



SIMULATION OF NEGATIVELY BUOYANT FOUNTAINS USING DATA MINING METHODOLOGY

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ABSTRACT

Data mining and boundary visualization techniques were employed to model the hydrodynamics of negatively buoyant fountains. Experimental and numerical results of two different fountain types of axisymmetric and plane fountains were selected from the literature for numerical simulation. Fountain characteristics such as penetration height y_m , fountain width x_w , and the thickness of the temperature layer ΔT were considered. Experimental studies in the literature indicated that the fountain characteristics can be correlated with non-dimensional parameters such as Froude, Reynolds, and Prandtl numbers. All proposed empirical models from the literature are nonlinear functions of Froude and Reynolds numbers. This nonlinearity causes a considerable prediction error. The M5P model tree was used for prediction of fountain characteristics such as penetration height and penetration width of negatively buoyant fountains. The selected model restructures non-linear correlations into a tree of linear models. It was found that all model trees accurately predict fountain parameters with maximum 3% error for y_m and x_w . Different fountain shapes were formed based on variations of Froude and Reynolds numbers. Regime classification was performed using boundary visualization. Different classifiers in Weka software were tested for boundary visualization to define regime boundaries. It was found that the Logistic classifier can properly define the boundary of different fountain flow regimes.

Keywords: Data mining, Weka, Negatively buoyant jets, fountains, modeling

1. INTRODUCTION

Fountain structure forms when a dense fluid is injected upward into another fluid of different density. In this condition the buoyancy force opposes the momentum of the flow. The injected fluid penetrates a distance into the ambient fluid before stagnating and falling back around itself. The characteristics of the fountain flow, such as the penetration height and mixing strength, depend on the Froude and Reynolds numbers (Williamson et al., 2008). Round fountain structure can be classified using Froude, Reynolds and Prandtl numbers as

$$[1] \quad Fr = \frac{u_o}{\sqrt{R_o g}}$$

$$[2] \quad Re = \frac{u_o R_o}{\nu}$$

$$[3] \quad \text{Pr} = \frac{\nu}{k}$$

where R_o is the radius of the nozzle, u_o is the characteristic velocity, g' (i.e., $g' = g(\rho_o - \rho_a)/\rho_a$) is the reduced gravity between the fountain source with a density of ρ_o and the ambient fluid with a density of ρ_a , g is acceleration due to gravity, ν is the kinematic viscosity of the fluid at the fountain source and k is the thermal diffusivity.

Fountain flow are categorized as weak and forced fountains. Weak fountains are generally steady and symmetric (Lin and Armfield, 2000a) and forced fountains are unsteady with stronger mixing capability with the ambient fluid (Turner 1966, Friedman et al., 2007). For a fountain with a weak discharge rate, the discharge momentum is less than the negative buoyancy and the flow is characterized when $\text{Fr} \leq 1.0$. Forced fountains occur for $\text{Fr} \geq 3$ (Kaye & Hunt 2006, Lin and Armfield, 2010). Transition fountains are classified for $1 < \text{Fr} < 3$. Weak fountains occur in the replenishment of cold water in solar ponds, in the melting of magma chamber roofs as well as many other environmental and industrial settings. Weak fountains are further classified to as axisymmetric and plane fountain (Lin and Armfield, 2000a). The main differences between axisymmetric and plane fountains are that the latter penetrate to a greater height, have a greater spread and take longer to achieve a steady state (Williamson et al., 2008, Lin and Armfield, 2000b).

