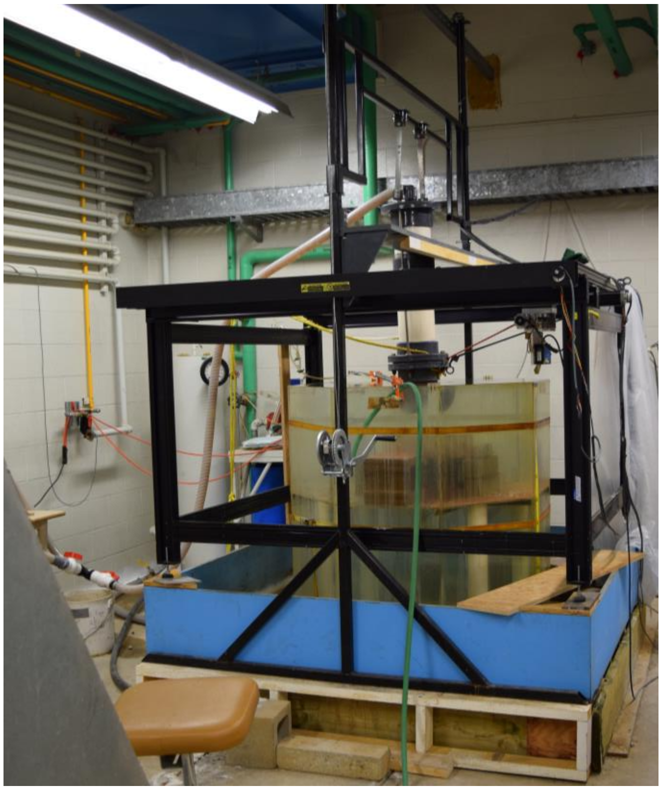

Figure 2: Experimental apparatus

For measurements of the entire scour profile, an optoNCDT 1700-750 Laser Triangulation Displacement Sensor was mounted above the jet tank on a two-dimensional computer-controlled motorized traverse system that could move up to 1.2 m in each direction of the horizontal plane. The scour profile measurements could be taken automatically by setting the limits of the measurements. The system was controlled using LabView. Measurements were taken on a 2 mm by 2 mm grid and the exact position of the laser was tracked using encoders. Details of the measurements are respectively given in Cossette (2016) and Amin (2016).

In the first set of experiments, each soil sample was contained within a 152 mm diameter stainless steel cylinder. Jet testing was conducted on 14 natural samples collected from rivers in Ontario, 4 samples from a field in Saskatoon, Saskatchewan, and 10 samples of manufactured pottery clays. Details of the samples properties are given in Cossette (2016). A nozzle diameter of 8 mm was used, with the impingement height ranging from 59-123 mm and a velocity at the nozzle, U_o, in the range of 1.1-14.8 m/s. In the natural samples, measurements of the centerline scour depth and maximum scour depth were taken at times of 5 min, 10 min, 20 min, 30 min, 60 min, 2 h, 4 h, and 8 h from the beginning of testing. After this, measurements were taken at about 24-48 h intervals until the scour hole was deemed to have reached an equilibrium state. For the pottery clays, the testing sequence was modified to include more measurements early in a test, with measurements at 5 min, 10 min, 15 min, 20 min, 30 min, 40 min, 60 min, 1.5 h, 2.5 h, 4.5 h, and 8.5 h from the beginning of testing. After this, measurements were taken at about 24-48 h intervals until the scour holes were deemed to have reached equilibrium. For the decision about whether a scour hole had reached equilibrium, the scour depth data was plotted with time and graphical assessment of whether the scour hole was continuing to grow was made (see the discussion on this type of decision in Amin (2016)). The time to equilibrium ranged from 42 to 759 h, with most tests taking about 240 h. After equilibrium was reached and the test was stopped, the entire profile of the scour hole was measured using the laser displacement meter.

In the second set of experiments, using the laser displacement meter, the entire scour hole profile was measured at times of 5 min, 10 min, 15 min, 20 min, 30 min, 40 min, 50 min, 1 h, 1.5 h, 2 h, 4 h, 8 h, 16 h, 24 h, and then after every 24 h thereafter until the equilibrium condition was achieved. During the test, the eroded volume was plotted against the logarithm of time to assess whether the resulting curve started to become parallel to the time axis, thus indicating that a state of equilibrium had been reached (the graphical approach used by Amin (2016)). For this testing, 10 samples of pottery clays were used. The jet impingement height was 85 mm, the jet diameter was 7.76 mm, and the jet velocity at the nozzle ranged from 4.4-11.4 m/s. Further details of these experiments are given in Amin (2016).

