Leadership in Sustainable Infrastructure

Leadership en Infrastructures Durables

Vancouver, Canada

May 31 – June 3, 2017/ *Mai 31 – Juin 3, 2017*

STATISTICAL MODELING OF EXTREME RAINFALL PROCESSES (SMEXRAIN): A DECISION SUPPORT TOOL FOR CONSTRUCTING INTENSITY-DURATION-FREQUENCY RELATIONS FOR URBAN WATER INFRASTRUCTURE DESIGIN

Nguyen, Truong-Huy[1,2], and Nguyen, Van-Thanh-Van[1]
[1] McGill University, Canada
[2] huy.nguyen5@mail.mcgill.ca

Abstract: In recent years, it has been recognized that society has become more vulnerable to extreme storm events. Many studies have been carried out to investigate the variability of these extreme storms. Of particular interest for urban water infrastructure design is the investigation of the probability of occurrence of the extreme rainfalls using frequency analysis method. Many probability distributions have been proposed to model the extreme rainfall processes. However, there is no general agreement as to which distribution should be used. Consequently, in practice, a number of popular distributions are often selected and their descriptive and predictive abilities are then investigated and compared. This task requires a significant investment of time to analyze a large amount of available rainfall data of different time scales and record lengths for a number of different sites. This paper presents therefore the development of a decision-support tool for statistical modeling of extreme rainfall processes (SMExRain). The proposed tool can assist stakeholders and decision-makers in the selection of the most suitable distribution(s) for accurate estimation of the extreme rainfalls. More specifically, the SMExRain can be used to evaluate the descriptive and predictive abilities of ten commonly-used probability models, Beta-K, Beta-P, Generalized Extreme Value, Generalized Logistic, Generalized Normal, Generalized Pareto, Gumbel, Log-Pearson Type III, Pearson Type III, and Wakeby, for their accuracy and robustness in the estimation of annual maximum rainfalls (AMRs). Results of a numerical application using daily and sub-daily AMR data available from a network of 84 stations in Ontario have indicated the feasibility and accuracy of the proposed decision support tool SMExRain.

1 INTRODUCTION

The design and management of various water infrastructure systems require an adequate knowledge of extreme rainfall events of high return periods and for different durations (from several minutes to hours or days). In most cases, these extreme rainfalls corresponding to the desired return periods could not be extracted directly from the observed data since the available historical rainfall record is usually too short. In current engineering practice, the estimated design rainfall values are accomplished based on statistical frequency analyses of annual maximum rainfall data where available rainfall records of adequate lengths could be used to estimate the parameters of a selected probability distribution (Chow 1964, Kite 1977). The selected distribution model is then used to estimate rainfall intensities corresponding to return periods greater than or less than those of the recorded storm events. Accurate estimation of extreme rainfall could help alleviate the damage caused by these extreme storms and it can help achieve more efficient design and management of water infrastructure systems.

In general, selection of an appropriate distribution to representing annual extreme rainfalls is the most difficult and time-consuming task since there are many probability models available in the literature as well as in the design guidelines from different countries (Chow 1964, Kite 1977, Stedinger et al. 1993, Hosking and Wallis 1997, Rao and Hamed 2000, WMO 2009). Hence, the choice of a suitable model still remains as one of the major problems in engineering practice since there is no general agreement as to which distribution, or distributions, that should be used for the frequency analysis of extreme rainfalls. The selection of an appropriate model, thus, depends mainly on the characteristics of the available rainfall data at a particular site. Therefore, it is necessary to evaluate many available distributions in order to find a suitable model that could provide accurate extreme rainfall estimates for a given location.

In view of the above issues, the main objective of the present study is to propose a decision-support tool (hereafter referred to as SMExRain – Statistical Modelling of Extreme Rainfall processes) that can be used to identify the most appropriate distribution(s) for estimating accurately the extreme rainfalls for design purposes. The proposed tool can provide a systematic and objective assessment of both the descriptive and predictive abilities of various distributions in the estimation of the extreme rainfalls using different graphical and statistical assessment criteria. In addition, this tool is highly efficient and convenient for the analysis of a large database of extreme rainfall data for different durations at a given location as well as for a large number of sites. The structure of the SMExRain and the procedure for identifying the best distribution is described in Section 2. An illustrative application of this decision-support tool using daily and sub-daily annual maximum rainfall data for Ontario region is presented in Section 3. Results of this numerical application have indicated the feasibility and accuracy of the proposed SMExRain. Finally, it can be noted that the results of the extreme rainfall frequency analyses using the SMExRain can be displayed in the popular form of Intensity-Duration-Frequency (IDF) relations for a given location of interest.

2 THE DECISION-SUPPORT TOOL: SMEXRAIN

2.1 General Description

The decision-support tool SMExRain has been coded in Matlab environment and equipped with a user-friendly ribbon interface. This software runs independently without the need of installation of a Matlab version. However, as a standalone application compiled from the Matlab environment, SMExRain requires the installation of the Matlab Compiler Runtime (MCR) v9.0 corresponding to the Matlab R2015b version (Mathworks 2016). The MCR v9.0 is free of charge and can be easily downloaded from the Matlab's website. The structure of SMExRain is depicted in Figure 1a.

Regarding the data screening and preliminary analysis, SMExRain provides users with many commonly used statistical criteria, including maximum, minimum, mean, standard deviation, skewness, and kurtosis of the input extreme data series. In addition to these numerical values, it also provides users with many useful graphs for statistical analyses, including the histogram plot for empirical probability density function analysis, the time series plot for trend analysis, and the boxplot for outlier detection.

