Leadership in Sustainable Infrastructure
Leadership en Infrastructures Durables

Vancouver, Canada
May 31 – June 3, 2017/ *Mai 31 – Juin 3, 2017*

TESTING EVOLUTIONARY ALGORITHMS FOR OPTIMIZATION OF WATER DISTRIBUTION NETWORKS

Moosavian, Naser [1,3], and Lence, Barbara[2]
[1] Graduate Research Assistant, Department of Civil Engineering, University of British Columbia, Canada
[2] Professor, Department of Civil Engineering, University of British Columbia, Canada
[3] naser.moosavian@alumni.ubc.ca

Abstract: Water distribution networks (WDNs) are one of the most important elements in the urban infrastructure system and require large investment for construction. Design of such networks is classified as a large combinatorial discrete non-linear optimization problem. The main concerns associated with optimization of water distribution networks are related to the nonlinearity of the discharge-head loss relationships for pipes and the discrete nature of pipe sizes. This paper compares different techniques, all based on evolutionary algorithms (EAs), which yield optimal solutions for design of water distribution networks. The fundamental concept of EAs is that they search for the global optimum with populations of solutions, rather than by improving a single solution, as in Newton-based and other search methods. EAs start with an initial population and use different operators on these populations to change their composition and improve their performance over repeated iterations, or generations. In this paper, six EAs are applied for design of two benchmark pipe networks, the Two-Loop and Hanoi networks, and one real water distribution system for the City of Farhadgerd, Iran. Results show that as the size of the network increases, the soccer league competition algorithm increasingly becomes the most efficient among these algorithms, and consistently converges to the global optimum.

1 INTRODUCTION

EAs are becoming increasingly popular for their use in solving engineering decision problems and in practical applications such as calibration, because they: (i) are based on rather simple concepts and are easy to implement; (ii) do not require gradient information; (iii) can consider and bypass local optima; and (iv) can be utilized in a wide range of problems covering different disciplines. Generally, the main structure and searching process of different EAs are similar, however, their operators may vary. Over the last three decades, many heuristic optimization techniques have been successfully used to identify water network designs, see, e.g., applications of genetic algorithms (Murphy and Simpson 1992); simulated annealing (Cunha and Sousa 2001); harmony search (Geem 2006); shuffled frog leaping (Eusuff and Lansey 2003); ant colony optimization (Maier et al. 2003); particle swarm optimization (Suribabu and Neelakantan 2006); cross entropy (Perelman and Ostfeld 2007); scatter search (Lin et al. 2007); differential evolution (Vasan and Simonovic 2010); self-adaptive differential evolution (Zheng et al. 2013); soccer league competition (Moosavian and Roodsari 2014); and improved genetic algorithms (Bi et al. 2015).

Because these heuristic algorithms were originally developed to address specific engineering problems, there is no guarantee that the global optimum may be found or that the method will be efficient in the solution of specific problems, such as the design of water distribution systems. Wolpert and Macready (1997) cite the No-Free-Lunch (NFL) theorem, assert that no one optimization algorithm may be suited for solving all kinds of optimization problems, and underscore the need for new algorithms that may address a wide range of problems or improve on efforts to reach global optima. In the area of water distribution systems, some comparisons of EA algorithms have been undertaken (Zheng et al. 2013; Moosavian and Roodsari 2014; Bi et al. 2015), though these studies focus on a select few methods. This paper presents a rigorous comparison of six EAs in application to the optimum design of WDNs, and evaluates the algorithms in terms of the best solution obtained, the speed of convergence, and the numbers of function evaluations.

In the optimization of water distribution systems, the primary objective is to minimize the total cost of the network. This cost is a function of pipe size diameters and these pipe diameters in turn affect the network pressures. The main constraint of the problem is to satisfy minimum required pressure at all network nodes. In applications of EAs, at each evaluation of the objective function, hydraulic analysis of the network must be conducted, which exacts a high computational burden, and is in fact the most computationally intensive portion of the optimization process. Therefore, the optimal solution, for a fixed number of function evaluations, is a suitable comparator for EAs. In this paper, six meta-heuristic algorithms are compared in terms of the WDN design selected for three example networks, under different stopping criteria based on the allowable number of function evaluations. To guard against selection of local optima, for each criterion and each pipe network, these algorithms are implemented 20 times. The algorithms include the: genetic algorithm (GA); harmony search (HS); differential evolution (DE); particle swarm optimization (PSO); artificial bee colony (ABC); and soccer league competition (SLC) meta-heuristics. In the following sections, the general WDN optimization model is introduced, the algorithms are briefly described, their application to three example networks are assessed, and conclusions are drawn from this comparison.

