Leadership in Sustainable Infrastructure
Leadership en Infrastructures Durables

Vancouver, Canada
May 31 – June 3, 2017/ *Mai 31 – Juin 3, 2017*

A TWO-SURFACE PLASTICITY CONSTITUTIVE MODEL FOR SOIL-STRUCTURE INTERFACES

Saberi, Miad[1,3], Annan, Charles-Darwin[1], Konrad, Jean-Marie[1] and Lashkari, Ali[2]
[1] Department of Civil and Water Engineering, Université Laval, Canada
[2] Department of Civil and Environmental Engineering, Shiraz University of Technology, Iran
[3] miad.saberi.1@ulaval.ca

Abstract: An elasto-plastic constitutive model is proposed to simulate the behavior of interfaces between coarse granular soil media and structural materials (i.e. steel or concrete). The proposed model is formulated within the frameworks of the two-surface plasticity, critical state soil mechanics and state parameter. The interface constitutive model requires a single set of eight calibration parameters to predict both monotonic and cyclic responses of the interface behaviour under a wide range of normal stress and soil densities without a need for recalibration. The model is capable of simulating the stress path dependent behavior of granular soil-structure interface problems. All the model parameters have physical meaning and can readily be obtained from standard interface shear tests. The performance of the proposed model is evaluated by comparing the model predictions and laboratory test data.

1 Introduction

In soil-structure systems, including embankment dams, retaining walls, and underground structures, a thin transition shear zone known as an interface exists between the soil mass and the structural material (Hu and Pu 2004; Zhang and Zhang 2006). The significant difference in stiffness between soils and the structural material could lead to stress concentration and strain localization at the interface. Furthermore, discontinuous deformation such as sliding and separation can occur in this zone. Thus, the interface characteristics and behavior can influence the response of these soil-structure systems, and may need to be accounted for in their analysis.

Many experimental studies have been conducted to provide understanding of the fundamental constitutive behavior and deformation mechanics of granular soil-structure interfaces. These include direct shear tests (e.g. Desai et al. 1985; Mortara et al. 2007; DeJong and Westgate 2009), simple shear tests (e.g. Uesugi and Kishida 1986; Fakharian and Evgin 1997), ring torsion shear tests (Yoshimi and Kishida 1981; Yasufuku and Ochiai 2005), and annular shear tests (Brumund and Leonards 1973). Advanced numerical methods have also been used to investigate the nature of the soil-structure contact problem. Zero thickness elements developed by Goodman et al. (1968) were modified to thin-layer elements (Yasufuku and Ochiai 2005; Desai et al. 1984) to capture normal deformations of the interface. The advent of these thin-layer elements has led to the use of constitutive models in interface problems.

Different interface constitutive models based on non-linear elasticity (e.g. Clough and Duncan 1971; Desai et al. 1985; Desai and Nagaraj 1988) and elasto-plasticity (e.g. Ghaboussi et al. 1973; Zaman et al. 1984; Desai and Ma 1992; Shahrour and Rezaie 1997; Fakharian and Evgin 2000; Ghionna and Mortara 2002; Zeghal and Edil 2002; Mortara et al. 2002; Hu and Pu 2004; Liu et al. 2006; Liu and Ling 2008; Zhang and Zhang 2008; Lashkari 2012, 2013; D'Aguiar et al. 2011; Duriez and Vincens 2015; Lashkari and Kadivar 2016; Stutz et al. 2016) have been proposed to simulate the behavior of granular soil-structure interfaces. The elasto-plasticity models have been formulated in different frameworks such as damage plasticity (e.g.

Hu and Pu 2004; Zhang and Zhang 2009), disturbed state concept (DSC) (e.g. Desai and Ma 1992; Fakharian and Evgin 2000; Desai et al. 2005), generalized plasticity (e.g. Liu et al. 2006, 2014; Liu and Ling 2008), two-surface plasticity (e.g. Shahrour and Rezaie 1997; Mortara et al. 2002; Lashkari 2012, 2013; Lashkari and Kadivar 2016). The two surface plasticity theory (Dafalias and Popov 1975; Krieg 1975; Dafalias 1986) is known to be most suitable for simulating cyclic behavior of materials under complex loading history in the plastic range. Critical State Soil Mechanics (CSSM) is also an important concept that has been introduced into interface constitutive formulations (Liu et al. 2006, 2014; Liu and Ling 2008; Lashkari 2013; Lashkari and Kadivar 2016) to simulate the state of the interface under large shear deformations.

A great majority of the existing interface constitutive models were formulated for sandy soil – structure interfaces and are characterized by a large number of parameters. However, a large number of geo-structures such as embankments, concrete face rockfill dams and speed-railways have critical interfaces between coarse granular soils (gravel and rockfill) and structural material. The mechanical behavior of this interface type, such as stress-displacement relationship, volumetric behavior under shear cycles and cyclic strength properties, can be significantly different from that of sandy soil-structure interfaces. For example, unlike sandy soil-structure interfaces, the softening behavior in gravelly soil-structure interfaces is negligible (Zhang and Zhang 2006, 2008, 2009).

