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# KINEMATIC MODEL FOR STRAIN EVALUATION INDENTED PIPELINES USING MULTIDIMENSIONAL B-SPLINE INTERPOLATION

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# **ABSTRACT:**

Current trends in the analysis of dents in pipelines are turning the focus of pipeline dent integrity management schemes on the localized strain distribution rather than the traditional dent depth based criterion for reasons well documented in history and research. The strain-based evaluation of dent severity is limited by the available techniques for evaluating the strain distribution in affected regions of the pipeline. In this study, a mathematical model is introduced for the strain evaluation of dented pipelines based solely on data relating the coordinates of the deformed profile. This novel approach employs the use of third-order cubic B-Spline functions to interpolate the deformed profile of the dented pipeline. The mathematical model developed discretizes the displacement components into the radial, circumferential and the longitudinal direction based on the linear elastic shell theory. The discretized displacement components create a platform that provides operators the liberty to constrain or release the strain based model from underlying assumptions and allows for flexibility in the choice of the strain measure. A good correlation is observed in the strains predicted by the mathematical models and those predicted by numerical models generated with finite element analysis (FEA). This agreement indicates the possibility of performing detailed strain analysis on dented pipelines without having to resort to FEA.

Keywords (Strain, Pipelines, Dents, FEA)

# 1 INTRODUCTION

Dents which are a form of mechanical damage in pipelines can be initiated by a host of factors including but not limited to third-party interactions, ground movements, and secondary loads imposed on the pipelines due to settlement. All have the potential to lead to serviceability or integrity concerns for pipelines (Rosenfeld *et al.* 1998, Panetta *et al.* 2001, Dinovitzer *et al.* 2002 and Hanif and Kenny 2014). The common premise for evaluating the severity of a plain dent (i.e. one without any feature interactions) in a pipeline is the depth based criterion, which has been proposed in various pipeline codes and standards including the Canadian pipeline codes (CSA Z662-2016). CSA Z662 stipulates that dents exceeding 6% of the outer diameter (OD) for pipes greater than 101.6mmm and dents exceeding depths of 6mm for pipes with 101.6mm OD or smaller shall be considered to be defects unless determined by an engineering assessment to be acceptable. An advisory (Erikson 2010) was released by the National Energy Board to pipeline operators under its jurisdiction regarding incidents where the codified stipulations failed to indicate the impending failure of said pipelines. This was to serve as an indication of the need to revise the current pipeline integrity management and assessment strategies as the depth alone does not provide sufficient information for judging the severity of a defect. The report also indicates that for cases of unconstrained

dents where fatigue cycling is expected there is an increased risk of crack propagation all of which are factors that could potentially result in the failure of the piping system. It is thus necessary that for a thorough dent defect assessment, the localized stresses and strains are to be investigated. In certain studies, (Ironside and Carrol 2002, Baker 2004, Dawson *et al.* 2006 and Rafi *et al.* 2012), it is also stated that the depth of the dent is not the sole factor in determining the severity of a dent and other phenomena such as the localised strain distributions can govern the failure of the pipe.

Information about strains in the deformed pipeline can be obtained either by FEA or analytically. The former being a rigorous procedure involving numerical modeling of the indentation and predicting the strains via the stiffness matrix and the shape functions derived from a numerical model of a defect of similar geometry (Belangar and Ram 2008). However, repeated FEA runs have to be performed to attain a geometric match in the actual dent and the numerical model. The analytical procedure for evaluating the strain in a dent is based on the dent length and amplitude or the thin shell small deformation model (Rosenfeld 1998). A procedure currently being used as an alternative to FEA in the American Society of Mechanical Engineers standard (ASME B31.8-2007) gives a set of closed-form expressions for analytically evaluating strains in dents. The three major components of strains for this procedure are the circumferential bending strain, the longitudinal bending strain, and the longitudinal membrane strain. The bending strains are proportional to the radius of curvature of the deformed pipe wall and vary linearly along the thickness of the pipe wall. The expressions for evaluating the bending strains in the circumferential and the longitudinal directions are shown in equations 1 and 2 respectively.

$$[1] \varepsilon_1 = \frac{t}{2} \left( \frac{1}{R_0} - \frac{1}{R_1} \right)$$

$$[2]\varepsilon_2 = -\frac{t}{2} \left(\frac{1}{R_2}\right)$$

where t is the pipe wall thickness,  $R_0$  is the radius of the un-deformed pipeline, and  $R_1$  and  $R_2$  are the external radius of curvature of the dent in the circumferential and longitudinal directions respectively.

In the longitudinal direction, the deflections are relatively large and the strains associated with the extension of the mid-surface of the pipeline can be evaluated analytically. Equation 3 presents the closed form expression for evaluating the longitudinal membrane strains as per ASME B-31.8 stipulations.

$$[3] \varepsilon_3 = \frac{1}{2} \left(\frac{d}{L}\right)^2$$

where d is the depth of the dent and L is length of the dent.