2 PROBLEM FORMULATION

A water distribution system is a collection of many components such as pipes, reservoirs, pumps and valves which are connected in order to provide water to consumers. The optimal design of such a system can be defined as the best combination of component sizes and component settings (e.g., pipe size diameters, pump types, pump locations and maximum power, and reservoir storage volumes) that gives the minimum cost for the given layout of network, such that hydraulic laws governing continuity of flow and energy are maintained and constraints on quantities and pressures at the consumer nodes are fulfilled. In this paper, water distribution system design is formulated as a least-cost optimization problem with the selection of pipe sizes as the decision variables, while pipe layout and its connectivity, nodal demand, and minimum pressure head requirements, are imposed. The optimization problem is stated mathematically as:

[1] $$Min\ C = \sum_{k=1}^{np} c_k\left(D_k\right) \times L_k$$

where $c_k\left(D_k\right) \times L_k$ = the cost of pipe k with length L_k and diameter D_k, and np = the number of pipes in the network. This objective function is minimized under the following constraints:

Flow continuity at nodes

For each node, flow continuity must be satisfied,

[2] $$\sum Q_{in} - \sum Q_{out} = q_j, \qquad \forall k \epsilon nn$$

where q_j = demand at node j (meters^3/second); nn = number of nodes; and Q_{in} and Q_{out} = flow into and out of node j (meters^3/second), respectively.

Energy conservation in loops

The total head loss around a closed pipe loop should be equal to zero, or the head loss along a loop between two fixed head reservoirs should be equal to the difference in water level of the reservoirs:

$$[3] \quad \sum_{k \in loop\ l} hf_k = \Delta H, \qquad \forall l \epsilon nl$$

where ΔH = difference between nodal pressures at both ends of a path (meters), and $\Delta H = 0$, if the path is closed; nl = number of loops; and hf_k = head loss due to friction in the pipe *k* (meters) which is obtained from following equation:

$$[4] \quad hf_k = H_i - H_j = R_k Q_k^n$$

where H_i and H_j = the nodal heads at the start node and the end node of the pipe (meters); R_k = the resistance coefficient of the *k*th pipe with flow rate Q_k (second/ meters^2); and *n* = a constant depending on the head loss equation, and is 1.852 for the most common expression for head loss, the Hazen-Williams head loss formulation.

Minimum pressure at nodes

For each junction node in the network, the pressure head should be greater than the prescribed minimum pressure head:

$$[5] \quad H_j \geq H_j^{min}, \forall\ j \in nn$$

where H_j = the pressure head at node *j* (meters); *nn* = the number of nodes; and H_j^{min} = the minimum required pressure head (meters).

In this work, the Global Gradient Algorithm, GGA (Todini and Pilati 1988), in a MATLAB environment, was applied to conduct the hydraulic analysis of network. GGA satisfies the continuity and energy conservation equations (Eq 2-4), while calculating the pressure head H_j at each junction node and the discharge Q_k in each pipe.

Pipe size availability

The diameter of the pipes should be available from a commercial size set:

$$[6] \quad D_k = \{D(1), D(2), ..., D(K)\}, \quad \forall k \in np$$

where *K* = the number of candidate diameters.

3 EVOLUTIONARY ALGORITHMS (EAs)

Generally, EAs are implemented in five main steps, including: (i) creation of a random initial population; (ii) evaluation of the cost function or fitness; (iii) selection of two or more solution vectors for the evolution process; (iv) implementation of an evolution strategy with different operators; and (v) selection of good solutions, and replacement into the population. Differences in EAs most often include variations in how they evolve new solution vectors, how they select these solutions from the population, and how they replace solutions into the updated population. In this section, a summary of the different EAs compared in this paperis presented.

3.1 Genetic Algorithm (GA)

Implementation of the GA (Holland 1972) begins with identification of a random set, or population, of solutions (chromosomes). The cost, or fitness, of each solution is determined by evaluation of the objective function and compared to determine those solutions, or population members, that are allowed to evolve. Typically the comparison process employs tournament selection, which involves a series of competitions, or "tournaments," among pairs of solutions selected at random from the population. The winner of each tournament is the solution with the best fitness. These winners are selected to undergo evolution with crossover and mutation operations, probabilistic-based mechanisms, through which new solutions are constructed. The new solutions are then evaluated and may be used to further evolve the population, if they provide better fitness (i.e., improved objective function value), than other population members. The process is continued for a large number of generations or until no further improvement in the bestobjective function is obtained.