For selecting a best-fit probability distribution, various numerical and graphical criteria could be employed to evaluate the best fit of the model to the selected data. This descriptive ability assessment includes the use of the popular L-moment ratio diagram (Hosking and Wallis 1997), the application of different statistical goodness-of-fit tests, and the use of various graphical displays such as probability plots and quantile-quantile plots to determine whether the fitted distributions are consistent with the given set of observations (Stedinger et al. 1993). In addition, the SMExRain provides necessary tools for evaluating the predictive ability of a model. This assessment is important to assess the capability of the selected model for describing accurately future extreme rainfall events. Often in selecting a particular distribution, one may be tempted to select a distribution with a high number of parameters. Generally, these distributions could provide a better fit to the existing data than those models with a fewer number of parameters, but they could be quite rigid and consequently are not able to provide an accurate estimation of future extreme rainfall events that are beyond the range of the available data. Nonparametric data resampling schemes such as bootstrap (Efron and Tibshirani 1994) is hence included in the SMExRain to permit an evaluation of the predictive ability of each distribution model. Detailed description of probability distributions, parameter estimation methods, as well as the graphical and numerical criteria to assess the descriptive and predictive abilities of a distribution are presented in the following sections. Notice also that, for convenience, the SMExRain has been designed to allow users to perform the assessment and comparison of up to 12 probability distributions simultaneously rather than to evaluate a single distribution at a time. Depending upon the number of distributions selected for assessment as well as the number of generated bootstrap samples considered, the total simulation process can be completed within a short period (from seconds to minutes on a laptop computer).

2.2 Probability Distributions and Parameter Estimation Methods

SMExRain includes common two-to-five parameter probability distributions that have been selected based on their popularity in hydrologic frequency analyses: Beta-K (BEK), Beta-P (BEP), Generalized Extreme Value (GEV), Generalized Normal (GNO), Generalized Logistic (GLO), Generalized Pareto (GPA), Gumbel (GUM), Log-Pearson Type III (LP3), Pearson Type III (PE3), and Wakeby (WAK) distributions. Other special cases of these distributions, such as exponential (EXP) and normal (NOM) were also included in the software (Chow 1964, Kite 1977, Stedinger et al. 1993, Hosking and Wallis 1997, Rao and Hamed 2000, WMO 2009).

Regarding the estimation of the distribution parameters, the SMexRain software include some common estimation procedures such as the method of moments, the maximum likelihood method, the method of L-moments (Stedinger et al. 1993, Rao and Hamed 2000), and the method of non-central moments (NCMs) (Nguyen et al. 2002). These methods differ in the weights they give to different elements in the selected data set. The maximum likelihood method yields asymptotically optimal estimators of the parameters for some distributions; however, it often involves tedious computation, and it is very sensitive to the computational techniques considered. The L-moments estimators are unbiased and discriminate the behavior of skewed data, which is ideal for parameter estimation of hydrologic data. They are more robust than conventional moments to outliers in the data and sometimes yield more efficient parameter estimates than the maximum likelihood estimates (Stedinger et al. 1993, Hosking and Wallis 1997). The method of NCMs has been shown to be able to consider some scale-invariance property of the NCMs of extreme rainfall data for different durations (Nguyen et al. 2002, Nguyen and Nguyen 2008, Nguyen et al. 2007). More specifically, in the SMExRain, the method of L-moments is used for all distributions (Hosking and Wallis 1997) except for the BEK and BEP models that are estimated by the method of maximum likelihood (Mielke and Johnson 1974). It is noted that the method of L-moments could be used for BEK (Murshed et al. 2011), but it is not preferable since the estimation procedure is more complicated and more tedious than the method of maximum likelihood and the results are almost the same. GEV parameters are estimated by both the L-moments (denotes as GEV) and non-central moments (denotes as GEV*) methods.

2.3 Goodness-of-Fit Tests for Assessing the Descriptive Ability of a Distribution

To visually assess the adequacy of a fitted distribution, SMExRain provides probability plots or quantile-quantile (Q-Q) plots as a means of comparing observed to the fitted (or estimated) data (see Figure 1b). To support the decision-making, SMExRain is equiped with many commonly-used empirical plotting position (EPP) formulas described in literature, for example, Bloom’s, Cunnane’s, Weibull’s formulas and so on (Nguyen et al. 1989, Helsel and Hirsch 2002). In addition, it also provides a general EPP formula which can be customized by users.

In addition to the visual assessment, SMExRain also provides different numerical assessment criteria to indicate the accuracy of the best fit. Four popular criteria are included in the SMExRain, including the root mean square error (RMSE), the relative root mean square error (RRMSE), the maximum absolute error (MAE), and the correlation coefficient (CC) as described in the following equations:

$$[1]\ \mathrm{RMSE}=\left\{\sum\frac{(x_i-y_i)^2}{(n-m)}\right\}^{1/2}$$

$$[2]\ \mathrm{RRMSE} = \left[\frac{1}{(\mathrm{n}-\mathrm{m})}\sum\left\{\frac{(\mathrm{x_i}-\mathrm{y_i})}{\mathrm{x_i}}\right\}^2\right]^{1/2}$$

$$[3]\ \mathrm{MAE} = \max(|\mathrm{x_i} - \mathrm{y_i}|)$$

$$[4]\ \mathrm{CC} = \frac{\sum\{(\mathrm{x_i}-\bar{\mathrm{x}})(\mathrm{y_i}-\bar{\mathrm{y}})\}}{\{\sum(\mathrm{x_i}-\bar{\mathrm{x}})^2\sum(\mathrm{y_i}-\bar{\mathrm{y}})^2\}^{1/2}}$$

where $x_i, i = 1,2, \ldots, n$ are the observed values and $y_i, i = 1, 2, \ldots, n$, are the values estimated from an assumed probability distribution for the same probability level; m is the number of distribution parameters; $\bar{x}$ and $\bar{y}$ denote the average value of the observations and estimated quantiles, respectively.