This paper proposes a new constitutive model for gravel-structure interfaces in the framework of two-surface plasticity, and compatible with the concept of CSSM. The proposed model uses a unified formulation for both monotonic and cyclic loading and for gravelly interfaces over a wide range of densities. The following sections present the constitutive equations, detailed procedures for calibration of the model parameters and validation using experimental data.

2 Constitutive Equations

The proposed interface constitutive model is based on the soil model proposed by Dafalias and Manzari (2004), and interface model proposed by Lashkari (2013). The interface elasto-plastic constitutive is formulated based on a two dimensional (*2D*) plane strain problem and dry condition assumption. For rough soil-structure interfaces, the thickness (t) of the interface zone is assumed to be between 5 to 10 times the mean effective diameter (D_{50}) of adjacent soil particles (Uesugi et al. 1988; Hu and Pu 2004; DeJong and Westgate 2009).

Stress vector in the interface model consist of normal stress (σ_n) and tangential stress (τ), and the stress vector increments on the interface plane can be defined as follows:

$$[1] \quad d\boldsymbol{\sigma} = \begin{Bmatrix} d\sigma_n \\ d\tau \end{Bmatrix}$$

Incremental strain vector is expressed in terms of elastic and plastic strains in Eq. 2

$$[2] \quad d\boldsymbol{\varepsilon} = \begin{Bmatrix} d\varepsilon_n \\ d\varepsilon_t \end{Bmatrix} = \begin{Bmatrix} d\varepsilon_n \\ d\varepsilon_t \end{Bmatrix}^e + \begin{Bmatrix} d\varepsilon_n \\ d\varepsilon_t \end{Bmatrix}^p = \frac{1}{t}\left(\begin{Bmatrix} du_n \\ du_t \end{Bmatrix}^e + \begin{Bmatrix} du_n \\ du_t \end{Bmatrix}^p\right)$$

where t is the interface thickness, and $d\varepsilon_n$, $d\varepsilon_t$, du_n and du_t represent the increments of the normal strain, tangential (shear) strain, normal displacement and tangential displacement respectively. The superscripts, *e* and *p*, represent the elastic and plastic parts, and contraction is assigned a positive sign.

The stress and elastic strain increment vectors are linked through the elasticity matrix $[\boldsymbol{D}]^e$ as:

$$[3] \quad \begin{Bmatrix} d\sigma_n \\ d\tau \end{Bmatrix} = [\boldsymbol{D}]^e \begin{Bmatrix} d\varepsilon_n \\ d\varepsilon_t \end{Bmatrix}^e = \begin{bmatrix} D_n & 0 \\ 0 & D_t \end{bmatrix} \begin{Bmatrix} d\varepsilon_n \\ d\varepsilon_t \end{Bmatrix}^e = \begin{bmatrix} D_{n0}\sqrt{\sigma_n/p_{atm}} & 0 \\ 0 & D_{t0}\sqrt{\sigma_n/p_{atm}} \end{bmatrix} \begin{Bmatrix} d\varepsilon_n \\ d\varepsilon_t \end{Bmatrix}^e$$

where D_{n0} and D_{t0} are model parameters, and p_{atm} represents the atmospheric pressure, taken as 101(kPa) (Lashkari 2013).

The elastic deformation is limited by a yield surface, which is characterized by an open-wedge in normal stress (σ_n)-tangential stress (τ) plane as illustrated in Figure 1. The function of this yield surface (f) is expressed as (Dafalias and Manzari 2004; Lashkari 2012):

[4] $$f = \left(\frac{\tau}{\sigma_n} - \alpha\right)^2 - m^2 = (\mu - \alpha)^2 - m^2 = 0$$

where μ is the stress ratio at yield, which is the ratio of the normal stress to tangential stress (σ_n/τ), and α is the back stress ratio, a kinematic hardening variable controlling the movement or response of the yield surface in the σ_n-τ space. As illustrated in Figure 1, m controls the size of opening of the wedge-type yield surface, and α corresponds to the slope of the yield surface bisector (i.e. $\alpha=\mu-m$). Since the size of the elastic region for granular soils is relatively small, selecting a constant value for m of about $0.01\mu^{cs}$-$0.05\mu^{cs}$ has proven to yield satisfactory accuracy (Papadimitriou and Bouckovalas 2002; Dafalias and Manzari 2004; Taiebat and Dafalias 2008). The parameter μ^{cs} is the slope of the critical state surface in the σ_n-τ space (Figure 1), and will be discussed in in the following.

