



PARTICLE BREAKAGE AND CONSTITUTIVE MODELING OF GRANULAR SOIL-STRUCTURE INTERFACES

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Abstract: Particle breakage occurs in granular soil-structure interface zones in different soil-structure interaction (SSI) systems under cyclic loading, and it largely affects the performance of these structures. The effect of particle breakage on mechanical behavior of granular soil-structure interfaces should be taken into account in an interface constitutive model for accurate simulating the SSI problems with critical interfaces. In this study, the effect of particle breakage on the mechanics of granular soil-structure interface is presented. An approach is then proposed to improve an elasto-plastic interface constitutive models to be capable of simulating particle breakage under shear loads. The effectiveness of the approach is evaluated using an interface constitutive model to predict the volumetric behavior of a granular soil-structure interface under different stress path. The model predictions are then compared with experimental observations. It is illustrated that the particle breakage has significant effect on the compressibility behavior of the granular soil-structure interface, and it can be well addressed within the constitutive formulations of an interface plasticity model.

1 Introduction

In geo-structures such as shallow and deep foundations, retaining walls, underground tunnels, buried pipelines and embankment dams, the transition zone between the soils and the structure, known as interface, may present a critical load transfer mechanism. This mechanism involves the interaction between the soils and the structure, and could play an important role in evaluating the performance of many soil-structure interaction systems. A trace of the particle behavior during stress transformation in contact zones between granular soils and structural materials indicated that the interface consists of a shear zone with a thickness of about 5-10 times the average grain size of the granular soil (D_{50}) (Uesugi et al. 1988; Hu and Pu 2004; DeJong and Westgate 2009). The design and performance of these geo-structures largely depend on mobilized shear strength and volumetric behavior at the interface between the soil and the structure. One of the important phenomenon in behavior of soil-structure interface systems is breakage of granular soil particles under shearing. The significance of particle breakage in granular soil-structure interface under cyclic loading has been recognized by some researchers as it results in additional contraction of the interface zone during shear cycles (Uesugi et al. 1989, 1990, Zhang and Zhang 2006, 2009). Thus, understanding the mechanics of particle breakage in granular soil (sandy and coarse grained)-structure interface under different loading conditions and its effect on the volumetric behavior and stress transfer mechanism between the soil and structure may be of important elements for the design of soil-structure interaction (SSI) systems. Moreover, in the numerical study of SSI problems, the proper simulation of this interface zone and its accumulative contraction leads to a better prediction of the responses to cyclic loading.

The existing soil-structure interface constitutive models for numerical simulation of these zones can be categorized into two main groups: nonlinear elastic (e.g. Clough and Duncan 1971; Desai et al. 1985; Desai and Nagaraj 1988) and elasto-plastic (e.g. Ghaboussi et al. 1973; Desai and Nagaraj 1988; Desai and Ma

1992; Shahrour and Rezaie 1997; Fakharian and Evgin 2000; Ghionna and Mortara 2002; Mortara et al. 2002; Zeghal and Edil 2002; Hu and Pu 2004; Liu et al. 2006; Liu and Ling 2008; Zhang and Zhang 2008; D'Aguiar et al. 2011; Lashkari 2012, 2013; Lashkari and Kadivar 2016; Saberi et al. 2016; Stutz et al. 2016). However, a few of these interface constitutive models have been proposed to simulate the effects of particle breakage (Zeghal and Edil 2002; Liu and Ling 2008; Liu et al. 2014).

In this study, the effect of particle breakage on the mechanics of granular soil-structure interfaces is explained. Then, an approach is proposed to improve an elasto-plastic constitutive model to be capable of considering particle breakage in numerical simulation. The effectiveness of this approach in predicting the volumetric behavior of a granular soil-structure interface under different stress paths is evaluated by comparing with available laboratory observations.

2 Particle Breakage in Granular Soil-Structure Interface

Breakage of granular soil particles is an important phenomenon in both monotonic and cyclic behavior of soil-structure interface systems. Particle breakage affects the strength, compressibility, volumetric behavior, and permeability of soils (Lade et al. 1996; Zeghal and Edil 2002). Factors such as mineralogy, grain size, shape and angularity, grain size distribution, stress path and stress level have been identified to play important role in aggregating particle breakage (Lade et al. 1996; Yasufuku and Ochiai 2005; Daouadji and Hicher 2010). As the particles begin to break, the voids between grains are filled by crushed particles and the grain size distribution changes. Based on experimental observations, badly-graded granular soils experience more particle breakage compared with well-graded ones (Daouadji et al. 2001). This is due to their lower relative density and less number of contacts between particles (Lade et al. 1996; Daouadji and Hicher 2010). Furthermore, as the size of particles in granular soils increases, the amount of particle breakage also increases; larger particles are known to possess significant internal flaws which make them more prone to breakage under stress (Lade et al. 1996; Yasufuku and Ochiai 2005).