In this paper, experimental data for weak negatively buoyant fountains were modeled using data mining methodology. Experimental data of Williamson et al. (2008) and numerical simulation of Lin and Armfield (2000c) were used for data mining simulation. Williamson et al. (2008) performed experiments by injecting salt water into fresh water. The saline water is fed from a header tank to the base of the fresh-water tank as shown in Figure 1. The water is injected from a sudden start and maintained at a constant flow rate throughout the experiment. The fountain inlet flow rate, inlet pipe diameter and the salinity of the inlet fluid were varied to cover a wide range of Reynolds and Froude numbers. The volume flow rates varied from 0.06 to 20 cm^3/s . Nozzle diameters ranging from 0.54 mm to 4.80 mm were used. The density ratio range was varied with $0.004 < (\rho_o - \rho_a)/\rho_a < 0.16$. The kinematic viscosity of saline water was varied in the range of $1.01 \times 10^{-6} < \nu < 1.4 \times 10^{-6} \text{ m}^2/\text{s}$. The flow was recorded with two digital cameras at a rate of 15 frames per second. For very weak fountains ($0.2 \leq \text{Fr} \leq 1.0$) a vertical cylindrical container considered for axisymmetric fountains whereas for a plane fountain it was a rectangular container. Numerical data of Lin and Armfield (2000c) have been carried out for $0.0025 \leq \text{Fr} \leq 0.2$ with $\text{Re} = 200$ and $\text{Pr} = 7$ to study the effects of Froude number. The thickness of the temperature layer ΔT was determined from temperature profile and it normalized with the nozzle radius.

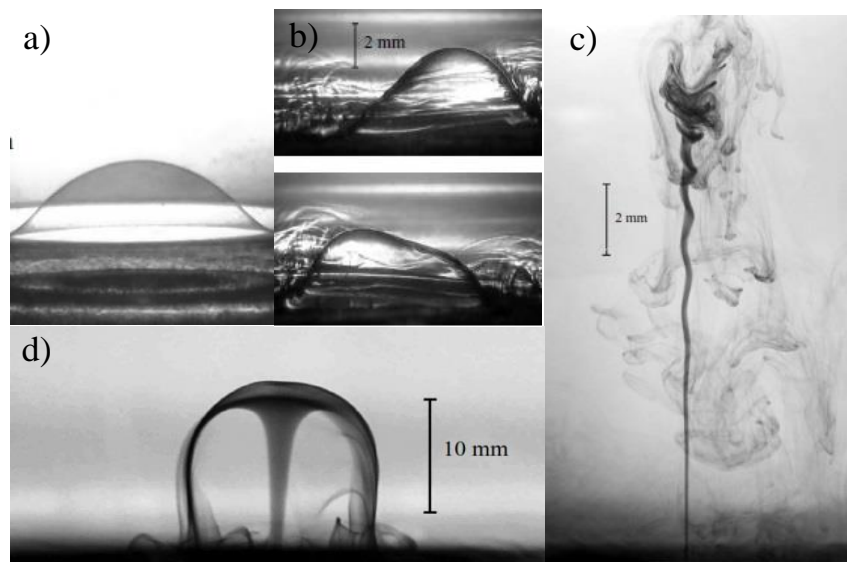


Figure 1. Visualization of axisymmetric fountain at different flow conditions; a) weak steady axisymmetric fountain ($\text{Re} = 21$ and $\text{Fr} = 0.71$); b) flapping fountain ($\text{Re} = 72$ and $\text{Fr} = 1$); c) sinuous behaviour ($\text{Re} = 105$ and $\text{Fr} = 105$); d) fountain behaviour ($\text{Re} = 26$ and $\text{Fr} = 26$). Images were taken from (Williamson et al., 2008).

2. MODELLING BACKGROUND

Knowledge Discovery in Data (KDD) or data mining is a computational process to extract meaningful patterns and rules from large number of variables and data using Weka software. Weka software written in Java language and it has a collection of machine learning algorithms to extract the relationship between different parameters. The classifiers in Weka enable to prepare classification, regression algorithms and estimate the accuracy of the proposed model. Different classifiers were implemented in Weka for data classification named as Bayesian, Trees, Rules, Functions and Lazy classifiers (Witten and Frank, 2005). Attributes in classifiers are categorized into numeric and nominal. Numeric attributes are variables such as Froude, Reynolds and Prandtl numbers and nominal attributes can be defined to be certain such as different shapes and regimes of negatively buoyant fountains.