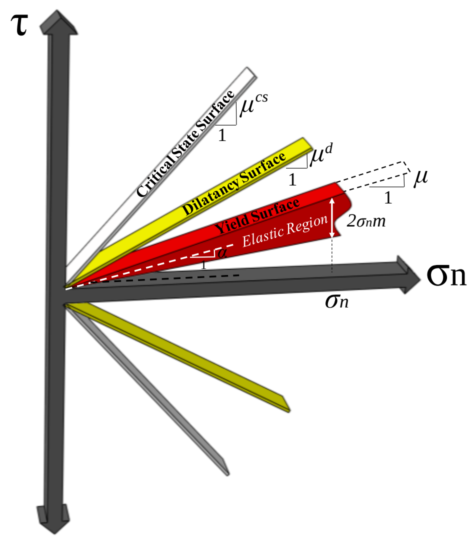

Figure 1: Model surfaces in the plane of σ_n-τ

In addition to the yield surface, the proposed interface model includes two other surfaces; dilatancy surface and critical state surface, as illustrated in Figure 1. These surfaces are used to predict the plastic behavior of the interface under monotonic and cyclic loading conditions. Based on the basic theory of plasticity (Chen and Baladi 1985; Dafalias 1986), the elasto-plastic incremental stress-strain relationships can be derived. The plastic strain rate is given by

[5] $$d\boldsymbol{\varepsilon}^p = \begin{Bmatrix} d\varepsilon_n \\ d\varepsilon_t \end{Bmatrix}^p = \langle \Gamma \rangle \boldsymbol{R} = \langle \frac{\boldsymbol{n} d\boldsymbol{\sigma}}{K_p} \rangle \boldsymbol{R}$$

where Γ is loading index, a scaler value and the operator < > is the Macaulay brackets defining $< \Gamma >= \Gamma$ if $\Gamma > 0$, and $< \Gamma >= 0$ if $\Gamma \leq 0$. $\boldsymbol{n}$ is a vector normal to the yield surface and defines the loading direction through Eq. 6, $\boldsymbol{R}$ is the direction of plastic strain rate vector given by Eq. 7 (Lashkari 2013), and K_p is the plastic modulus.

[6] $$\boldsymbol{n} = \begin{Bmatrix} \partial f/\partial \sigma_n \\ \partial f/\partial \tau \end{Bmatrix}$$

[7] $$\boldsymbol{R} = \begin{Bmatrix} D \\ \partial f/\partial \tau \end{Bmatrix}$$

As can be observed from Eqs. *6* and *7* , $\boldsymbol{R}$ is not a vector normal to the yield surface (i.e. $\boldsymbol{R} \neq \boldsymbol{n}$). Thus, the non-associated flow rule was adopted in this constitutive model which is the most efficient flow rule for predicting the rate of plastic strain in granular soils. *D* is the dilatancy coefficient (Eq. 8), which is proposed as a function of the distance (d^d) between the current stress state and its image on the dilatancy surface.

$$[8] \quad D = \frac{d\varepsilon_n^p}{\left|d\varepsilon_t^p\right|} = \frac{du_n^p}{\left|du_t^p\right|} = A^d(d^d) = A^d(\mu^d - s\mu) = A^d\left(\mu^{cs} exp\left(\frac{K^d\psi}{W_{p-in}^{\chi}+1}\right) - s\mu\right)$$

where, A^d is a positive model parameter and *s* is the auxiliary parameter for which *s*=+1 if $\mu - \alpha \geq 0$, and *s*=-1 if $\mu - \alpha < 0$. μ^d is the slope of the phase transition line, which is a line in stress space starting from the origin and locating the positions of transition phase points for different confining pressures in which compression behavior changes to dilatant and vice versa (Lade and Ibsen 1997). This line and its slope (μ^d) in σ_n-τ plane are called dilatancy surface and dilatancy stress ratio, respectively, and shown in Figure 1. K^d is also a model parameter and W_{p-in} is the value of total plastic work (W_p) at the initiation of the most recent loading, and χ is a constant parameter that can be set to 0.18 as a default value. W_p is calculated using the modified plastic work expression (Hu et al. 2011) given by:

$$[9] \quad W_p = \frac{1}{t}\int \left(\sigma_n \langle du_n^p \rangle + \sigma_n du_t^p\right)$$

where, *t* is the interface thickness.