3.2 Harmony search (HS)

The HS algorithm (Geem, 2006) is conceptualized based on the musical process of searching for a ‘perfect state’ of harmony, such as in jazz improvisation. Jazz improvisation seeks a best state (fantastic harmony) determined by aesthetic estimation, and analogously HS seeks a best state (global optimum) determined by evaluating the objective function. Aesthetic estimation is evaluated by the set of pitches played by each instrument, and in HS the objective function evaluation is performed by the set of values assigned by each decision variable. The harmony quality is enhanced with practice, and in HS the solution quality is enhanced over iterations. Each new harmony or solution vector is generated based on three rules: memory consideration, pitch adjustment, and random selection. If this new harmony is better than the existing worst harmony in memory, the new harmony is included in memory and the worst harmony is excluded from memory. This procedure is repeated until what is defined as a fantastic harmony is found.

3.3 Differential Evolution (DE)

DE (Price et al. 2005) is an improved version of GA. Similar to the GA, there are three important operators involved in the DE algorithm including the mutation, crossover, and selection operators. The main difference between GA and DE is that GA relies on its crossover operator to exchange information among solutions, while DE primarily relies on its mutation operator to form new solution vectors. Mutation is based on the fitness difference of randomly sampled pairs of solutions in the population. DE automatically adapts the mutation increments (i.e., step size of allowable variable changes), based on the stage of the evolutionary process. At the beginning of the evolution process, the mutation operator favors exploration and as evolution progresses, it favors exploitation. The algorithm uses a uniform crossover operator that can take child vector parameters from one parent more often than from the other. By using components of existing population members to construct trial vectors, the crossover operator efficiently shuffles information about successful combinations, enabling the search for an optimum to focus on the most promising area of the solution space.

3.4 Particle swarm optimization (PSO)

PSO (Kennedy and Eberhart 1995) is an evolutionary optimization algorithm which originated as a simulation of a simplified social system such as birds flocking and fish schooling. Similar to GA, PSO is also population-based and searches for optimal solutions by updating generations. However, unlike GA, PSO possesses no evolution operators. Instead, PSO relies on the exchange of information between individuals, or particles, of the population, or swarm. In order to select fitter particles for further populations, each particle adjusts its trajectory towards its current position, towards its previous best position and towards the current best position attained by any other member in its neighborhood. Compared with GA, PSO presents the advantage of being conceptually very simple and fast. However, its main disadvantage is the risk of a premature search convergence.

3.5 Artificial Bee Colony (ABC)

In the ABC algorithm (Karaboga 2005), the position of a food source represents a possible solution to the optimization problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution. Search processes start based on three kinds of bees: employed bees; onlooker bees; and scout bees. The number of the employed and onlooker bees is equal to the number of solutions considered in the population. The employed bee whose food source has been exhausted by the bee colony becomes a scout bee. Each cycle of the search procedure consists of the following actions: employed bees are sent onto their food sources and their nectar amounts are evaluated; employed bees share the nectar information of food sources with the observer bees, and based on this information, onlooker bees select among food source regions, and evaluate the nectar amount of the selected food sources. This process continues iteratively, and iff a solution represented by a food source is not improved by a pre-determined number of iterations, then the food source is abandoned and the employed bee for that source is converted to a scout, and is sent randomly onto possible new food sources. In this case, a new solution vector is randomly created and replaces the abandoned solution.

3.6 Soccer League Competition (SLC) algorithm

The SLC algorithm (Moosavian and Roodsari 2014) is inspired from professional soccer leagues. It involves different teams, or collections of solutions, where each solution is a team member, and a number of operators that act on the team members to perform an effective search for finding the near optimal solution. SLC mimics matches between teams and determines the winners based on their relative power, where the winner of a match has a higher probability of increasing its power for future matches. After each match, all players (decision vectors) on the winning team undergo operations that change their decisions, and thus their power (i.e., strengthening or weakening each player), producing modified team members. Finally, the original strength of each player on this winning team is compared with its modified strength and the player, i.e., decision vector, with the greater strength, is allowed to play on the team in the next match.

An iteration, or round, is defined as a set of matches that allows each team, or modified team, to play with each other team in the league. Thereafter, the next round is played with these new teams. The user specifies the stopping criteria to be either based on a limit of the number of rounds undertaken, or the number of function evaluations made.