Weka software used the model tree algorithm for flow classification. The main goal of the model tree approach is the process of dividing complex problems into smaller problems (Bhattacharya et al., 2007). Model trees are accurate, understandable, and easy to train and it can be employed as a robust method for classification, prediction and dealing with missing data (Witten and Frank, 2005; Jung et al., 2010). One of the most famous approaches of model tree simulation is M5 algorithm which was initially introduced by Quinlan (1992). Two main procedures are involved in the algorithm as building the tree and inferring knowledge from it (Jung et al., 2010; Etemad-Shahidi et al., 2010). M5P is an improved version of the M5 algorithm which is proposed by Wang and Witten (1997). The new version has a similar structure to the M5 algorithm which produces easier trees and effectively deals with missing values and enumerated attributes (Jung et al., 2010; Etemad-Shahidi et al., 2010).

The M5P algorithm generally consists of three main steps as building the tree, pruning the tree and smoothing it. One distinct advantage of this mechanism is more transparent than other machine learning algorithms such as artificial neural network (ANN), k nearest neighbouring (kNN) and logistic regression. Therefore, it is easy to follow a tree structure to understand how a decision has been made (Pedrycz and Sosnowski, 2001). Other advantages of model trees are that they are more accurate than ANN, easy to train and robust when dealing with missing data (Witten and Frank, 2005). Figure 2 is an example of regime classification of negatively buoyant fountain from experimental study of (Williamson et al., 2008) based on variations of Reynolds and Froude numbers.

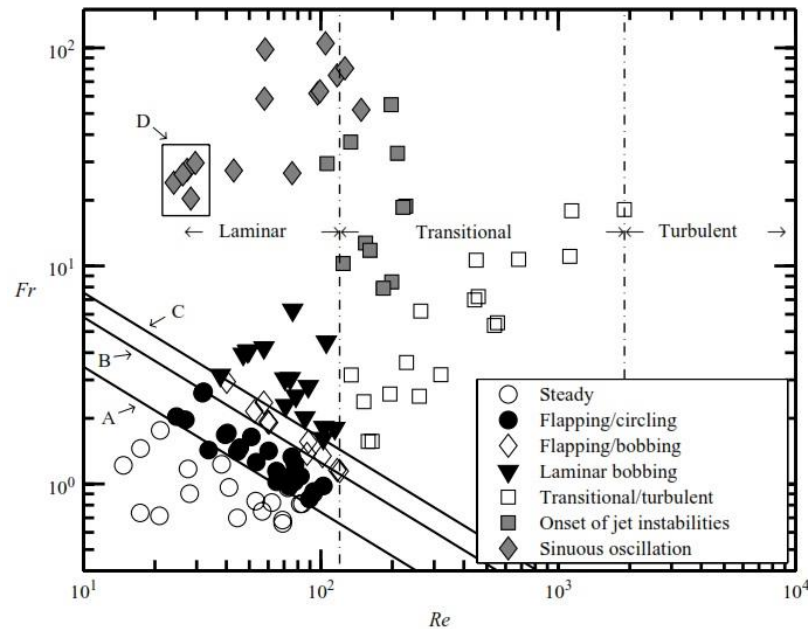


Figure 2. Regime classification of fountain behaviour with non-dimensional parameters of Re and Fr . Solid lines $FrRe^{2/3}=C$ where the constant C is: $A=16$; $B=27$; $C=35$. (From Williamson et al., 2008).

3. RESULTS AND DISCUSSION

3.1 Fountain Classification using Boundary Visualization

Boundary visualization technique in Weka software was employed for regime classification of negatively buoyant fountains. Many classifiers are available in Weka software such as Bayesian, Trees, Rules, Functions and Lazy classifiers (Witten and Frank, 2005). It is important to select a proper classifier for boundary visualization. Figure 3 shows two boundary visualization maps of data points in the study of Williamson et al. (2008). Figure 3a shows the boundary visualization using one of Trees classifiers (i.e., J48 classifier). Comparison of the boundary visualization with J48 and experimental data (see Figure 2) indicates that the J48 classifier is not able to properly define regime boundaries. Many classifiers from different categories in Weka software were tested to find the best match between experimental regime map and boundary visualization. Figure 3b shows the boundary visualization results using function/logistic classifier. As can be seen, the classifier properly defined the boundaries of each fountain regimes.