For ease of computation and assessment the descriptive ability of a probability distribution, a ranking scheme has also been developed to judge the overall goodness-of-fit of each distribution by comparing the four categories of test criteria mentioned above. Rankings are assigned to each distribution for every test category according to the relative magnitude of the statistical test results. A distribution with the lowest RMSE, RRMSE, MAE or highest CC would be given the rank of 1. In case of a tie, average ranks are assigned to those distributions. It should be pointed out that equal ranks could also be used in case of ties with almost identical results. SMExRain has not been equipped with the case of equal ranks; however, since Microsoft Excel has supported this function, users could estimate the equal ranks using the Excel built-in function after exporting the results from SMExRain without any difficulty.

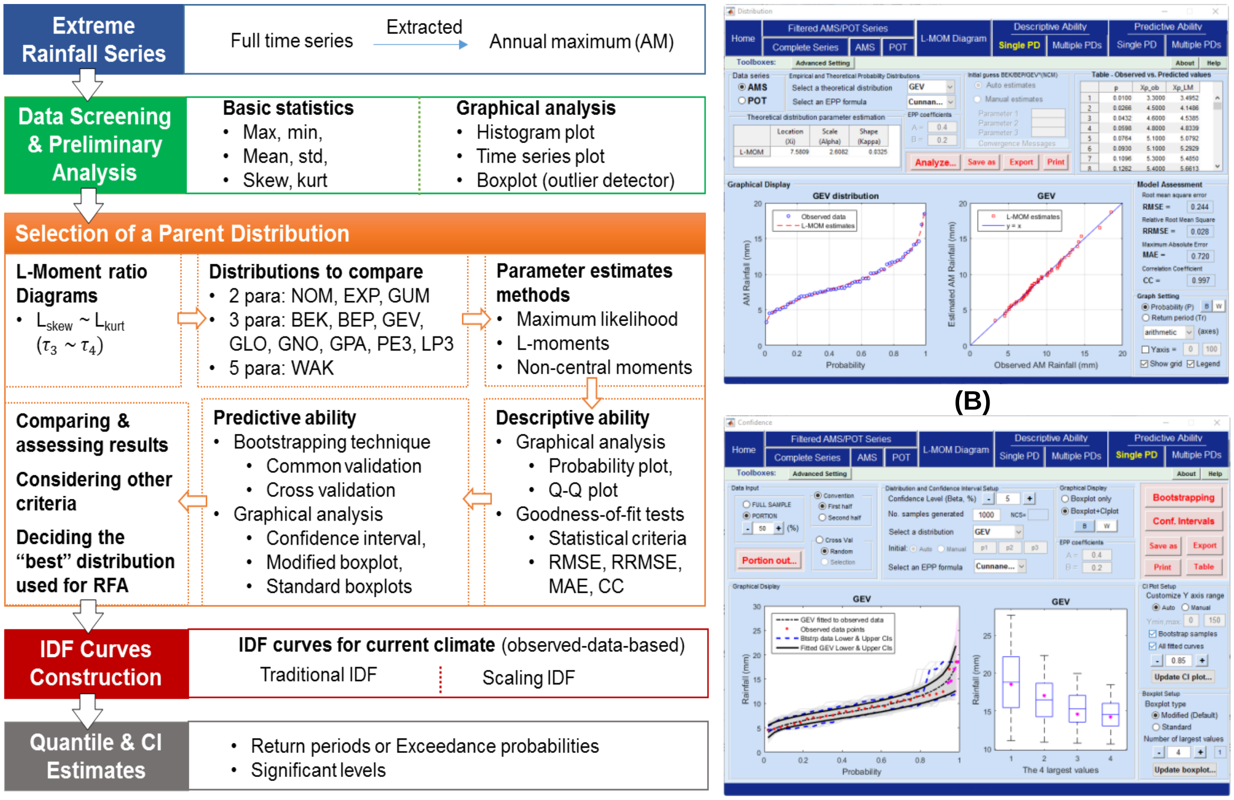
2.4 Bootstrap Method for Assessing the Predictive Ability of a Distribution

In general, it is expected that models with more parameters could fit better to the observed data; however, their parameter estimates could be more complex; and more importantly, their prediction abilities may not be better than those models with fewer parameters. Additionally, the performance of a distributions in extrapolating beyond the data is often of primary interest in real applications. To better evaluate the performance of various distributions at predicting extreme right-tail data, a bootstrap method was used in the SMExRain. This is a nonparametric approach to quantify the estimation uncertainty with statistical sampling procedure that yields multiple synthetic samples of the same sizes as the original observations (Efron and Tibshirani 1994). It is found that the distribution of sample statistics computed from the bootstrap samples is a good representation of the respective distribution of the observed statistics (Vogel 1995).