4 APPLICATIONS TO WDN DESIGN

The performance of the six above-mentioned EAs is tested for three water distribution networks, the Two-Loop and Hanoi benchmark networks, and the WDN for the City of Farhadgerd, Iran.

For consistency in comparison, three stopping critera are defined as: (i) the maximum number of function evaluations is set to 1000. Given this criteria, the initial speed of convergence and performance of the algorithms may be evaluated; (ii) the maximum number of function evaluations is set to 10000. Here, the performance of the algorithms is evaluated for a more mature evolution, compared with (i); and (iii) the maximum number of function evaluations is set to 40000, and the evolution of the algorithms is considered to be fully mature. For each of these criteria, all algorithms are executed 20 times, i.e., 20 sets of initial populations of solutions are generated based on 20 different random number sequences.

Statistical analyses of the results for all algorithms are performed and are presented as box plots for the 20 different executions of the algorithm. All of the computations are implemented in the MATLAB programming language environment with an Intel(R) Core(TM) 2Duo CPU P8700 @ 2.53GHz and 4.00 GB RAM.

4.1 Two-loop benchmark network

This network, shown in Figure 1, is a hypothetical benchmark which has seven nodes and eight pipes with two loops, and is fed by a reservoir with a 210-m fixed head (Alperovits and Shamir 1977). The pipes are all 1,000 m long with a Hazen-Williams coefficient of 130. The minimum pressure limitation is 30 m above ground level for all nodes. There are 14 available commercial pipe diameters. The degree of candidate diameter is an indicator of the size and complexity of the network problem, and is defined as the number of candidate diameters divided by the number of pipes. For this case, the number of candidate diameters is equivalent to *14 / 8 =1.75*. Geem (2006) determines the minimum cost of this network to be $419,000 US.

The performance of the algorithms, in terms of the minimum total cost obtained, under the three stopping criteria, is shown in the box plots in Figures 2, 3, and 4, for the 1000-, 100000, and 40000-function evaluation stopping criteria, respectively. The results show that, for the first stopping criterion, the PSO alsgorithm exhibits the best convergence speed, and over the 20 executions of the algorithms, its minimum and mean cost is $423,000 and $435,000 US, respectively. Of the remaining algorithms, the GA and HS algorithmsresult in the lowest and second lowest mean and minimum costs, respectively, and the DE algorithm exhibits the worst performance among all algorithms.

As the stopping criterion is relaxed to 10000 function evaluations, the performance of the ABC, SLC, HS, and DE algorithms improves significantly, and each of these algorithms achieve the global optimal minimum cost. Here, the ABC, DE, and SLC algorithms have the least standard deviation, in this order. These small standard deviation values show that the algorithms perform the search process consistently, for the breadth of different initial random populations. In contrast, the PSO algorithm also achieves the global optimum, but its average performance over 20 executions is relatively poor.

Finally, for the stopping criterion of 40000 function evaluations, the box plot shown in Figure 4 indicates that the high-performance algorithms are ABC, DE, SLC, and HS, in decreasing order of performance.

4.2 Hanoi benchmark network

The Hanoi network, shown in Figure 5, consists of 32 nodes, 34 pipes with 3 loops, and is fed by gravity from a reservoir with a 100-m fixed head. All pipes have a Hazen-Williams coefficient of 130 and the minimum head limitation at all nodes is 30 m above ground level (Alperovits and Shamir 1977). There are six possible pipe diameters and 34 pipes in the system, thus the degree of candidate diameter is equivalent to *6 / 34 =0.1765*. The global optimum solution has the total cost of $6.1 million US (Alperovits and Shamir 1977).

Optimization results for this network are provided in Figures 6, 7, and 8 for the 1000-, 10000-, and 40000-function evaluation stopping criteria, respectively. For the first criterion, the SLC algorithm exhibits the highest performance. with a the minimum and mean least cost equal to $7.0 and $8.3 million, respectively. It also has the smallest standard deviation. The PSO and GA algorithm obtain minimum costs of $11 and $8.4 million US, respectively. For the 10000-function evaluation criterion, the SLC also exhibits the best performance, and has a minimum and mean least cost of $6.1 and $6.3 million US, respectively. The DE algorithm has the smallest standard deviation, but does not find the lowest cost overall. The GA and DE algorithms are the second and third best algorithms, respectively. A similar trend is exhibited for the 40000-function evaluation criterion, where the SLC and DE algorithms exhibit the highest performance, in this order, and PSO exhibits the worst performance. This indicates that the convergence properties of the PSO algorithm do not improve when the number of iterations increases. A comparison of the Hanoi and Two-Loop benchmark networks indicates that as the degree of candidate diameter decreases, the convergence potential of the SLC algorithm improves significantly.