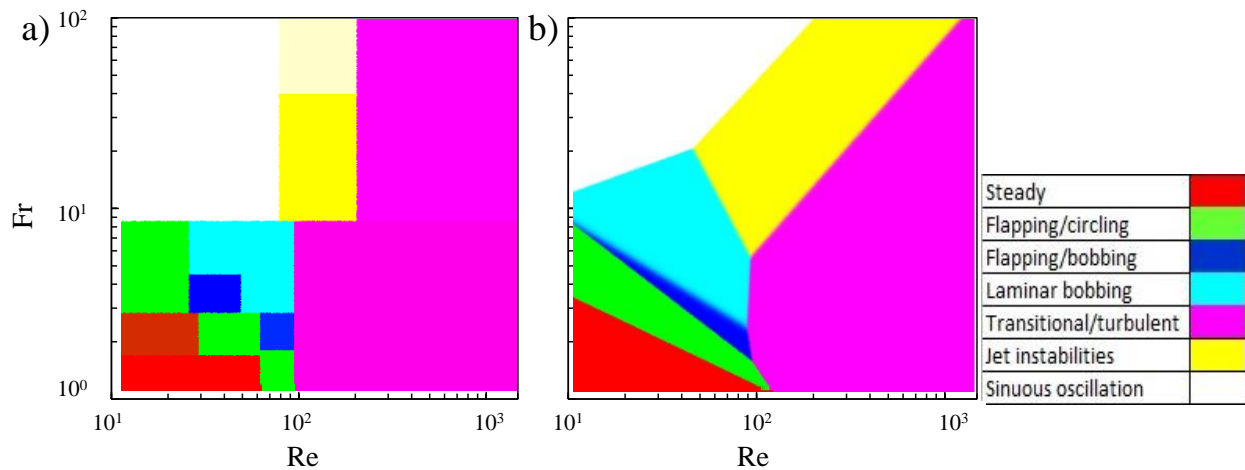


Figure 3. Boundary visualization map of fountain behaviour with Re and Fr numbers by Weka software, a) Tree-J48 classifier, b) Function-Logistic classifier.

3.2 Model Tree and Fountain Characteristics

The M5P model was used for correlating the fountain's characteristics with the mentioned non-dimensional parameters. Fountain height y_m , fountain width x_w and the thickness of temperature layer ΔT were correlated with Re, Pr, and Fr. The number of branches in model tree indicate the complexity of correlation between measured parameters and non-dimensional numbers. Figure 4 shows the model trees of y_m , x_w and ΔT based on non-dimensional parameters. As can be seen in Figure 4, linear modeling of the fountain height is more complex than fountain width since M5P defined five linear models for y_m whereas for x_w and ΔT , two linear equations were defined. As can be seen in Figure 4a, three linear models (LM1, LM4, and LM5) were classified based on Fr. For $Fr \leq 0.078$, the proposed linear models were classified based of the value of Re. As can be seen in Figure 4a, Prandtl number had no impact on regime classification and Reynolds number effect became important for very low values of Froude number. Coefficients of linear models (i.e., η_1 , η_2 , η_3 and C) for prediction of y_m , x_w , and ΔT for both axisymmetric and plane fountains are listed in Tables 1, 2, and 3.

As can be seen in Table 1, y_m is correlated with Fr and Re in all five linear equations whereas for x_w , linear equations are a function of Froude number (see Table 2). Correlation of the thickness of temperature layer ΔT with non-dimensional numbers indicates that for $Re \leq 150$, the thickness of temperature layer is only a function of Re whereas for $Re > 150$, ΔT correlates with Re, Fr, and Pr.