It should be noted that the underlying analysis of the jet test does not restrict the conditions to those used in the ASTM standard or Hanson and Cook (2004).

3 JET ERODIBILITY TEST ANALYSIS TO OBTAIN THE SOIL ERODIBILITY PARAMETERS

3.1 Determining the Critical Shear Stress of the Soil

Following Hanson and Cook (2004), to estimate the critical shear stress of a soil from JET data, first, the shear stress created by the flow on the soil surface must first be estimated. For this work, the variation of the velocity with distance from the nozzle is considered. At the nozzle, it is assumed that the velocity profile is uniform and equal to U_o. A shear layer forms at interface between the edge of the jet and the surrounding fluid as the jet exits the nozzle. At a certain distance from the nozzle, x_p, the shear layer penetrates through the entire diameter of the jet. The distance x_p is the length of the potential core of the jet. If the distance from the nozzle along the jet centreline, x, is greater than the length of the potential core (*i.e.* $x > x_p$), the maximum velocity of the jet varies with distance from the nozzle as (Rajaratnam 1976):

[1]
$$\frac{u_m}{U_o} = C_d\left(\frac{d}{x}\right)$$

where u_m is the maximum velocity at a distance x from the nozzle and C_d is a diffusion coefficient, which has a value of $C_d \approx 6.3$ (Rajaratnam 1976). This therefore assumes that the velocity of the jet is decreasing with distance from the nozzle in a similar way to a free circular jet (*i.e.,* one that is not impinging on a boundary or is affected by a boundary). It is known that this relationship (Eq. 1) remains valid to about 86

% of the distance from the nozzle to the flat, rigid boundary for an impinging jet (Beltaos and Rajaratnam 1974).

What one would estimate as the velocity of the jet at the boundary using Eq. 1 is converted to an estimate of the maximum shear stress on the bed, τ_{om}, using the local skin friction coefficient, c_f:

[2] $$\tau_{om} = c_f \rho u_m^2$$

where ρ is the eroding fluid density. The value of $c_f = 0.00416$, which was determined by Hanson *et al.* (1990) in a study of the shear stresses created by a circular impinging jet along a smooth, flat boundary. This value of c_f compared favourably with $c_f = 0.00403$ found by Beltaos and Rajaratnam (1974), who did not assume free jet behavior in developing their estimate but did assume that the jet is at a large impingement height (H/d>8.3) (Beltaos and Rajaratnam 1977). Use of $c_f = 0.00416$ assumes that the soil bed is smooth, non-porous, and rigid. It is known, however, that c_f for an impinging jet depends on the roughness of the bed (it is larger for a rough bed) (Rajaratnam and Mazurek 2005).

For a bed that has scoured to a depth of ε_{cl}, which is the depth of scour at the centreline relative to the original, unscoured soil surface, with an original height of the jet above the soil bed of H, the total distance from the jet to the scoured-out bed is $H+\varepsilon_{cl}$ (see Figure 1). The maximum shear stress on the bed at this depth is then:

[3] $$\tau_{om} = c_f \rho \left[C_d U_o \frac{d}{H+\varepsilon_{cl}} \right]^2$$

When the scour depth reaches equilibrium, or $\varepsilon_{cl} = \varepsilon_{cl\infty}$, it is assumed that the maximum shear stress on the bed, τ_{om}, is equal to the critical shear of the soil, τ_c, or:

[4] $$\tau_c = \tau_{om} = c_f \rho \left[C_d U_o \frac{d}{H+\varepsilon_{cl\infty}} \right]^2$$

Hanson and Cook (2004) did not run their tests until the scour hole reached an equilibrium depth, since doing so can take several hundred hours and this is impractical. Instead, the equilibrium scour depth is estimated by the curve-fitting approached developed by Blaisdell *et al.* (1981). A hyperbolic equation is fitted to the scour data from the jet test to estimate the scour depth at very long times, which is taken to be the equilibrium depth. Blaisdell *et al.* (1981) provide a step-by-step methodology for this technique, which is also described in Hanson and Cook (2004) and Cossette (2016).