4.3 Farhadgerd network

The Farhadgerd WDN serves a population of approximately 8200 in the town of Farhadgerd, Iran, a residential community near the regional capital, Mashhad. The network shown in Figure 9, comprises 64 pipes and 53 nodes, has one reservoir with a head of 510 m, and a minimum pressure requirement of 20 m at all nodes. There are nine possible pipe diameters and thus the degree of candidate diameter for this network is equivalent to *9 / 64 =0.14*. The recorded optimal cost for Farhadgerd is $120 million US.

Optimization results for this network are provided in Figures 10, 11, and 12 for the 1000-, 10000-, and 40000-function evaluation stopping criteria, respectively. For the first criterion, the SLC surpasses the other algorithms with a minimum and mean least cost of $210 and $240 million US, respectively. The ABC algorithm exhibits the worst performance by far with respect to the others under this criterion. Among the other algorithms, the mean result of the PSO algorithm indicates that it performs second-best, with a minimum and mean least cost of $260 and $290 million US, respectively. For the 10000-function evaluation criterion, the SLC algorithm again out-performs all other algorithms, with a minimum and mean least cost of $120 and $160 million US, respectively. The GA algorithm performs second-best, under this criterion. The DE algorithm results are comparable with other algorithms, and exhibit the smallest standard deviation, however, it is unable to find a global optimum solution. These findings are similar for the cases with a stopping criterion of 40000 function evauations. The minimum and maximum least cost solutions obtained by the SLC algorithm are $120 and $140 million US, respectively, which are less than all solutions obtained by other algorithms. This confirms that for the range of networks investigated in this paper, the performance of the SLC improves as the degree of candidate diameter decreases, though the DE algorithm has the lowest standard deviation.

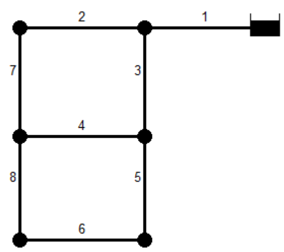

Figure 1: Two-Loop benchmark network

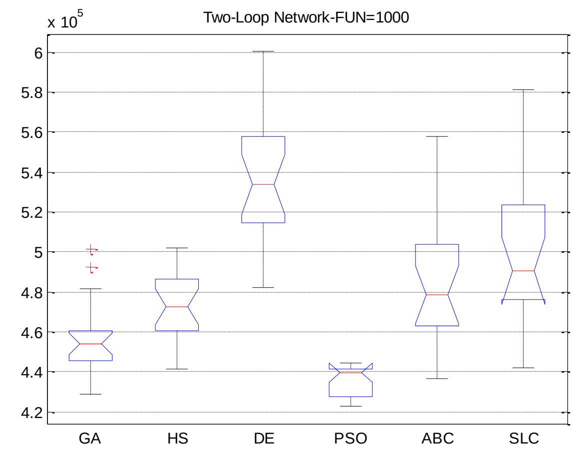

Figure 2: Box-plot of least cost solutions for the Two-Loop benchmark network: 1000 function evaluations

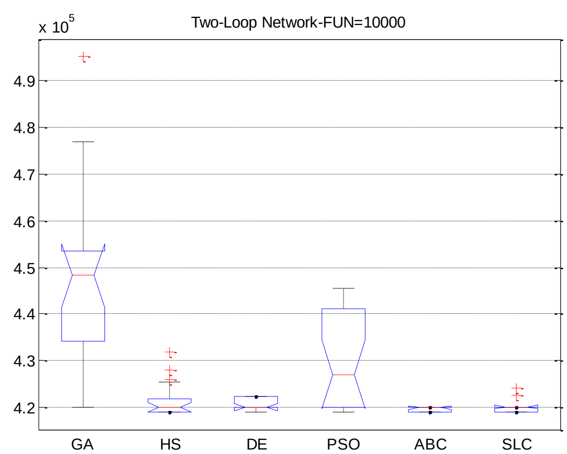

Figure 3: Box-plot of least cost solutions for the Two-Loop benchmark network: 10000 function evaluations