Based on laboratory observations (Evgin and Fakharian 1996; Liu et al. 2006; DeJong and Westgate 2009), a granular soil-structure interface under large shear displacement reaches an ultimate state in which although the stress ratio ($\mu = \tau/\sigma_n$) is unchanged, the shear deformation increases with no changes in volume. This ultimate state is called a critical state in soil-structure interface problems, similar to the concept of critical state in soil mechanics. In Eq. 8, μ^{cs} is the critical state stress ratio, which is the slope of the critical state line in σ_n-τ plane and ψ is the state parameter originally introduced by Been and Jefferies (Been and Jefferies 1985) in the framework of CSSM for sands. In this study, the modified state parameter (ψ) (Liu et al. 2006; Liu and Ling 2008; Lashkari 2013) is used, as defined by Eq. 10:

$$[10] \quad \psi = e - e_{cs} = e - \left(e_0 - \lambda ln\left({}^{\sigma_n}/_{p_{atm}}\right)\right)$$

where *e* is the void ratio at the current state and e_{cs} is the critical state void ratio corresponding to current value of normal stress. e_{cs} is determined using e_0 and λ which are model parameters. As $\psi<0$, the interface is in dense state and experiences contraction/expansion (dilation) (i.e. $\mu^d<\mu^{cs}$) while it experiences full contraction when the $\psi>0$ and the interface is in loose state (i.e. $\mu^d>\mu^{cs}$). Since the state parameter (ψ) is constantly changing during loading, μ^d changes until it coincides with the critical state stress ratio (μ^{cs}) at $\psi=0$.Using this concept, the proposed interface model can simulate the behavior of an interface with different density with a single set of model parameters.

In Eq. 5, K_p is plastic modulus which relates stress increment to plastic strain increment based on plasticity theory. Following the concept of the two-surface plasticity theory (Dafalias and Popov 1975; Dafalias 1986), the following relationship is defined for a stress ratio dependant plastic modulus (K_p).

$$[11] \quad K_p = K_{p0}\frac{D_{t0}\sqrt{\sigma_n/p_{atm}}}{|\mu - m|}(\mu^{cs} - s\mu)$$

where K_{p0} is a model parameter and all other parameters have already been defined.

Based on Eqs. [1, 2, [*3*, 5, 6, 7 and 11, the relationship between increments of total stress vector and increments of total strain vector can be expressed as

$$[12] \quad d\boldsymbol{\sigma} = [\boldsymbol{D}]^{ep}\, d\boldsymbol{\varepsilon} = \left[[\boldsymbol{D}]^e - \frac{[\boldsymbol{D}]^e\, \boldsymbol{R}\, \boldsymbol{n}^{\boldsymbol{T}}\, [\boldsymbol{D}]^e}{K_p + (\boldsymbol{n}^{\boldsymbol{T}}\, [\boldsymbol{D}]^e\, \boldsymbol{R})}\right] d\boldsymbol{\varepsilon}$$

where $[\boldsymbol{D}]^{ep}$ is the elasto-plastic stiffness matrix.

3 Model Calibration

The proposed constitutive model requires eight parameters in total: two for elasticity (D_{t0} and D_{n0}), three for critical state (e_0, λ and μ^{cs}), two for dilatancy (A^d and K^d) and one for hardening (K_{p0}). The parameters all have physical meaning and can be readily determined through standard interface shear tests: Constant Normal Load (CNL) and Constant Normal Stiffness (CNS) shear test. In CNL tests, normal stress remains constant during shear deformation. However, in CNS tests, the normal stiffness imposed on soil-structure interface systems as confinement condition remains constant during the test.

The D_t and D_n are evaluated as the initial slopes of the shear stress-tangential displacement under CNL condition and normal stress-normal displacement respectively. By applying D_t and D_n in Eq. 3, the elasticity parameters D_{t0} and D_{n0} are obtained. If the shear stresses at critical state (τ_{cr}) are plotted against the corresponding normal stresses ($\sigma_{n\text{-}cr}$), the slope of the best fitted line starting from the origin is obtained as μ^{cs}. For capturing e_0 and λ, one can plot the critical void ratios (e_{cr}) at different normal stresses (σ_n) against $ln(\sigma_n/p_{atm})$. The slope of the best fitted line of the points represents λ and the intersection of the fit line with the e_{cr} axis is given as e_0.

The parameter A^d is calculated using the data of monotonic interface shear tests in normal displacement (u_n)-tangential displacement (u_t) plane. Assuming the elastic parts of normal and tangential displacements are negligible, the following equation can be drawn from Eq. 8.

$$[13] \quad A^d \approx \frac{(du_n/|du_t|)}{\mu^d - \mu}$$

Using the data obtained from different constant normal load or constant normal stiffness tests, different values of A^d are calculated by Eq. 8. The average value of A^d is recommended for use in the proposed model.

By the data at phase transformation state, the parameter K^d is calculated from Eq. 14 below.

$$[14] \quad K^d = \frac{(W_{p-in}^{0.18} + 1) ln\left(\frac{\mu^d}{\mu^{cs}}\right)}{\psi_{phts}}$$

where ψ_{phts} is the state parameter at phase transformation state. For monotonic loading, the $W_{p\text{-}in}$ in Eq. 14 is equal to zero (no reversal of load). Thus, it is evident that K^d can be determined as the slope of the best fitted line passing through the origin by plotting ψ_{phts} against $ln(\mu^d/\mu^{cs})$ at different constant normal load or constant normal stiffness interface shear tests.