In order to generate bootstrap samples, a portion size n of the original dataset size N, for instance, n= 50% or half, must be extracted first. SMExRain provides users with two options: common validation and cross validation. In the former option, users can select the first or second halves to bootstrap. In the latter option, a portion of the selected size n is extracted with the starting point selected randomly. After extracting the sample of size n, users can enter the number of bootstrap samples to be generated, for example, hundreds to thousands samples etc. The default value in SMExRain is 1000 samples for stable results and efficient computation costs. Each candidate distribution is then fitted to the generated bootstrap samples and is extrapolated to estimate the right-tail quantiles corresponding to the k largest (k=4 by default) observed rainfall amounts in the full data set (see Figure 1c). The variability in the estimation of these extrapolated quantiles is presented in SMExRain in the form of modified boxplots by default. However, users can also switch to the standard boxplots (Helsel and Hirsch, 2002). The standard boxplot generally portrays the median, interquartile range, and outliers of the investigated data set. In this paper, the modified boxplots are used to show the robustness of each distribution predictive ability. The middle line of a modified box is the sample mean, the box height is twice standard deviation, the upper and lower whisker extend to the maximum and minimum value of the sample respectively (see Figure 1c). Large box widths or long whiskers imply high uncertainty in the estimation of these extreme values. If the observed values fall outside the box, then the distribution fitted to the bootstrap samples has overestimated or underestimated the true values and is therefore not commendable. It is important to know that, rather than considering each distribution at a time, SMExRain allows user to compare the predictive performance of up to 12 probability distributions simultaneously using the same generated bootstrap samples to ensure a fair comparison.

2.5 Construction of IDF relations

In standard engineering practice, the results of at-site rainfall frequency analyses are often summarized and presented in the form of depth-duration-frequency (DDF) relations or intensity-duration-frequency (IDF) relations for each rain-gauge site with adequate rainfall records. In SMExRain, these IDF or DDF curves are available in both tabular and graphical forms for estimated rainfall intensities (or depths) at investigated durations (generally from five minutes to one day) and for return periods of interests (generally from two to a hundred years). Depending upon the choice of an empirical regression equation representing IDF relations, the coefficients are also computed for the ease of use and of interpolation of rainfall intensities at unobserved durations. SMExRain supports many popular regression equations (WMO 2009, Green et al. 2016) in both real-space (with two or three coefficients) and log-space (with polynomial up to order 6). The coefficients are estimated based on the least-square technique.



(A)

(B)

(C)

Figure 1: (a) SMExRain structure and functions, (b) SMExRain user interface for the descriptive ability test, and (c) for the predictive ability test

3 ILLUSTRATIVE APPLICATION USING ONTARIO DATA

3.1 Database

For purposes of illustrative application, this paper presents the utilization of SMExRain to identify the "best" distribution(s) for representing the distribution of sub-daily and daily annual maximum rainfalls in Ontario. A total of 252 datasets for three different rainfall durations (5 minutes, 1 hour, and 24 hours) from a network of 84 stations were used. The selection of these stations relied on the quality of the data, the adequate length of available historical extreme rainfall records (containing at least 20 years), and the spatial distribution of the rain-gauges to represent the different climatic conditions in this study area.

3.2 Decision-Support Process

3.2.1 Descriptive Ability Assessment Results

The Q-Q plots of all 252 AMS shows that all distributions can describe well the left-tail and central parts. The right-tail parts, however, are less well described and there are no obvious trends. These values can be accurately estimated, over-estimated, or under-estimated by any of the 11 candidate distributions. For purposes of illustration, only 1-hour AMS from one of the longest record and highest data quality stations – Toronto Int. Airport, is presented here as shown in Figure 2. From the visual standpoint, all distributions seem to perform well in this case, except the BEK and GPA distributions. However, the significance of the differences between the remaining models is difficult to judge merely based on the graphical display, as the differences are minor. A more objective evaluation using numerical comparison criteria is thus necessary.

Rankings of the 11 candidates at each of the 84 stations based on the four statistical criteria were performed. Ranking from number 1 to 11 indicates the gradual decrease from the best to the worst distributions. On the basis of these goodness-of-fit numerical comparison results, it was found that no unique distribution ranked consistently best for all locations and for all three selected rainfall durations. The overall ranks of each distribution were obtained for every test category by summing the individual point rank at each location. The rank sum results of the 84 stations of three durations for different record lengths (i.e. stations contain at least 40 years, 30 years, and 20 years), presented in Figure 3, show that WAK model outperforms the others in describing the distribution of daily and sub-daily AMS. The GEV, GNO, and PE3 models also performed well overall and their scores are close to each other. This can be expected since these models are advocated for use in frequency analyses of hydrologic extreme variables by many previous studies. It is also noticed that the PE3 model performed slightly better than GEV and GNO models for 5-min duration data. However, for data set of longer durations – 1-hour and 24-hour, GEV and GNO are slightly better (see Figure 3). The GLO, LP3, and GEV* distributions moderately performed and they stand among middle positions (see Figure 3). It is also interesting to notice that if only RRMSE criterion is considered, LP3 is the best candidate for data of all durations.