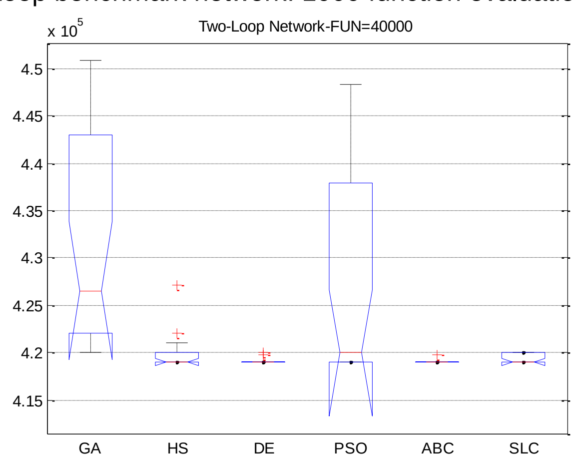

Figure 4: Box-plot of least cost solutions for the Two-Loop benchmark network: 40000 function evaluations

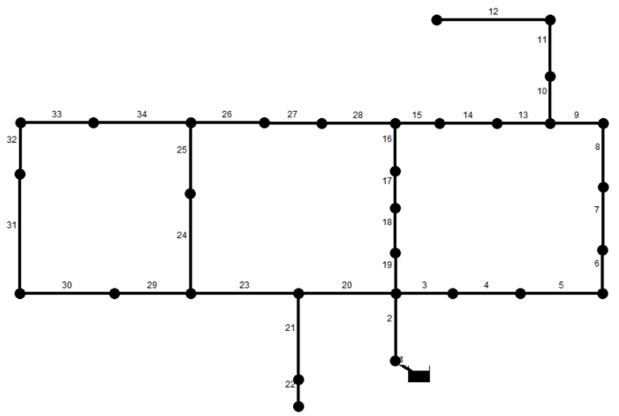

Figure 5: Hanoi benchmark network

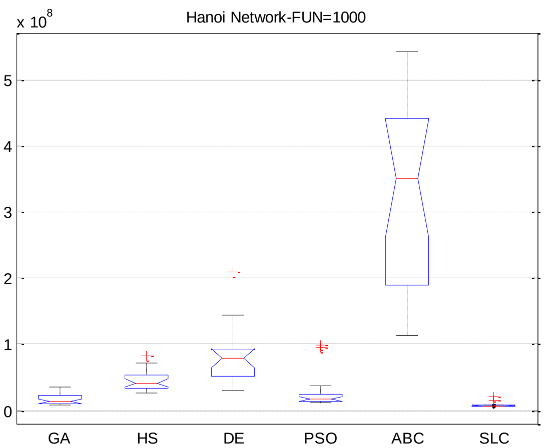

Figure 6: Box-plot of least cost solutions for the Hanoi benchmark network: 1000 function evaluations

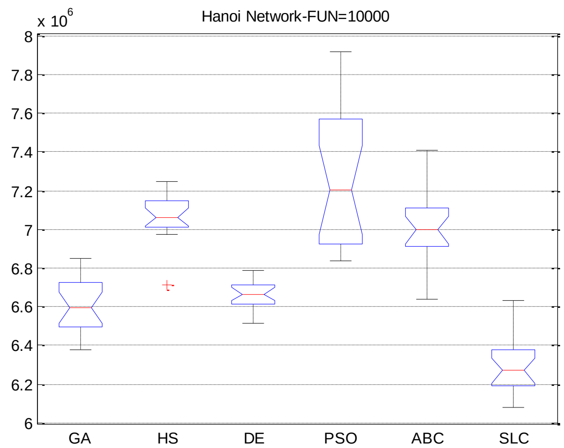

Figure 7: Box-plot of least cost solutions for the Hanoi benchmark network: 10000 function evaluations

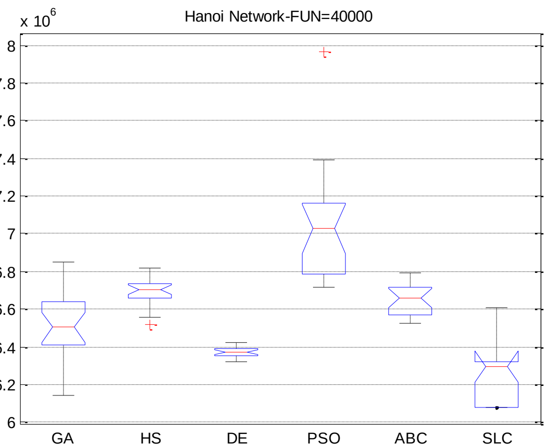

Figure 8: Box-plot of least cost solutions for the Hanoi benchmark network: 40000 function evaluations

5 CONCLUSION

In this paper, six EAs are evaluated for their performance in finding the optimal least-cost design of water distribution networks, under three pre-defined stopping criteria based on function evaluations. The performance is shown to be dependent on the stopping criteria, and on the degree of candidate diameter, which is an indication of the size and complexity of the network. For larger benchmark networks, the SLC, ABC, and DE algorithms result in better performance. For the Farhadgerd WDN, with a large number of pipes, and small candidate diameter, the SLC algorithm exhibits the best performance in terms of mean and minimum total cost.