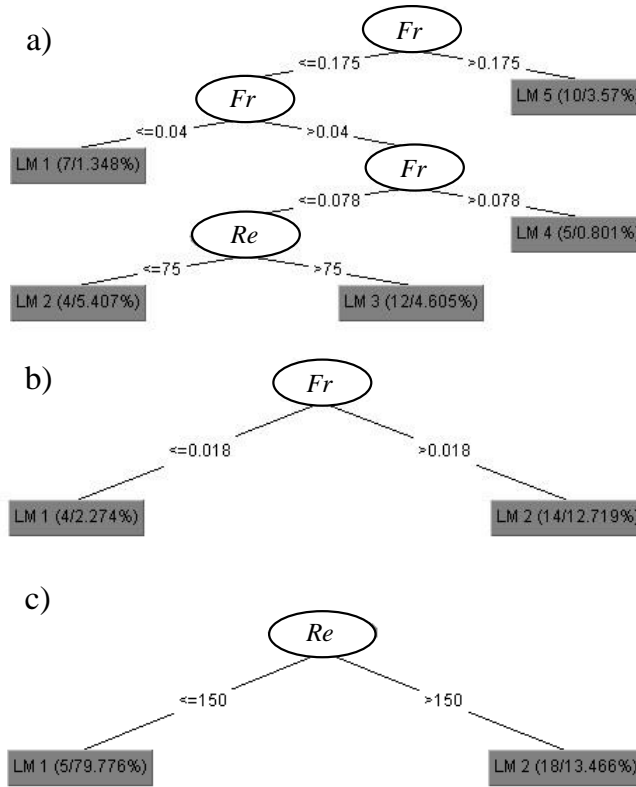


Figure 4. Structure of model trees constructed by M5P and linear models based on numerical data of Lin and Armfield (2000c); a) Model tree for y_m , b) Model tree for x_w , c) Model tree for ΔT .

Table 1. Performance of M5P classification model in form of series of linear models to predict y_m .

Parameter	Linear model	η_1	η_2	C
y_m (axisymmetric)	LM1	-0.0001	2.1335	0.085
	LM2	-0.0002	1.3624	0.1334
	LM3	-0.0001	1.3624	0.1277
	LM4	-0.0001	1.703	0.1107
	LM5	-0.0001	1.227	0.1695
y_m (plane)	LM1	-0.0001	3.0256	0.1242
	LM2	-0.0002	2.0304	0.1893
	LM3	-0.0001	2.0304	0.1817
	LM4	-0.0001	2.5476	0.1538
	LM5	-0.0001	1.9695	0.2225

* $LM = (\eta_1 Re) + (\eta_2 Fr) + C$

Table 2. Performance of M5P classification model in form of series of linear models to predict x_w .

Parameter	Linear model	η_1	C
x_w (axisymmetric)	LM1	1.1913	1.087
	LM2	0.8869	1.1001
x_w (plane)	LM1	2.4042	1.1203
	LM2	1.7612	1.1499

* $LM = (\eta_1 Fr) + C$

Table 3. Performance of M5P classification model in form of series of linear models to predict ΔT .

Parameter	Linear model	η_1	η_2	η_3	C
ΔT (axisymmetric)	LM1	-0.0011	0	0	0.2411
	LM2	-0.0002	-0.0102	0.2668	0.1874
ΔT (plane)	LM1	-0.0015	0	0	0.3069
	LM2	-0.0002	-0.0137	0.357	0.2397

* $LM=(\eta_1 Re)+(\eta_2 Pr)+(\eta_3 Fr)+C$

Performance of M5P model tree to simulate fountain height of both axisymmetric and plane fountains is shown in Figure 5. Comparison of the M5P predictions with numerical data of Lin and Armfield (2000c) indicated that the M5P model is able to accurately predict the fountain height with $\pm 0.5\%$. The M5P model over-predicts y_m for fountains with very small Fr values (i.e., LM1). Figure 6 shows the performance of M5P model for fountain width for both negatively buoyant axisymmetric and plane fountains. As can be seen, the M5P model over-predicts the experimental data by 2%.