The parameter K_{p0} can be estimated by a trial and error approach or by equalizing the normal-constant-load shear test for the interface zone to a uniaxial test for soil. In the latter method, the following relation between increment of tangential displacement and increment of shear stress under constant normal load (CNL) condition is derived.

$$[15] \quad d\varepsilon_t = \left(\frac{1}{D_t} + \frac{1}{K_p}\right) d\tau$$

By submitting Eq. 11 into Eq. 15, the parameter K_{p0} can be evaluated as:

$$[16] \quad K_{p0} = \frac{1}{\left(\frac{\mu^{cs} - m}{\mu - m} - 1\right)\left(\left(D_t \frac{d\varepsilon_t}{d\tau}\right) - 1\right)}$$

4 Model Evaluation

In this section, the performance of the proposed interface constitutive model is evaluated by compering the model predictions to monotonic and cyclic experimental test data obtained by Zhang and Zhang (2008) for gravel-steel interfaces under constant normal load (CNL) condition. The properties of the interface materials, and the evaluated values of model parameters are presented in Table 1 and Table 2 respectively.

Table 1. Properties of gravelly interfaces

Interface Type	Property		
	Average Grain Size D_{50} (mm)	Interface Thickness t (mm)	Dry Unit Weight γ_d (kN/m^3)
Gravel-Steel Interface	7	50	17.5

Table 2. Parameter values of the gravel-steel interface constitutive model

	Elasticity		Critical state			Dilatancy		Hardening
Interface Type	D_{t0} (MPa)	D_{n0} (MPa)	e_0	λ	μ^{cs}	A^d	K^d	K_{p0}
Gravel-Steel Interface	3.5	4.1	0.548	0.0437	0.777	0.39	5.2	0.53

As illustrated in Figure 2, in both shear stress-tangential displacement plane and normal displacement-tangential displacement plane, there is very good agreement between the experimental results for gravelly soil-steel plate interface and the proposed model prediction. The model predicts well the initial compressive and then dilative behavior of gravelly interface under low normal stresses (σ_n=100 kPa and σ_n=200 kPa), as well as the fully contractive behavior under high normal stresses (σ_n=400 kPa and σ_n=700 kPa).

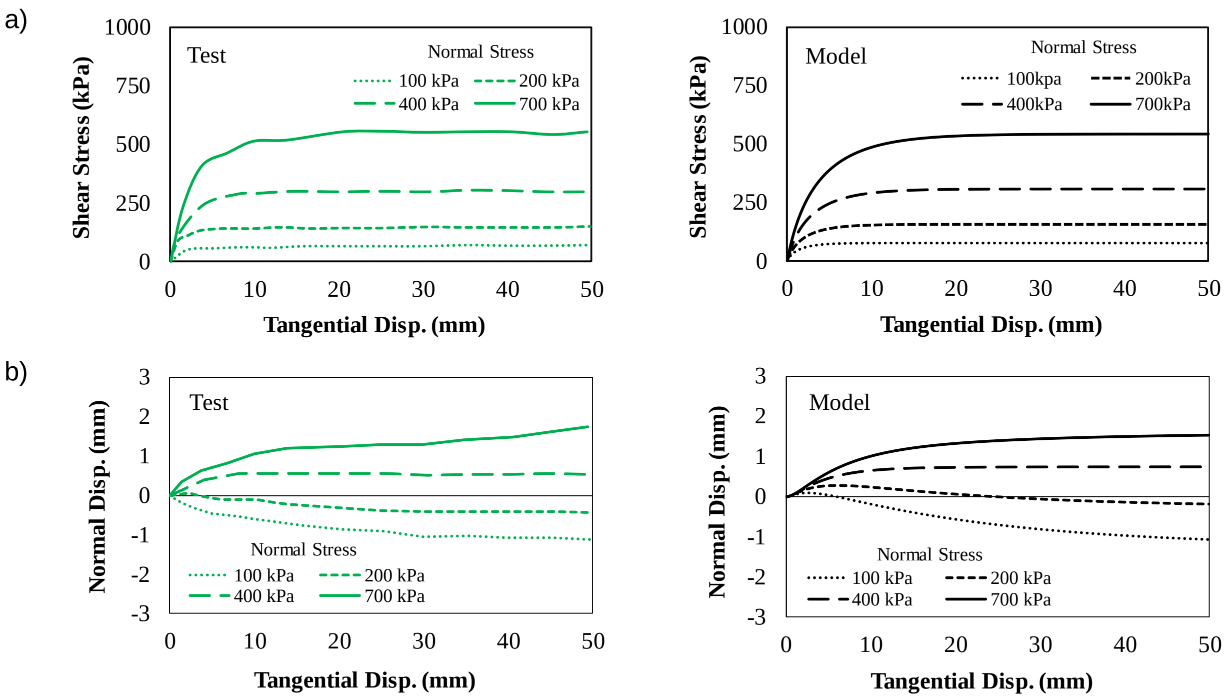

Figure 2: Comparison between experimental data (Zhang and Zhang 2008) and model prediction for monotonic behavior, a) shear stress-tangential displacement, and b) normal displacement-tangential displacement.