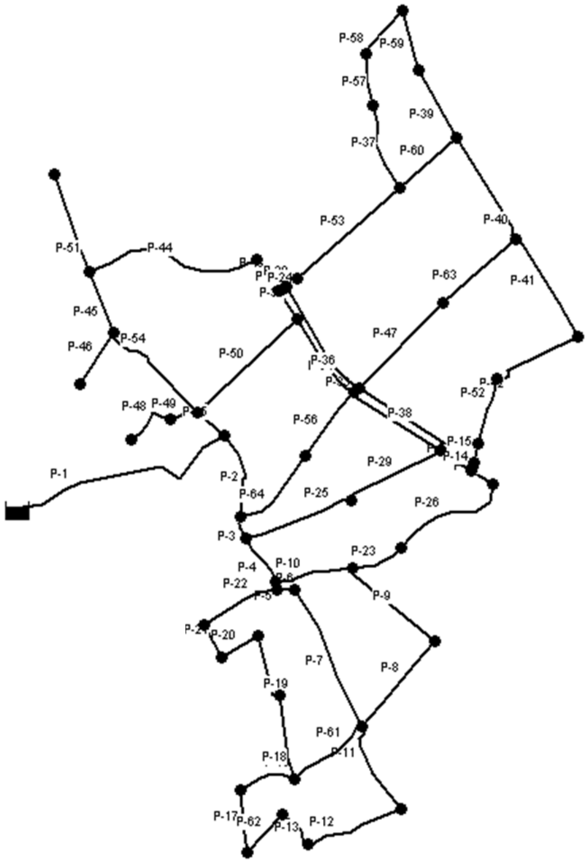

Figure 9: City of Farhadgerd network

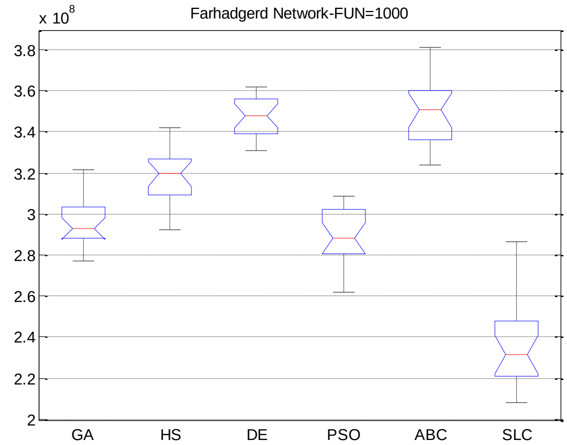

Figure 10: Box-plot of least cost solutions for the City of Farhadgerd network: 1000 function evaluations

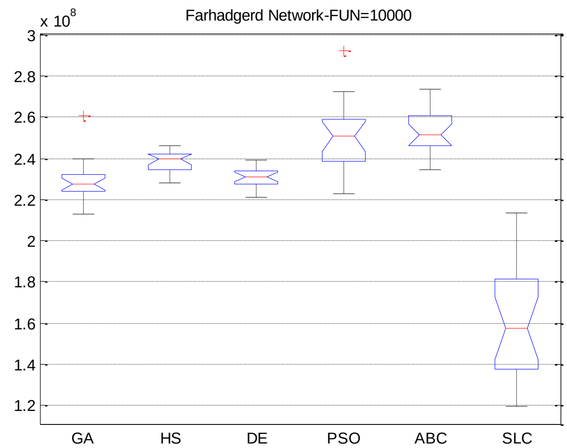

Figure 11: Box-plot of least cost solutions for the City of Farhadgerd network: 10000 function evaluations