Quantifying the degree of particle breakage of both individual grains and the soil medium could help better understand soil and interface properties under loading. To do this, a number of approaches have been proposed on the basis of particle size changes (Marsal 1967; Lee and Farhoomand 1967; Hardin 1985; Lade et al. 1996; Yasufuku and Ochiai 2005). Marsal (1967), and Lee and Farhoomand (1967) suggested an index for degree of particle breakage based on single grain size changes. Hardin (1985) proposed another index for quantifying the degree of particle breakage based on changes of overall soil grain size distribution. The aforementioned particle breakage indexes which have been widely used in different studies are defined based on grain size distribution. Thus, correlating those indexes with basic soil properties such as shear strength is not meaningful. For instance, a soil with a specific grain distribution could have different densities and consequently different shear strength (Lade et al. 1996). To overcome this drawback in indexes for degree of particle breakage, Lade et al. (1996) stated that both confining pressure and shearing affect the level of stresses, strains and consequently degree of particle breakage. Since both stresses and strains are considered for measuring the total input energy per unit volume, the magnitude of total input energy per volume might be a very good factor for quantifying the degree of particle breakage. By using the data of experimental observations, Lade et al. (1996) proposed the degree of particle breakage as a hyperbolic function of total input energy per unit volume of specimen. In granular soil-structure interface systems, Uesugi et al. (1989) and Zeghal and Edil (2002) proposed that the degree of particle breakage is proportional to the total input energy or plastic work of the system. Their investigations in interface systems were in good agreement with the finding of Lade et al. (1996) for granular soil medium. Zhang and Zhang (2006, 2009) introduced parameter (D_c) as "thickness of crushing band" in coarse-grained soil-structure interface systems during shear cycles for representing the degree of particle breakage. The parameter (D_c) semi-quantitatively estimates the thickness of the zone with significant particle breakage. D_c increases by an increase in number of shear cycles and then stabilized after a number of cycles (Zhang and Zhang 2006, 2009).

Yasufuku and Ochiai (2005) proposed a parameter σ'_n/σ_{sf} for quantifying the degree of breakability in granular soils. σ'_n is normal effective stress and σ_{sf} is particle fragmentation strength which is particle strength at the mean effective grain size (D_{50}). In this definition, the soil grain breakability is an increasing function of ratio σ'_n/σ_{sf} . Yasufuku and Ochiai (2005) using the data of ring shear tests on the behavior of granular soil-structure interface revealed the interrelationship between the surface roughness and particle breakage on interface friction angle. This interrelationship is schematically illustrated in Figure 1. As can be

seen from Figure 1, the behavior of granular soils with low breakable particles in contact with smooth structural surfaces is like elastic-perfectly plastic, as revealed in other studies (e.g. Uesugi and Kishida 1986b; Fakharian 1996; Shahrour and Rezaie 1997; Frost et al. 2002; Hu and Pu 2004). By an increase in breakability of the granular soils, the residual friction angle mobilized at the interface increases up to internal friction angle of adjacent soil mass. This phenomenon is in good agreement with Uesugi et al. (1989). In smooth surfaces, during particle breakage, the broken particles fill the voids between larger grains. It can result in an increase in relative roughness and consequently the raise of residual interface friction angle. As can be observed in Figure 1, by increasing relative surface roughness, the softening behavior is observed and the peak and residual interface friction angles increases. However, the interface friction angles are similar to internal friction angle of adjacent soils, as confirmed in other studies (e.g. Uesugi and Kishida 1986a; Uesugi et al. 1988, 1989; Koval et al. 2011). Thus, particle breakage does not have significant effect on the friction angle of the interfaces with rough surfaces.