The performance of the model for simulating cyclic behavior of the interface was also performed and compared with laboratory observations. As can be observed from Figure 3, there is very good agreement between predictions by the proposed model and test observations for shear stress-tangential displacement

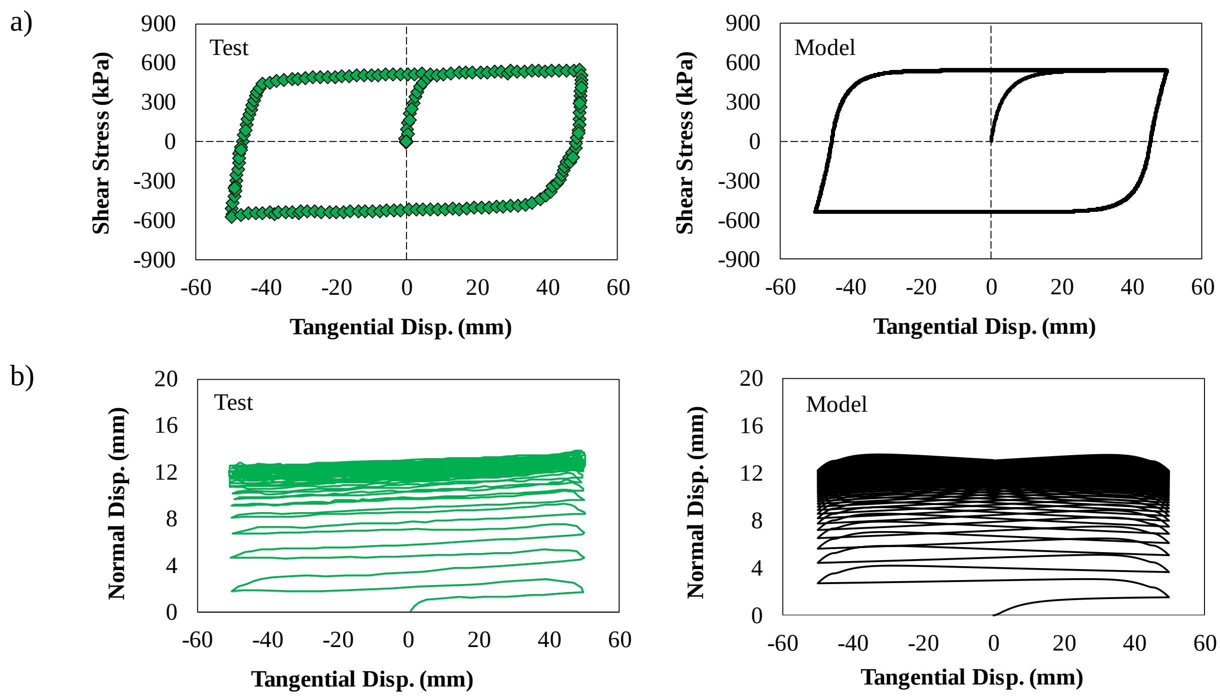

Figure 3: Comparison between experimental data (Zhang and Zhang 2008) and model prediction for cyclic behavior under CNL condition with σ_n=700 kPa, a) shear stress-tangential displacement, and b) normal displacement-tangential displacement.

and normal displacement-tangential displacement cyclic behaviors under CNL stress path with σ_n=700 kPa. The model can also well simulate the accumulative contraction of the gravely interface when the number of cycles is increased (Figure 3-b). The stabilization of accumulative normal displacement after large number of cycles typical of gravelly interface was predicted very well by the proposed model. As can be seen from Figure 3-b, normal contraction of both excremental test and model prediction are stabilized around 13.5 mm.

5 Conclusion

This paper has proposed an elasto-plastic constitutive model for predicting the behavior of the interface between gravelly soils and structural materials. The model is compatible with critical state soil mechanics and was developed based on the two-surface plasticity theory. The performance of the model was evaluated using data of experimental investigations available in the literature. The following highlights the capabilities of the proposed interface model:

1) The proposed model requires only a set of eight parameters with physical meaning, which can be readily determined using conventional shear tests.
2) The proposed model is capable of predicting the monotonic and cyclic behavior of gravel-structure interfaces with different densities only using a single set of model parameters. That is, no further calibration is required for predicting cyclic behavior as well as the samples with different densities ranging from loose to dense..
3) The proposed model is capable of simulating phase transformation and the effect of normal stress on the behavior of gravelly soils - structure interfaces.
4) The proposed model simulates very well the cyclic accumulative contraction and cyclic stabilization of the gravelly soil – structure interfaces subjected to shear cycles.