3.2 Determining the Erodibility Coefficient of the Soil

The erodibility coefficient of the soil comes from the relationship of the soil erosion rate of the soil, $\dot{E}$, to the excess shear stress on the bed:

[5] $$\dot{E} = K(\tau_{om} - \tau_c)^n$$

where K is the erodibility coefficient and n is a power. To determine K, the data for time development of the centerline scour depth are used. The method of analysis was developed by Hanson and Cook (1997). It adapts the work of Stein *et al.* (1993) (see also Stein (1990)) for the time development of scour by obliquely impinging plane wall for the vertical circular jets used in the jet erodibility test.

First, the erosion rate of the soil is taken as the rate of change of the scour depth along the jet centerline, or:

[6] $$\dot{E} = \frac{d(H + \varepsilon_{cl})}{dt} = \frac{dH_t}{dt}$$

where t is the time from the start of scouring and $H_t = H + \varepsilon_{cl}$. Next, for cohesive soils, it has been assumed that n=1 for the jet erodibility test analysis, although Stein *et al.* (1993) did not require this assumption. They only noted that for the cohesive soil tested n=1 gave the best fit to the time development of scour data. Hanson and Cook (1997) showed that the relationship between the time of scouring measured from the start of a test and the scour depth can be expressed as

[7] $$t = T_r \left(0.5 \ln\left(\frac{1 + H_*}{1 - H_*} \right) - H_* - 0.5 \ln\left(\frac{1 + H_{i*}}{1 - H_{i*}} \right) - H_{i*} \right)$$

where T_r is a reference time given as $T_r = H_{t\infty}/(K\tau_c)$; $H_{t\infty}$ is value of H_t when scour has reached equilibrium (or $H_{t\infty} = H + \varepsilon_{cl\infty}$); H_* is the dimensionless scour depth, defined as $H_* = H_t/H_{t\infty}$; and H_{i*} is the initial height of the soil surface expressed in dimensionless form, where $H_{i*} = H/H_{t\infty}$.

To determine K, the critical shear stress was first determined by the method described above. Then, measurements for the time from the start of the test and centerline scour depth measured at that time, expressed as H_*, were plotted. Equation 7 is then fit to the data by minimizing the least square errors to find T_r using the Solver function in Excel. The K value is then found from T_r.

4 ISSUES WITH CONDUCTING A JET ERODIBILITY TEST

One of the main issues with conducting a jet erodibility test in its standard form is that it can be difficult to know what velocity to set the jet *a priori*. The test is conducted to determine the erodibility of the sediment so the erodibility of the material is therefore unknown. When using a sample taken from the field, there will be a limited diameter of the sample. It is very easy then to accidentally “blow out” the sample by setting the jet velocity too high. One of the ways the authors have avoided this is to make a visual assessment of the velocity to initiate erosion at the start of the test. At the beginning of the test, the flow rate is slowly stepped up every 5 minutes in small increments to visually assess the point of erosion. Cohesive soils tend to erode in chunks so that the observer is looking for the first chunk removal. From the flow conditions at this critical flow, either one can estimate a critical shear stress from Eq. 4 then use the formulas for estimating the equilibrium size of the scour hole presented in Mazurek *et al.* (2001) to assess whether the scour hole would fit within the sample container or simply multiply the flow by 1.2-1.3 times. Samples that scour to the edge of the container tend to be asymmetric and this means that the assumptions for the jet decay of velocity given in Eq. 1 are questionable in their application.

Cohesive soils tend to erode in chunks and the size of the chunks are small (on the order of a few millimeters in dimension) to several centimeters in dimension. When a large chunk is eroded, the scour depth tends to remain constant with time for several measurements. This can impact the quality of curve fitting for the time development of scour data. In that case, it may be necessary to cut the sample in some way and repeat the test if the enough of the sample is available for testing. Similar issues often occur with the time development of scour of a sample when the scouring ceases due to the presence of a rock or roots in natural samples. Although measurements of jet erodibility have been made for samples with roots used previously (Clark and Wynn 2007), there is some question of whether the scour test is representative for the entire sample, as the roots introduce heterogeneity in the sample.