After assessing how well each distribution fit to the overall data sets, the focus is on the right tail region of the distribution since this is the region of importance to engineering design and planning applications. The degree of fit on the right tail of the distributions was visually examined using the quantile-quantile plots to gain further appreciation of the overall fit of the distributions. From the visual standpoint, there was very little to choose from between the various distributions for representing the data used in this study since a very small variability was found in the comparison results for different locations as shown, for instance, in Figure 2 – Toronto Int. Airport

3.2.2 Predictive Ability Assessment Results

The sampling characteristics of extrapolated right-tail quantiles were investigated using the bootstrap procedure for all stations containing at least 30-year record length (i.e. 47 stations). In this study, one thousand bootstrap samples of size equal to half (i.e. 50%) of the actual sample size were drawn with replacement from the observations for 21 stations containing at least 40 years of record. For 26 stations contain between 30-40 years of record, approximate two third (i.e. 65%) of the actual sample size were used in order to make sure that the generated bootstrap samples contain at least 20 years of record. Each candidate distribution was then fitted to the generated bootstrap samples and used to extrapolate the right-tail quantiles corresponding to the four largest observed rainfall amounts in the full data set. The distribution of the extrapolated amounts obtained was presented in the form of the modified box plots. The performance of each distribution in estimating the four largest values was evaluated.

The modified boxplots of 141 AMS show that, generally, the Beta-K, Beta-P gave consistently the worst performance with large sampling variation and bias for all three rainfall durations. For instance, Figure 4 shows the modified boxplots of extrapolated right-tail bootstrap data for 5-min AMS at Toronto Int. Airport. Unlike the BEK and BEP models, the modified boxplots for the WAK model do not show large box widths; however, they reveal long upper whiskers (see Figure 4 as an example). In addition, results of modified boxplots reveal that, the LP3 model produced larger box widths than other remaining distributions, yet it was not as poorly performed as BEK, BEP or WAK (see Figure 4). Although the Gumbel distribution exhibited the lowest sample variation in most cases, it tends to overestimate or underestimate the observed values most frequently. The GEV, GEV*, GLO, GNO, GPA, and PE3 distributions produced satisfactory results at most stations where the box enclosed the observed right-tail values with a reasonable whisker spread and correlation with the observed values. In particular, the GEV, GNO, and PE3 produced almost identical results. Occurrences of over- or under-estimation of largest rainfall amounts did occur for all distributions at several locations.

3.3 Decision-Making Process

In general, it is observed that no one distribution performed the best at every station for each category. This could be due to the strong spatial variabilty of rainfall characteristics within this study region. While it is difficult to provide a clear physical interpretation of the regional variability of the probability distribution parameters, one is still able to rely on the proposed tool to identify the GEV, GNO, and PE3 as the best distributions for a large number of cases considered. Furthermore, it is easy to recognize distributions that perform less satisfactory, it is more difficult to identify the overall best distribution. These three models could be thus used alternately for the frequency analysis of annual extreme rainfalls for a given site as shown for instance in Figure 5 for Toronto Int. Airport station. The difference in extreme design rainfall estimates produced by the three distributions is also further investigated for all stations containing at least 30-year records. Results reveal that the estimated values for return periods within the twice sample lengths are almost identical for the three distributions. However, the GEV model tends to provide slightly higher values for high return periods, while the PE3 model tends to give slightly higher values for low return periods. The three models, therefore, could be used interchangeably in constructing IDF relations and estimating extreme design rainfalls for Ontario region. Nonetheless, if only one probability model is preferred for the entire region, other criteria should be thus considered in the choice of an appropriate distribution. For instance, the GEV model is based on a more solid theoretical basis than the other two distributions because it was derived from the statistical theory of extreme random variables. Therefore, the GEV could be considered as the most suitable distribution if only a unique probability model is required for describing the distribution of annual maximum rainfalls for this study area.

4 CONCLUSIONS

A decision-support tool (SMExRain) has been developed for evaluating systematically the performance of various commonly-used probability distributions in hydrologic frequency analyses in order to identify the most suitable model for representing the distribution of extreme rainfalls for a study region of interest. Based on a number of graphical and numerical criteria, and being equipped with a user-friendly ribbon interface, this tool can be used to assess in an efficient manner the descriptive and predictive abilities of each distribution for a large database of extreme rainfall data of different durations at a given location as well as for a large number of sites. More specifically, it is relied on the results of four goodness-of-fit tests, including root mean square error (RMSE), root mean square relative error (RRMSE), maximum absolute error (MAE), and correlation coefficient (CC), and is also supported by the visual comparison of the probability plots, quantile-quantile plots and the modified and standard box plots from the bootstrap samplings. Furthermore, SMExRain can provide IDF relations for the current climate (i.e. based on historical data) in both tabular and graphical forms for a given location of interest. These IDF relations can also be presented in mathematical formulas (in real or log-space) for the convenience of computation and application in practice.

The proposed SMExRain tool has been tested using extreme rainfall data for other regions in Canada. In this paper, it has been successfully applied to identify the best probability distributions that could provide accurate annual maximum rainfall estimates for the selected Ontario region. In particular, it was found that, among the eleven selected candidates, the GEV, GNO and PE3 provided the most consistent and the best performance for extreme rainfall data for different durations and for a number of locations in this study area. The estimated rainfall values for return periods within the twice sample lengths are almost identical for these three distributions. However, the GEV model tends to provide slightly higher values for high return periods, while the PE3 model tends to give slightly higher values for low return periods. The three models, therefore, could be used interchangeably in constructing IDF relations and for estimating extreme design rainfalls.

Nonetheless, for practical application purposes and for safety in urban water infrastructure design and management, the GEV could be preferable to the GNO and PE3 because it has a more solid theoretical basis and it produces higher estimates (or quantiles) for high return periods as compared to the others.