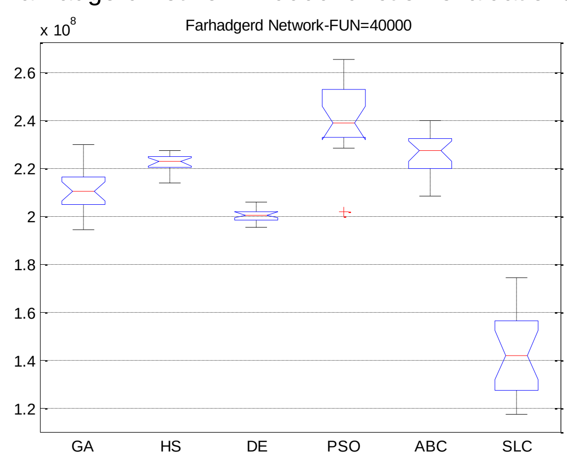

Figure 12: Box-plot of least cost solutions for the City of Farhadgerd network: 40000 function evaluations

6 REFERENCES

Alperovitz, E. and Shamir, U., 1977. Design of Optimal Water Distribution Systems. *Water Resour. Res.,* **13**(6): 885–900.

Bi. W., Dandy, G.C. and Maier, H.R. 2015. Improved Genetic Algorithm Optimization of Water Distribution System Design by Incorporating Domain Knowledge, *Environmental Modelling & Software,* **69**: 370–381.

Cunha, M.C. and Sousa, J. 2001. Hydraulic Infrastructures Design Using Simulated Annealing, *J. Infrastruct. Syst.,* **7**(1): 32–38.

Holland, J. H. 1975. Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence. Ann Arbor, MI: University of Michigan Press, 1975.

Eusuff, M.M. and Lansey, K.E. 2003. Optimisation of Water Distribution Network Design Using Shuffled Frog Leaping Algorithm, *J. Water Resour. Plan. Manag.,* **129**(3): 210–225.

Geem, Z.W. 2006. Optimal Cost Design of Water Distribution Networks Using Harmony Search, *Eng. Optim.*, **38**(3): 259–280.

Karaboga, D. 2005. An Idea Based On Honey Bee Swarm For Numerical Optimization, Technical Report-TR06, Erciyes University, Engineering Faculty, Computer Engineering Department, 2005.

Kennedy, J. and Eberhart, R.C. 1995. Particle Swarm Optimization, *in: Proceedings of the 1995 IEEE International Conference on Neural Networks*, **4**: 1942–1948.

Lin, M.D., Liu, Y.H., Liu, G.F. and Chu, C.W. 2007. Scatter Search Heuristic For Least-Cost Design of Water Distribution Networks, *Eng. Optim.,* **39** (7): 857–876.

Maier, H.R., Simpson, A.R., Zecchin, A.C., Foong, W.K., Phang, K.Y., Seah, H.Y. and Tan, C.L. 2003. Ant Colony Optimization For The Design of Water Distribution Systems, *J, Water Resour. Plan. Manag.,* **129** (3): 200–209.

Moosavian, N and Roodsari, B.K. 2014. Soccer League Competition Algorithm: A Novel Meta-Heuristic Algorithm For Optimal Design of Water Distribution Networks, *Swarm and Evolutionary Computation*, **17**: 14-24.

Murphy, L.J. and Simpson, A.R. 1992. Pipe Optimization Using Genetic Algorithms, Research Rep. No. **R93**, Dept. of Civil Engineering, Univ. of Adelaide, Adelaide, Australia.

Perelman, L. and Ostfeld, A. 2007. An Adaptive Heuristic Cross-Entropy Algorithm For Optimal Design of Water Distribution Systems, *Eng. Optim.,* **39** (4): 413–428.

Price, K.V., Storn, R.M. and Lampinen, J.A. 2005. (Eds.), Differential Evolution: A Practical Approach to Global Optimization, Springer Natural Computing Series.

Suribabu, C.R. and Neelakantan, T.R. 2006. Design of Water Distribution Networks Using Particle Swarm Optimization, *Urban Water J.,* **3** (2): 111–120.

Todini, E., and Pilati, S.1988. A Gradient Algorithm for the Analysis of Pipe Network, *Computer Applications in Water Wupply, **1**, System Analysis and Simulation,* B. Coulbeck and O. Chu-Hou, eds., Wiley, London, 1–20.

Vasan, A. and Simonovic, S. 2010. Optimization of Water Distribution Network Design Using Differential Evolution, *J. Water Resour. Plan. Manag.,* **136** (2): 279–287.

Wolpert, D.H. and Marcready W.G. 1997. No Free Lunch Theorems For Optimization, *IEEE Transactions On Evolutionary Computation*, **1**(1): 67-82.

Zheng, F., Zecchin, A.C. and Simpson, A.R. 2013. Self-Adaptive Differential Evolution Algorithm Applied to Water Distribution System Optimization, *J. Comput. Civil Eng.,* ASCE, **27**(2): 148–158.