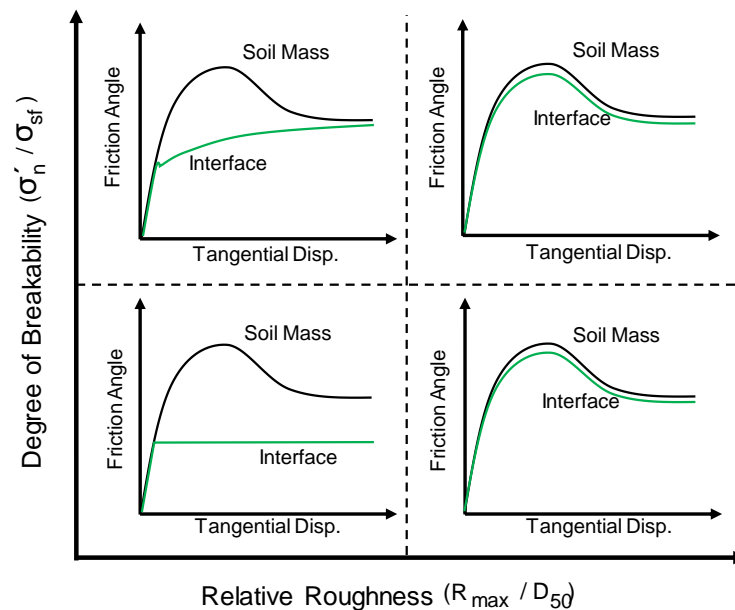


Figure 1: Schematic of the effect of particle breakage on the friction angle of granular soil-structure interface (after Yasufuku and Ochiai 2005).

Particle breakage has also considerable effect on volumetric behavior of granular soil-structure interfaces. Based on experimental observations (DeJong et al. 2003; DeJong and Westgate 2009), particle breakage reduces the normal dilatancy in interface zones as less work needs to break the particles than to rearrange them. Moreover, during cyclic loading, particle breakage reduces void ratio of the soil and increases accumulative contraction which is an important feature of granular soil-structure interface systems.

3 Constitutive Modeling of Particle Breakage

Experimental observations (Fakharian 1996; DeJong and Westgate 2009) revealed that a granular soil-structure interface similar to granular soils during monotonic loading experiences an ultimate state at large shear displacement. In this state, although the stress ratio ($\mu = \tau/\sigma_n$) is unchanged, the shear deformation increases with no changes in volume. This ultimate state is called a critical state in soil mechanics and it has been used for the first time by Liu et al. (2006) within the constitutive formulations of a sandy soil-structure interface model. Afterward, the concept of critical state soil mechanics (CSSM) for accurate simulation of interface behavior has been used in a number of constitutive model for the interface between granular soils and structural materials (Liu and Ling 2008; D'Aguiar et al. 2011; Lashkari 2012, 2013; Lashkari and Kadivar 2016; Liu et al. 2014; Saberi et al. 2016).

Previous studies have revealed that particle breakage in granular soils affects the position of the critical state line (CSL) in the plane of $e-\log p'$, where p' is the mean effective pressure (Konrad 1998; Daouadji et al. 2001). In triaxial compression tests, the CSL undergoes an abrupt change in slope at the onset of particle breakage as illustrated in Figure 2.

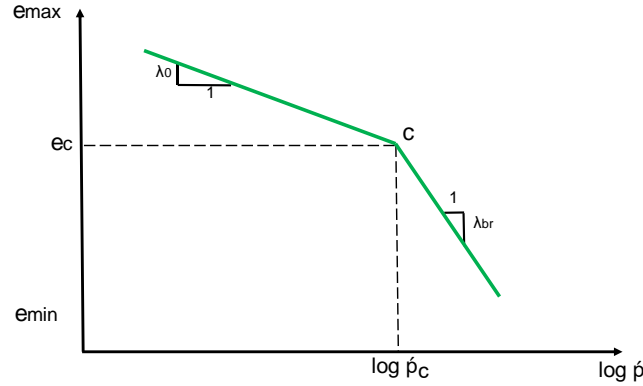


Figure 2: Particle breakage and critical state line under compression test (after Konrad 1998)

In triaxial tests, Daouadji et al. (2001) observed significant particle breakage by inducing deviatoric stress path. They found that as the uniformity coefficient ($C_u=d_{60}/d_{10}$) increases due to particle breakage, the CSL in the $e-\log p'$ plane translates downward towards smaller void ratio. This observation was also confirmed by Ghafghazi et al. (2014). That is, by inducing deviatoric stress path in triaxial tests, the CSL of granular soils in $e-\log p'$ plane moves downwards (translation towards lower void ratio) due to particle breakage. This CSL translation can be observed within the granular soils under low to medium stresses (Ghafghazi et al. 2014). In granular soil-structure interface systems, laboratory observations showed that the interface experiences a significant particle breakage under shear cycles even at low normal stresses (Zhang and Zhang 2006, 2009). Thus, particle breakage in granular soil-structure interfaces under shear cycles can result in CSL translation in $e-\ln(\sigma_n/p_{atm})$ plane.