References

Been, K., and Jefferies, M. G. 1985. A state parameter for sands. *Géotechnique*, 35(2), 99–112.

Brumund, W., and Leonards, G. 1973. Experimental Study of Static and Dynamic Friction Between Sand and Typical Constuction Materials. *Journal of Testing and Evaluation*, ASTM International, 1(2), 162–165.

Chen, W. F., and Baladi, G. Y. 1985. *Soil Plasticity: Theory and Implementation*, Elsevier.

Clough, G. W., and Duncan, J. M. 1971. Finite Element Analyses of Retaining Wall Behavior. *Journal of the Soil Mechanics and Foundations Division*, ASCE, 97(12), 1657–1673.

D'Aguiar, S. C., Modaressi-Farahmand-Razavi, A., dos Santos, J. A., and Lopez-Caballero, F. 2011. Elastoplastic constitutive modelling of soil-structure interfaces under monotonic and cyclic loading. *Computers and Geotechnics*, Elsevier Ltd, 38(4), 430–447.

Dafalias, Y. F. 1986. Bounding Surface Plasticity. I: Mathematical Foundation and Hypoplasticity. *Journal of Engineering Mechanics*, ASCE, 112(9), 966–987.

Dafalias, Y. F., and Manzari, M. T. 2004. Simple Plasticity Sand Model Accounting for Fabric Change Effects. *Journal of Engineering Mechanics*, 130(6), 622–634.

Dafalias, Y. F., and Popov, E. P. 1975. A model of nonlinearly hardening materials for complex loading. *Acta Mechanica*, 21(3), 173–192.

DeJong, J. T., and Westgate, Z. J. 2009. Role of Initial State, Material Properties, and Confinement Condition on Local and Global Soil-Structure Interface Behavior. *Journal of Geotechnical and Geoenvironmental Engineering*, American Society of Civil Engineers, 135(11), 1646–1660.

Desai, C. S., Drumm, E. C., and Zaman, M. M. 1985. Cyclic testing and Modeling of Interfaces. *Journal of Geotechnical Engineering*, 111(6), 793–815.

Desai, C. S., Lightner, J. G., and Siriwardane, h J. 1984. Thin-Layer Element for Interfaces and Joints. *International Journal for Numerical and Analytical Methods in Geomechanics*, 8, 19–43.

Desai, C. S., and Ma, Y. 1992. Modelling of joints and interfaces using the disturbed-state concept. *International Journal for Numerical and Analytical Methods in Geomechanics*, 16, 623–653.

Desai, C. S., and Nagaraj, B. K. 1988. Modeling for Cyclic Normal and Shear Behavior of Interfaces. *Journal of Engineering Mechanics*, 114(7), 1198–1217.

Desai, C. S., Pradhan, S. K., and Cohen, D. 2005. Cyclic Testing and Constitutive Modeling of Saturated Sand–Concrete Interfaces Using the Disturbed State Concept. *International Journal of Geomechanics*, 5(4), 286–294.

Duriez, J., and Vincens, É. 2015. Constitutive modelling of cohesionless soils and interfaces with various internal states: An elasto-plastic approach. *Computers and Geotechnics*, 63, 33–45.

Evgin, E., and Fakharian, K. 1996. Effect of stress paths on the behaviour of sand-steel interfaces. *Canadian Geotechnical Journal*, 33(6), 853–865.

Fakharian, K., and Evgin, E. 1997. Cyclic Simple-Shear Behavior of Sand-Steel Interfaces under Constant Normal Stiffness Condition. *Journal of Geotechnical and Geoenvironmental Engineering*, 123(12), 1096–1105.

Fakharian, K., and Evgin, E. 2000. Elasto-Plastic Modelling of Stress-Path- Dependent Behaviour of Interfaces. *International Journal for Numerical and Analytical Methods in Geomechanics*, 24(2), 183–199.

Ghaboussi, J., Wilson, E. L., and Isenberg, J. 1973. Finite Element for Rock Joints and Interfaces. *Journal of the Soil Mechanics and Foundations Division*, ASCE, 99(10), 849–862.

Ghionna, V. N., and Mortara, G. 2002. An elastoplastic model for sand—structure interface behaviour. *Géotechnique*, 52(1), 41–50.

Goodman, R. E., Taylor, R. L., and Brekke, T. L. 1968. A Model for the Mechanics of Jointed Rock. *Journal of the Soil Mechanics and Foundations Division*, ASCE, 94(3), 637–660.

Hu, L., and Pu, J. 2004. Testing and Modeling of Soil-Structure Interface. *Journal of Geotechnical and Geoenvironmental Engineering*, 130(8), 851–860.

Hu, W., Yin, Z., Dano, C., and Hicher, P.-Y. 2011. A constitutive model for granular materials considering grain breakage. *Science China Technological Sciences*, 54(8), 2188–2196.