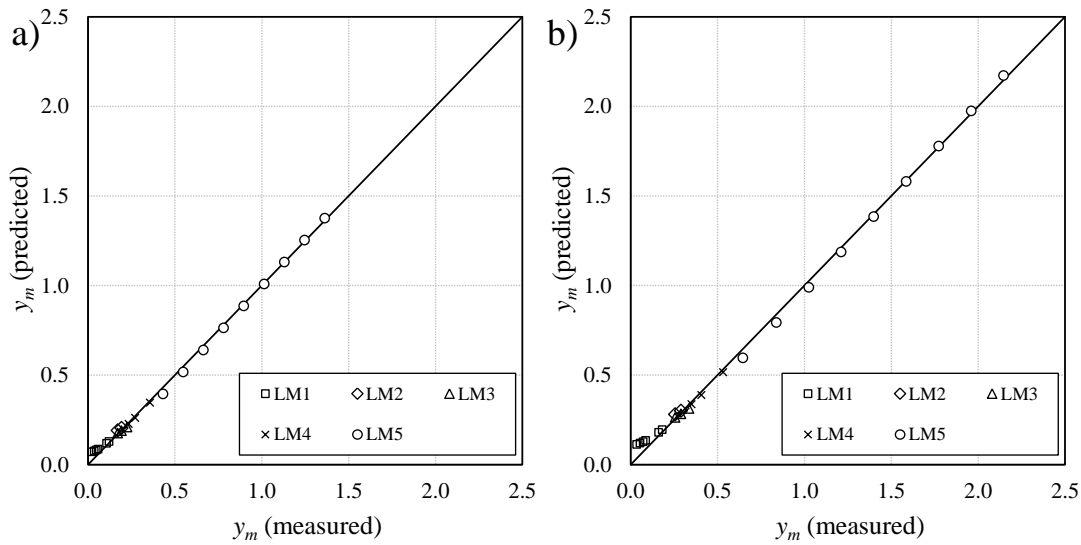


Figure 5. Performance of M5P model tree to simulate fountain height; a) axisymmetric fountain, b) plane fountain.

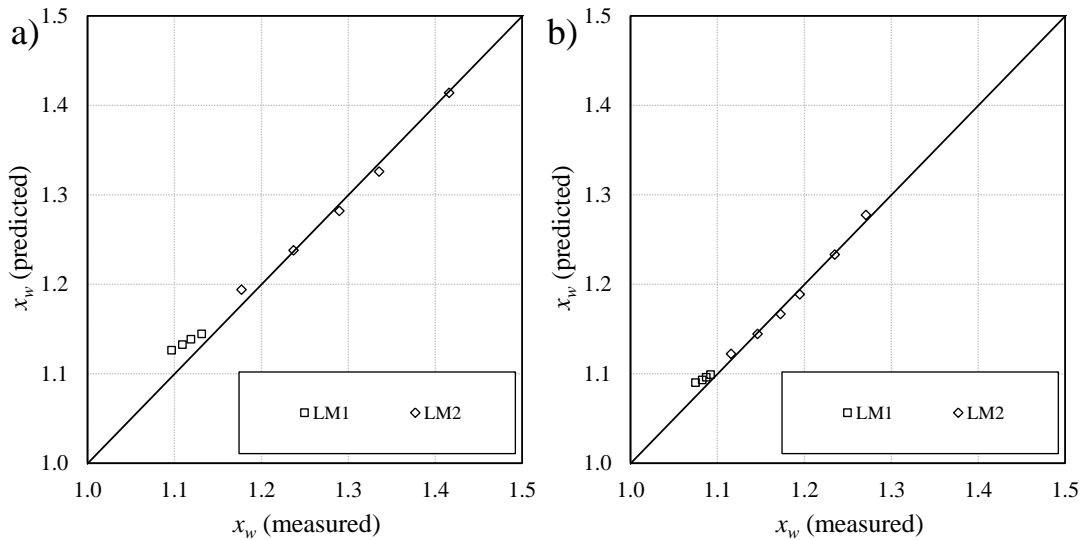


Figure 6. Performance of M5P model tree to simulate fountain width; a) axisymmetric fountain, b) plane fountain.