For simulating particle breakage by CSL downward translation in $e-\log p'$ plane within the constitutive equations, the interface constitutive model should be compatible with the concept of critical state soil mechanics (CSSM). In the interface constitutive models compatible within the framework of CSSM, the location of the CSL in the plane of $e-\ln(\sigma_n/p_{atm})$ must be defined. In an interface model, the CSL can be formulated as a linear relationship in the plane of $e-\ln(\sigma_n/p_{atm})$ which is a widely used relationship in soil mechanics (Liu et al. 2006; Liu and Ling 2008; Lashkari 2013; Lashkari and Kadivar 2016; Saberi et al. 2016) given by Eq. 1.

$$[1] \quad e_{cs} = e_0 - \lambda \ln\left(\frac{\sigma_n}{p_{atm}}\right)$$

where e_0 is the void ratio at atmospheric normal stress, while λ is the slope of the critical state line in $e-\ln(\sigma_n/p_{atm})$ plane.

Now, for simulating particle breakage, the CSL formulation (Eq. 1) should be changed in a way to address the downward translation in $e-\ln(\sigma_n/p_{atm})$ plane. To this aim, the CSL formulation (Eq. 1) of the model can be changed as given by Eq. 2.

$$[2] \quad e_{cs} = e_0(1 - B_r) - \lambda \ln\left(\frac{\sigma_n}{p_{atm}}\right)$$

where B_r is an index for quantifying the degree of particle breakage. B_r can be calculating through different approaches proposed by Marsal (1967), Lee and Farhoomand (1967), Hardin (1985) or Lade et al. (1996). As explained in Sec. 2, the index proposed by Lade et al. (1996) is the one of the high efficient available

indexes for quantifying the degree of particle breakage. By using this index, B_r is given by a hyperbolic formulation as Eq. 3.

$$[3] \quad B_r = \frac{W}{a+bW}$$

where a and b are two constants, and W can be total input energy (W_e) or total plastic work (W_p) (Lade et al. 1996). In this study, the modified total plastic work (W_p) (Eq. 4) proposed by Hu et al. (2011) is suggested.

$$[4] \quad W_p = \int (\sigma_n \langle d\varepsilon_n^p \rangle + \tau d\varepsilon_t^p)$$

where $d\varepsilon_n^p$ and $d\varepsilon_t^p$ are increment of plastic normal strain and plastic tangential strain respectively. The operator $\langle \rangle$ is the Macaulay brackets defining $\langle x \rangle = x$ if $x > 0$, and $\langle x \rangle = 0$ if $x \leq 0$.

4 Evaluation of an Interface Constitutive Model with Particle Breakage Simulation

In this section, the performance of an elasto-plastic constitutive model with and without considering particle breakage is evaluated. The interface model proposed by Saberi et al. (2016) was selected in this study. Saberi et al. (2016) model is an advanced interface constitutive model in the framework of two-surface plasticity (Dafalias and Popov 1975; Krieg 1975) and compatible with the concepts of CSSM. Particle breakage was simulated by translating CSL in $e-\ln(\sigma_n/\rho_{atm})$ plane as hyperbolic function of total plastic work (W_p) using the approach introduced in Sec. 3 and Eqs. 2-4. Then, the cyclic performance of the model in predicting the volumetric behavior of a gravelly soil-steel interface under two different stress paths was evaluated, as illustrated in Figure 3 and Figure 4. The experimental data are from laboratory observations by Zhang and Zhang (2006, 2009). Two widely used stress paths use in this study are Constant Normal Load (CNL) and Constant Normal Stiffness (CNS). In CNL stress path, normal stress remains constant during shear deformation. However, in CNS stress path, the normal stiffness (K) imposed on soil-structure interface systems as confinement condition remains constant during the test and this leads to variation of both shear and normal stresses during shearing.