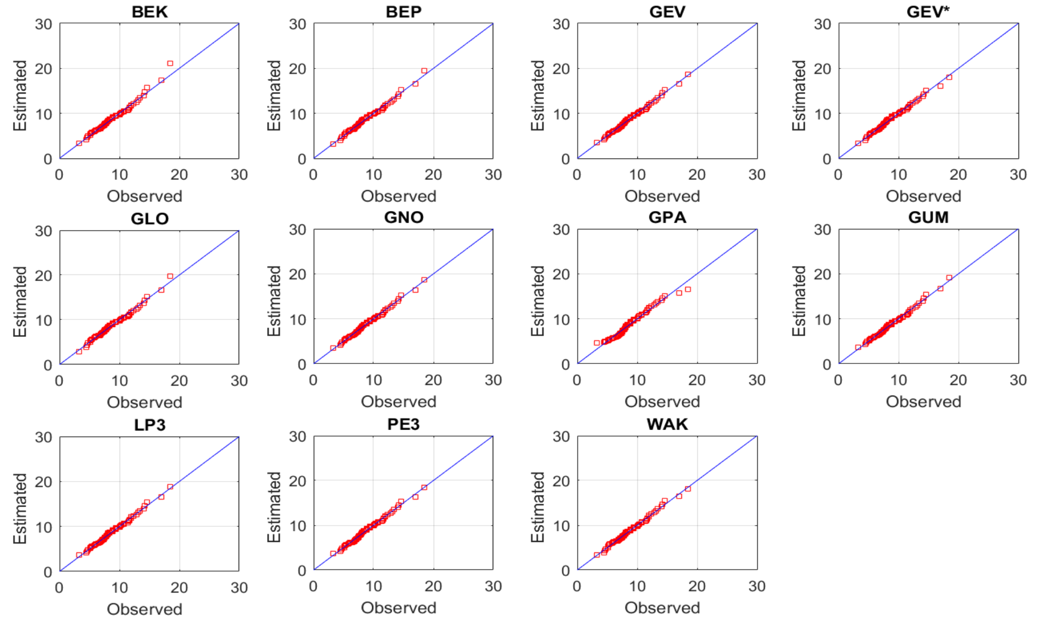
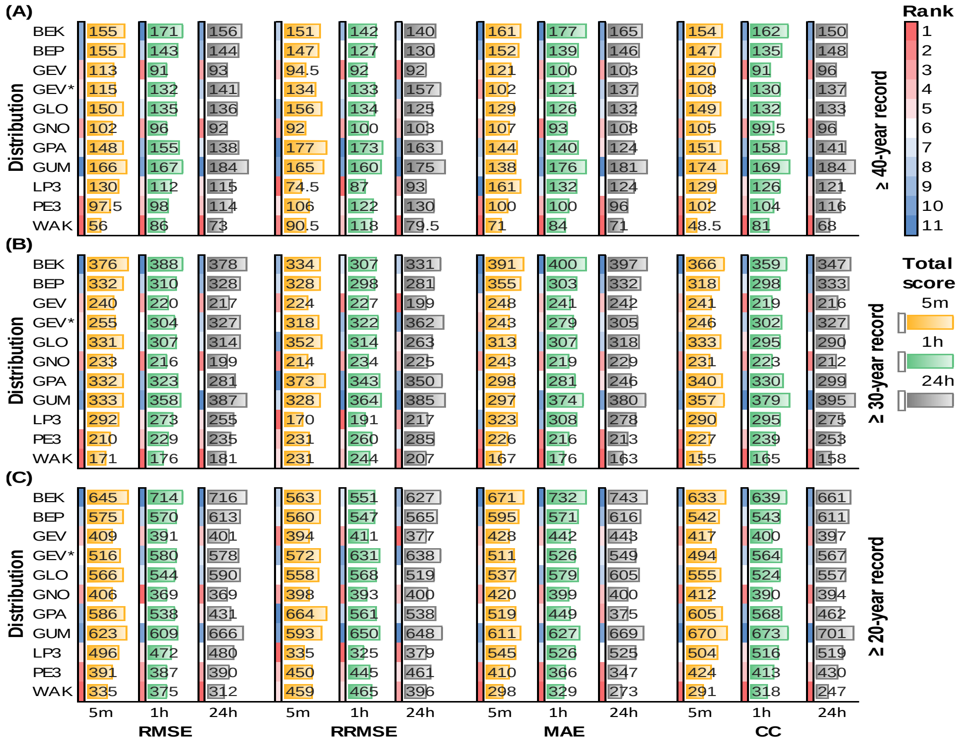


Figure 2: Q-Q plots for distributions fitted to 5-min AMS at Toronto Int. Airport station



(A)

Distribution	RMSE 5m	RMSE 1h	RMSE 24h	RRMSE 5m	RRMSE 1h	RRMSE 24h	MAE 5m	MAE 1h	MAE 24h	CC 5m	CC 1h	CC 24h
BEK	155	171	156	151	142	140	161	177	165	154	162	150
BEP	155	143	144	147	127	130	152	139	146	147	135	148
GEV	113	91	93	94.5	92	92	121	100	103	120	91	96
GEV*	115	132	141	134	133	157	102	121	137	108	130	137
GLO	150	135	136	156	134	125	129	126	132	149	132	133
GNO	102	96	92	92	100	103	107	93	108	105	99.5	96
GPA	148	155	138	177	173	163	144	140	124	151	158	141
GUM	166	167	184	165	160	175	138	176	181	174	169	184
LP3	130	112	115	74.5	87	93	161	132	124	129	126	121
PE3	97.5	98	114	106	122	130	100	100	96	102	104	116
WAK	56	86	73	90.5	118	79.5	71	84	71	48.5	81	68

(B)