5 ISSUES WITH ANALYZING A JET ERODIBILITY TEST

5.1 Estimates of the Equilibrium Depth of Scour

The most important quantity to determine from the jet erodibility test is the critical shear stress of the soil, which, as noted, is done by determining the equilibrium depth of scour and calculating the critical shear stress using Eq. 4. Determining the equilibrium scour depth is normally done using the curve fitting approach of Blaisdell *et al.* (1981), however, for the experiments described herein the scour experiments were conducted until the sample reached equilibrium. For the first set of experiments described herein, the equilibrium scour depth was estimated using Blaisdell *et al.* (1981) and then compared to the measured values.

In estimating the equilibrium scour depth using the Blaisdell et al. (1981) approach, it was found that the number of data points used in the curve-fitting procedure highly impacted results. For the soil sample called BS(1) (see Cossette (2016) for details of its characteristics), using all data points from the first 120 minutes of testing (8 points) gave a value for the equilibrium depth of scour 274 mm. However, using the first 10 data points from testing gives an equilibrium scour depth of 300 mm. Using all the depth readings collected during testing (20 points) gives a value for the equilibrium scour depth of 261 mm. Although the equilibrium scour depth varied by up to 39 mm depending on how many data points were used, Blaisdell *et al.*'s (1981) method still produced values over 123 mm above what was measured as equilibrium depth for BS(1), as seen in Figure 3. Figure 4 shows a similar finding for sample LC(1), where the addition of subsequent data points early on had a large impact on the equilibrium scour depth value. Using the first 120 minutes of testing (6 points) produced a value for the equilibrium scour depth that is about 1.3 times the magnitude of the measured depth, whereas using all the available data produced a value about 7 times the measured depth. Table 1 shows a comparison of the critical shear stress evaluated from the measured equilibrium scour depth, τ_{ce}, to the critical shear stress estimate from the Blaisdell *et al.* (1981) using the first 120 minutes of test data, τ_{cB}, as would normally be done in the standard jet test analysis, for 10 of the tested soil samples. Thus, it necessary to find an alternate model for estimating equilibrium scour from early scour measurements for the jet test.

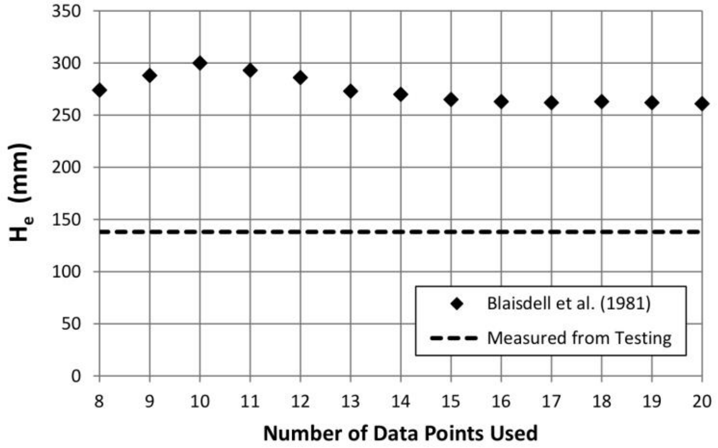

Figure 3: Effect of the number of data points on the equilibrium scour depth estimate for sample BS(1)

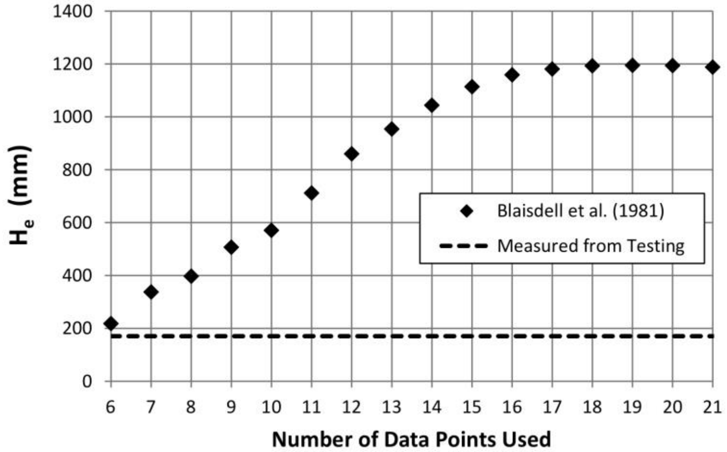

Figure 4. Effect of the number of data points on the equilibrium scour depth estimate for sample LC(1).