Figure 3 is for an interface between gravelly soil and steel under cyclic CNL stress path with normal stress (σ_n)=800 kPa and Figure 4 is under a CNS stress paths with initial normal stress (σ_{n0})=500 kPa and normal stiffness (K)=50 kPa/mm. As can be observed from Figure 3 and Figure 4, by neglecting particle breakage, the best possible prediction by the model for accumulative contraction is stabilized after a small number of cycles and it cannot well simulate the gradual contraction with increasing number of cycles (N). In addition, accumulative contraction of the interface is largely underestimated, especially for large number of cycles. The interface constitutive model by considering particle breakage is capable of well simulating the interface normal contraction up to large number of cycles.

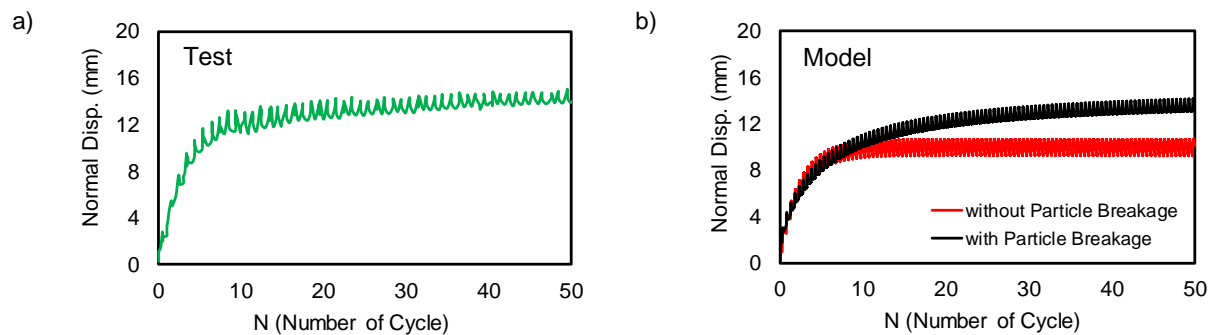


Figure 3: The effect of considering particle breakage on the performance of interface constitutive modeling for predicting normal displacement against number of cycle under CNL stress path.

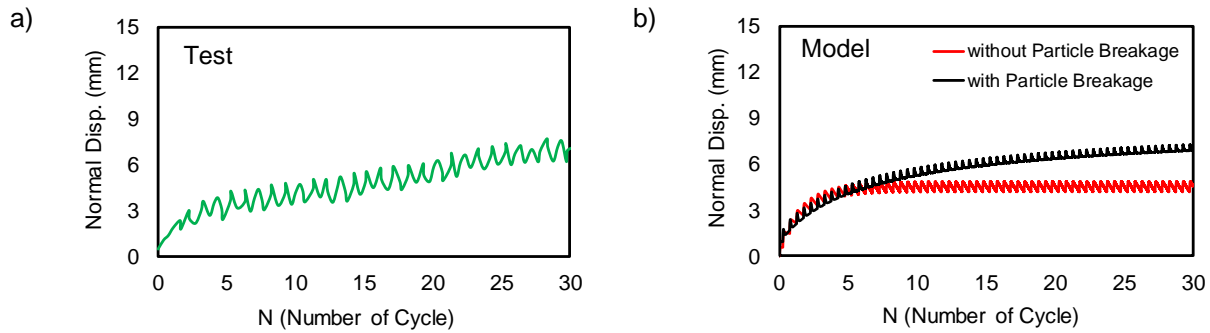


Figure 4: The effect of considering particle breakage on the performance of interface constitutive modeling for predicting normal displacement against number of cycle under CNS stress path.

5 Conclusion

In this study, the effect of particle breakage on the mechanics of granular soil-structure interface under monotonic and cyclic loading was introduced. Then, an efficient approach for simulating particle breakage within a constitutive formulation of an elasto-plastic interface model was proposed. The conclusions of the study are highlighted in the following:

1. In smooth surfaces, particle breakage increases surface roughness and it consequently leads to an increase in residual shear strength. However, particle breakage does not have significant influence on the stress-displacement behavior of rough surfaces.
2. Particle breakage has significant effect on the shear strength and volumetric behavior of granular soil-structure interfaces. During cyclic loading, particle breakage reduces void ratio of the soil and increases accumulative contraction in the interface zones.
3. The Formulation of an interface constitutive model compatible with the concept of CSSM can be improved to consider particle breakage by translating the CSL in $e-\ln(\sigma_r/\rho_{atm})$ plane towards smaller void ratio.
4. The Performance of an interface constitutive model is significantly increased by considering particle breakage. An interface model with considering particle well simulate cyclic accumulative contraction followed by a gradual stabilization by increasing the number of cycles.

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