More data scatter were found on correlation between model predictions and experimental data for ΔT since the thickness of the temperature layer is a function of all non-dimensional parameters (i.e., Re, Fr, Pr) in this study. Lin and Armfield (2000c) shows that the thickness of the temperature layer is a function of Re, Fr, and Pr as

$$[4] \quad \Delta T \approx \frac{Fr^{2/3}}{(Re Pr)^{2/3}}$$

Figure 7 shows the performance of M5P model on prediction of ΔT for both negatively buoyant axisymmetric and plane fountains. The average prediction uncertainty for LM2 was $12\% \pm 15\%$.

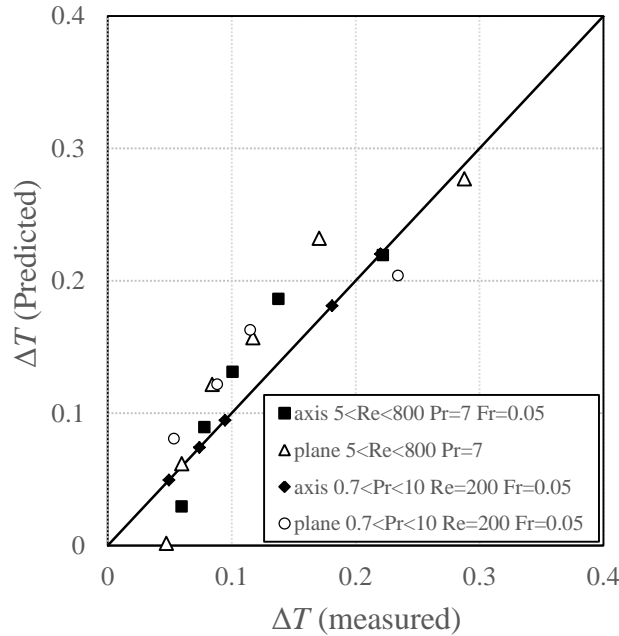


Figure 7. Performance of M5P model tree to simulate the thickness of the temperature layer, ΔT .

4. SUMMARY AND CONCLUSION

Data mining methodology was employed for regime classification and modeling the main characteristics of negatively buoyant fountains. For model validation, data points for both axisymmetric and plane fountains were used from experimental studies of Williamson et al. (2008) and numerical modeling of Lin and Armfield (2000c). Non-dimensional parameters such as Reynolds, Froude, and Prandtl numbers were defined as the numeric data entry for data mining using Weka software. Regime classes such as steady, flapping, bobbing, laminar bobbing, transitional, onset of jet instability, and sinuous oscillation were selected as nominal data entry. Boundary visualization technique in Weka software was shown that the accuracy of the regime map is directly related to the type of classifier. It was found that the function/logistic classifier can accurately determine the regime map.

Three fountain parameters named as fountain height y_m , fountain width x_w and the thickness of the temperature layer ΔT were selected to study the performance of a data mining model (i.e., Weka) to simulate fountain characteristics. A robust model tree (i.e., M5P) was selected amongst other model tree algorithms in Weka software to predict fountain characteristics. Each model tree provided different numbers of linear equation to cover the entire range of data. It was found that the M5P model can accurately simulate y_m and x_w with 0.5% variations. Higher uncertainty level of prediction to model the thickness of the temperature layer ΔT was found due to complexity of the linear model. It was found that the M5P model can predict the thickness of the temperature layer with $12\% \pm 15\%$ accuracy.

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