Krieg, R. D. 1975. A Practical Two Surface Plasticity Theory. *Journal of Applied Mechanics*, American Society of Mechanical Engineers, 42(3), 641.

Lade, P. V., and Ibsen, L. B. 1997. *A Study of the Phase Transformation and the Characteristic Lines of Sand Behaviour*. Geotechnical Engineering Group.

Lashkari, A. 2012. A plasticity model for sand-structure interfaces. *Journal of Central South University*, 19(4), 1098–1108.
Lashkari, A. 2013. Prediction of the shaft resistance of nondisplacement piles in sand. *International Journal for Numerical and Analytical Methods in Geomechanics*, 37(January 2012), 904–931.
Lashkari, A., and Kadivar, M. 2016. A constitutive model for unsaturated soil-structure interfaces. *International Journal for Numerical and Analytical Methods in Geomechanics*, 40(2), 207–234.
Liu, H., and Ling, H. I. 2008. Constitutive description of interface behavior including cyclic loading and particle breakage within the framework of critical state soil mechanics. *International Journal for Numerical and Analytical Methods in Geomechanics*, 32(12), 1495–1514.
Liu, H., Song, E., and Ling, H. I. 2006. Constitutive modeling of soil-structure interface through the concept of critical state soil mechanics. *Mechanics Research Communications*, 33(4), 515–531.
Liu, J., Zou, D., and Kong, X. 2014. A three-dimensional state-dependent model of soil-structure interface for monotonic and cyclic loadings. *Computers and Geotechnics*, 61, 166–177.
Mortara, G., Boulon, M., and Ghionna, V. N. 2002. A 2-D constitutive model for cyclic interface behaviour. *International Journal for Numerical and Analytical Methods in Geomechanics*, 26(11), 1071–1096.
Mortara, G., Mangiola, A., and Ghionna, V. N. 2007. Cyclic shear stress degradation and post-cyclic behaviour from sand–steel interface direct shear tests. *Canadian Geotechnical Journal*, 44(7), 739–752.
Papadimitriou, A. G., and Bouckovalas, G. D. 2002. Plasticity model for sand under small and large cyclic strains: a multiaxial formulation. *Soil Dynamics and Earthquake Engineering*, 22(3), 191–204.
Shahrour, I., and Rezaie, F. 1997. An elastoplastic constitutive relation for the soil-structure interface under cyclic loading. *Computers and Geotechnics*, 21(1), 21–39.
Stutz, H., Mašín, D., and Wuttke, F. 2016. Enhancement of a hypoplastic model for granular soil–structure interface behaviour. *Acta Geotechnica*. 11(6), 1249-1261.
Taiebat, M., and Dafalias, Y. F. 2008. SANISAND: Simple anisotropic sand plasticity model. *International Journal for Numerical and Analytical Methods in Geomechanics*, 32(8), 915–948.
Uesugi, M., and Kishida, H. 1986. Influential Factors of Friction between Steel and Dry Sands. *Soils and Foundations*, 26(2), 33–46.
Uesugi, M., Kishida, H., and Tsubakihara, Y. 1988. Behavior of sand particles in sand-steel friction. *Soils and Foundations*, 28(1), 107–118.
Yasufuku, N., and Ochiai, H. 2005. Sand-Steel Interface Friction Related to Soil Crushability. *Proceeding, Geomechanics: Testing, Modeling, and Simulation*, ASCE, Reston, VA, 627–641.
Yoshimi, Y., and Kishida, T. 1981. A Ring Torsion Apparatus for Evaluating Friction Between Soil and Metal Surfaces. *Geotechnical Testing Journal*, 4(4), 145–152.
Zaman, M. M., Desai, C. S., and Drumm, E. C. 1984. Interface Model for Dynamic Soil Structure Interaction. *Journal of Geotechnical Engineering*, 110(9), 1257–1273.
Zeghal, M., and Edil, T. B. 2002. Soil structure interaction analysis: modeling the interface. *Canadian Geotechnical Journal*, 39(3), 620–628.
Zhang, G., and Zhang, J.-M. 2006. Monotonic and Cyclic Tests of Interface between Structure and Gravelly Soil. *Soils and Foundations*, 46(4), 505–518.
Zhang, G., and Zhang, J.-M. 2008. Unified Modeling of Monotonic and Cyclic Behavior of Interface Between Structure and Gravelly Soil. *Soils and Foundations*, 48(2), 231–245.
Zhang, G., and Zhang, J.-M. 2009. Constitutive rules of cyclic behavior of interface between structure and gravelly soil. *Mechanics of Materials*, Elsevier Ltd, 41(1), 48–59.
Zienkiewicz, O. C., Best, B., Dullage, C., and Stagg, K. G. 1970. Analysis of Nonlinear Problems in Rock Mechanics with Particular Reference to Jointed Rock Systems. *2nd International Society of Rock Mechanics, Proceedings*, Belgrad, 501–509.