Distribution	RMSE 5m	RMSE 1h	RMSE 24h	RRMSE 5m	RRMSE 1h	RRMSE 24h	MAE 5m	MAE 1h	MAE 24h	CC 5m	CC 1h	CC 24h
BEK	376	388	378	334	307	331	391	400	397	366	359	347
BEP	332	310	328	328	298	281	355	303	332	318	298	333
GEV	240	220	217	224	227	199	248	241	242	241	219	216
GEV*	255	304	327	318	322	362	243	279	305	246	302	327
GLO	331	307	314	352	314	263	313	307	318	333	295	290
GNO	233	216	199	214	234	225	243	219	229	231	223	212
GPA	332	323	281	373	343	350	298	281	246	340	330	299
GUM	333	358	387	328	364	385	297	374	380	357	379	395
LP3	292	273	255	170	191	217	323	308	278	290	295	275
PE3	210	229	235	231	260	285	226	216	213	227	239	253
WAK	171	176	181	231	244	207	167	176	163	155	165	158

(C)

Distribution	RMSE 5m	RMSE 1h	RMSE 24h	RRMSE 5m	RRMSE 1h	RRMSE 24h	MAE 5m	MAE 1h	MAE 24h	CC 5m	CC 1h	CC 24h
BEK	645	714	716	563	551	627	671	732	743	633	639	661
BEP	575	570	613	560	547	565	595	571	616	542	543	611
GEV	409	391	401	394	411	377	428	442	443	417	400	397
GEV*	516	580	578	572	631	638	511	526	549	494	564	567
GLO	566	544	590	558	568	519	537	579	605	555	524	557
GNO	406	369	369	398	393	400	420	399	400	412	390	394
GPA	586	538	431	664	561	538	519	449	375	605	568	462
GUM	623	609	666	593	650	648	611	627	669	670	673	701
LP3	496	472	480	335	325	379	545	526	525	504	516	519
PE3	391	387	390	450	445	461	410	366	347	424	413	430
WAK	335	375	312	459	465	396	298	329	273	291	318	247

Figure 3: Overall rank for all stations containing at least (A) 40-year, (B) 30-year, and (C) 20-year records based on the four statistical tests for all three durations of 5-min, 1-hour, and 24-hour AMS (The lowest scores or the shortest bar indicates the best distribution)

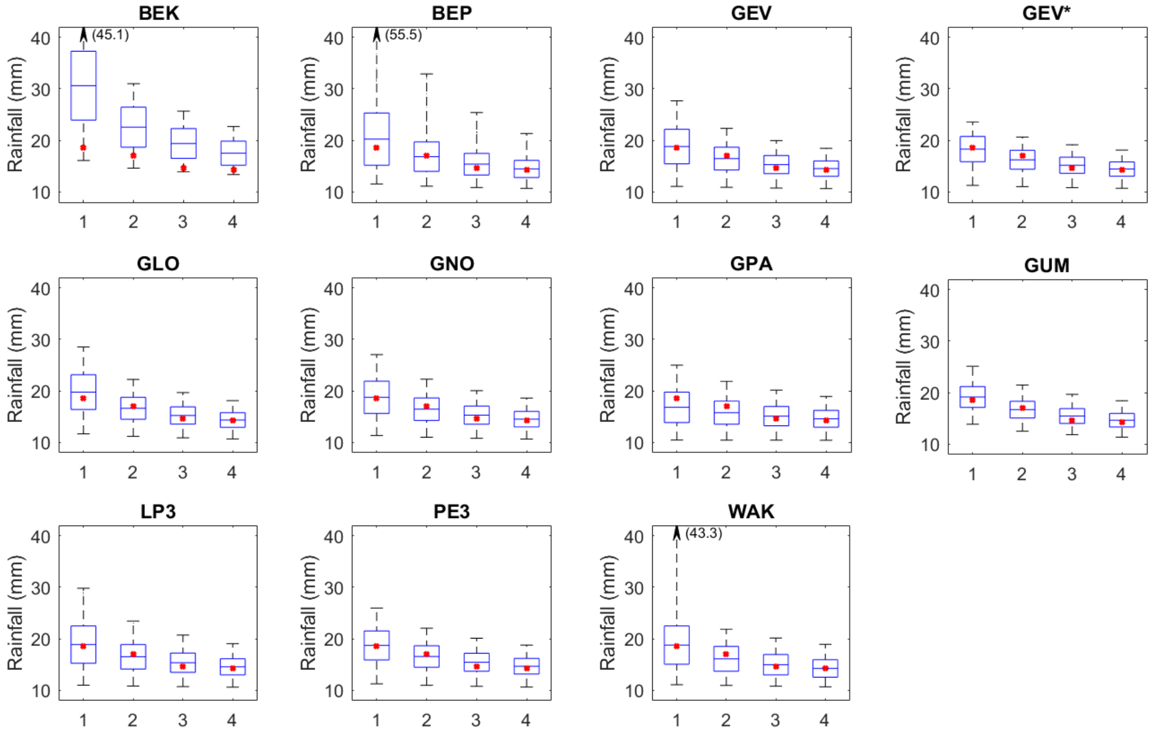


Figure 4: Boxplots of extrapolated right-tail bootstrap data for 5-min AMS at Toronto Int. Airport