5.2 Determination of the Equilibrium Depth of Scour

In the second series of experiments, where the entire scour hole profile was measured with time, it was observed that the scour hole did not obtain an equilibrium state even though it appeared that the centerline depth had reached that value (Amin 2016). The decision about equilibrium is better to result from measurement of scour depths at several locations instead of simply relying on measurements made along the jet centreline.

5.3 Assumption of Linear Relationship between Erosion Rate and Excess Shear Stress

One of the assumptions made for the jet test is that there is a linear relationship between erosion rate and excess shear stress. The basis for this original assumption was reviewed by Mazurek *et al.* (2016); there was data that suggested that the linear relationship might be true, but it was not rigorously verified. Recently, it has come into question *(e.g*. Knapen *et al.* 2007; Walder 2015).

The linear assumption was tested for the first set of experiments described herein. The assumption of linearity (n=1 in Eq. 5) was loosened and the equation for the time development of scour given in Eq. 7 was redeveloped for powers of n=0.5, 1.5, and 2.0. Details are given in Cossette (2016). It is seen that although n=1 was a reasonable fit, it was not the optimum value, which was found by minimizing the mean absolute relative error of the model from the data. The optimized n values are given for 10 of the samples tested in Table 1. In Cossette (2016), the best-fit n value was typically found to be n=2. Any erosion model considered in future for analyzing jet test data in future, therefore, needs to avoid the assumption of linearity between erosion rate and excess shear stress.

Table 1: Comparison of the values for the critical shear stress estimated from the measured equilibrium scour depth and that determined from the estimate using Blaisdell *et al.* (1981) and best-fit n value

Soil Sample	Critical Shear Stress from Equilibrium Depth, τ_{ce}, (Pa)	Critical Shear Stress from Blaisdell Method Estimates, τ_{cB} (Pa)	Optimized n value
SN(1)	8.8	2.8	1.0
WC(1)	3.8	2.0	2.0
LC(1)	1.3	0.8	1.0
SC(2)R	10.1	3.3	1.5
BB(1)	1.2	0.9	0.5
JR(1)	5.8	2.4	1.5
RR(1)	13.9	7.4	1.5
M390(2)	99.3	23.5	2.0
M370(1)	77.0	14.5	2.0
LW(1)	10.4	9.2	2.0

5.4 Estimation of Shear Stress on the Bed

The jet test analysis relies on estimation of shear stresses through estimation of jet velocities. The estimation of velocities relies on jet theory for a jet unaffected by any boundaries. It is known that the boundaries do affect jet shear stresses significantly (Rajaratnam *et al.* 1993; Camino *et al.* 2012; Weidner

2012; Weidner *et al.* 2012). When the jet is redirected upon itself because of the concave shape of a scour hole, the shear stresses that would otherwise be observed will be reduced.

Even if the estimated jet velocities were correct, an assumption in estimating shear stresses is that the bed is smooth. However, although the soil surface may be cut so that it is initially reasonably smooth at the start of testing, this does not remain so once erosion begins. The roughness of boundary impacts the bed shear stress (Rajaratnam and Mazurek 2005). The bed roughness will change as the sample erodes since this is usually in the form of chunks or lumps of soil for cohesive soils. As such, it appears necessary to incorporate some measurement of shear stresses into the jet test as the test is conducted..

6 CONCLUSIONS

There are many problems with the existing jet erodibility test analysis: (i) the assumption of linearity between erosion rates and excess shear stress on the bed; (ii) the estimation of the equilibrium scour depth; and (iii) the estimation of the stresses on the bed. Although the jet test has utility in erosion testing because of its ease of setup and operation and its potential for use for a broad range of sediment erosional strengths, it appears there are a number of ways that it could be improved to more accurately estimate sediment erodibility.

7 ACKNOWLEDGMENTS

The authors gratefully acknowledge funding support from the Natural Sciences and Research Council of Canada for this work. The authors also acknowledge the contributions of Colin Rennie of the University of Ottawa in collecting the Ontario soil samples used in the first set of experiments described herein.

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