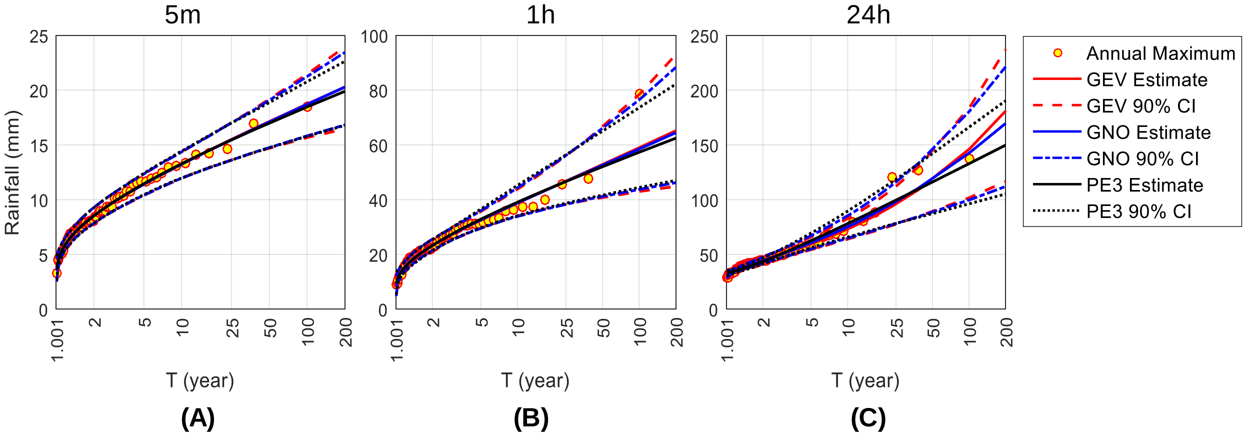


Figure 5: Frequency curves (solid lines) and 90% confidence limits (90% CI, dashed lines) of (a) 5-minute, (b) 1-hour, and (c) 24-hour annual maximum rainfalls (circle markers) at Toronto Int. Airport station using the top three distributions – GEV, GNO, and PE3

ACKNOWLEDGEMENTS

The authors would like to acknowledge the funding provided by the Faculty of Engineering at McGill University and the NSERC FloodNet Strategic Network for this project.

REFERENCES

Chow, V.T. 1964. *Handbook of Applied Hydrology*. McGraw-Hill, New York, USA.

Efron, B. and Tibshirani, R.J. 1994. *An introduction to the bootstrap*. CRC Press, New York, USA.

Environment Canada. 2014. *Engineering Climate Dataset*. Rainfall Intensity-Duration-Frequency (IDF) Tables and Graphs V2.3. http://climate.weather.gc.ca/prods_servs/engineering_e.html. Accessed on Jan 2016.

Green, J., Johnson, F., Beesley, C., The, C. 2016. *Design Rainfall. Book 2, Chapter 2 in Australian Rainfall and Runoff - A Guide to Flood Estimation*, Commonwealth of Australia.

Helsel, D.R. and Hirsch, R.M. 2002. *Statistical Methods in Water Resources Techniques of Water Resources Investigations, Book 4, chapter A3*. U.S. Geological Survey, US.

Hosking, J.R.M. and Wallis, J.R. 1997. *Regional Frequency Analysis: An Approach Based on L-Moments*. Cambridge University Press, Cambridge, UK.

Kite, G.W. 1977. *Frequency and Risk Analyses in Hydrology*. Water Resources Publications, Fort Colling, Colorado, USA.

Mathworks. 2016. *Matlab Compiler Runtime (MCR).* https://www.mathworks.com/products/compiler/mcr/. Accessed on Jan 2016.

Mielke, P.W. and Johnson, E.S. 1974. Some generalized beta distributions of the second kind having desirable application features in hydrology and meteorology. *Water Resources Research,* **10** (2): 223-226.

Murshed, M.S., Kim, S., Park, J.-S. 2011. Beta-κ distribution and its application to hydrologic events. *Stochastic Environmental Research and Risk Assessment*, **25**(7): 897-911.

Nguyen, V-T-V., Nguyen, T-D., Ashkar, F. 2002. Regional frequency analysis of extreme rainfalls. *Water Science and Technology Journal*, **45**(2): 75-81.

Nguyen, V-T-V., Inna, N., Bobee, B. 1989. New Plotting-Position Formula for Pearson Type-Iii Distribution. *Journal of Hydraulic Engineering-Asce*, **115**(6): 709-730.

Nguyen, V-T-V. and Nguyen, T-D. 2008. A spatial-temporal statistical downscaling approach to estimation of extreme precipitations for climate-related impact studies at a local site, *World Environmental and Water Resources Congress 2008*: Honolulu, HI.

Nguyen, V-T-V., Nguyen, T.D., Cung, A. 2007. A statistical approach to downscaling of sub-daily extreme rainfall processes for climate-related impact studies in urban areas, In: Fang, H.H.P., Lee, J.H.W. (Eds.), *Water Science and Technology: Water Supply*, pp. 183-192.

Rao, A.R. and Hamed, K.H. 2000. *Flood frequency analysis*. CRC Press, Boca Raton, London.

Stedinger, J.R., Vogel, R.M., Foufoula-Georgiou, E., 1993. *Frequency Analysis of Extreme Events, chapter 18 in Handbook of Hydrology* edited by D. A. Maidment. McGraw-Hill, New York, USA.

Vogel, R.M. 1995. Recent advances and themes in hydrology. *Reviews of Geophysics*, **33**(S2): 933.

WMO. 2009. *Guide to hydrological practices, volume II: Management of water resources and application of hydrological practices.* 6th edition, World Meteorological Organization, Geneva, Switzerland