These directional strain components are then combined to evaluate an equivalent strain in the dented section as shown in equations 4 and 5.

[4] 
$$\varepsilon_i = \sqrt{(\varepsilon_1)^2 + \varepsilon_2(\varepsilon_2 + \varepsilon_3) + (\varepsilon_2 + \varepsilon_3)^2}$$

[5] 
$$\varepsilon_o = \sqrt{(\varepsilon_1)^2 + \varepsilon_1(\varepsilon_3 - \varepsilon_2) + (\varepsilon_3 - \varepsilon_2)^2}$$

where  $\varepsilon_i$  and  $\varepsilon_o$  are the strains in the inner and outer surfaces of the pipe wall respectively. The codes further stipulate a maximum allowable strain of 6%.

The circumferential membrane strains are ignored in the ASME stipulations as this quantity cannot easily be obtained from the dent profile and its contribution to the entire strain state of the pipeline is assumed not to be significant as the cross section can be ovalized during indentation. These formulations in the ASME equations are however oversimplified in assumptions and some of the concerns that have been outlined in (Baker 2004 and Noronha *et al.* 2010) include the fact that the longitudinal membrane strain equation is an empirical estimate benchmarked against a limited number of finite element runs, and a state of plane strain is assumed in the pipeline. The latter implies that the radial strain component is ignored, a condition that does not comply with deformation plasticity. Certain works by (Lukasiewicz *et al.* 2006) seek to improve the

accuracy of this method of evaluating strains by predicting the problematic membrane strains using FEA and evaluating the bending components of the strains analytically as per the codified stipulations. This procedure however still relies on the input of FEA.

Described herein is a procedure for the analytical evaluation of strains in plain restrained dents (void of stress concentrators or discontinuities) directly from the deformation data obtained from inline inspection, without the need for the material properties or complex nonlinear FEA. In this study, the response of the mid-surface of the pipeline is constrained to be governed by the kinematic strain-displacement equations from shell theory (discussed in Love 1888, Reissner 1952, Naghdi and Nordgren 1963, and Sanders 1963). Shell theory imposes the Kirchhoff-Love's hypothesis of straight normals on models as the displacements in the radial, circumferential and the longitudinal directions are obtained. The corresponding strain components can then be evaluated via performing derivatives with respect to the undeformed state of the pipeline. The strain distribution obtained using the technique presented herein is benchmarked against the results from the numerical models developed with FEA.

# 2.0 METHOD

The proposed technique comprises a series of steps outlined in this section.

#### 2.1 FEA Simulation

A series of elastoplastic analysis was performed on pipelines subjected to a denting displacement using the commercially available ABAQUS 6.14 numerical solver. The implicit analysis codes were equipped with large deformation and rotation capabilities. The pipeline was modeled as an elastoplastic material with a Poisson ratio of 0.3 and Young's Modulus value of 200 GPa. The plastic regime of the pipeline was modeled assuming isotropic hardening using a post yield stress-strain response of a typical X52 gas pipeline. The meshing algorithm employed discretizes the entire structure into a series of 8-node linear brick elements with hourglass control. A mesh convergence analysis was performed to validate the meshing scheme used and computation time was reduced by taking advantage of the symmetry of the dented pipeline and also by using the reduced integration option in the solver. The simulation was displacement controlled in which an indenter was displaced into the pipeline to correspond to the dent depth required. The contact surfaces were modeled using the master-slave algorithm so as to ensure that the pipeline deformed locally relative to the indenter at the region of contact. Two types of dents were considered in this study, sharp dents resulting in a steep change in the curvature of the dent profile and dents resulting in a smooth change in the curvature of the dent profile. The sharp dents were created with a dome-shaped indenter of radius 60mm and the smooth dents were created with a 300mm long flat indenter. In total, eight numerical models were generated for this study by varying indentation depth of the models. A unique name was chosen for the numerical models of the sharp dent: DS2, DS6, DS10 and DS12. A similar nomenclature was used for the smooth dents: FS2, FS6, FS10, and FS12, where alphabetic prefix represents the type of indenter used, DS for dome shaped and FS for flat shaped indenters and the numeric suffix is the dent depth represented as a percentage of the OD of the pipeline. The pipeline model was restricted from translation and rotation at its ends and the vertical support provided by the soil in buried pipelines was idealized by preventing the vertical displacement of the pipeline. The pipe wall was 8mm thick and the length of the pipeline was chosen to be approximately ten times the radius of the pipeline so as to prevent interactions between the dent and the end restraints of the pipeline.

# 2.2 Spline Interpolation

The coordinates of the dent profile are extracted from the nodes of the numerical model and B-Spline functions are used to generate a series of parametric curves that define the dent profile. The interpolation is done in the cylindrical coordinate system that fits the morphology of the pipeline. The piecewise nature of the B-Spline functions allows for smooth differentiable curves between neighboring nodes, thus a mathematical representation of the dent contours is developed equipped with second order continuity. The

radial displacement is a function of the longitudinal length of the dent, Z with interval  $(-L \le Z \le L)$  (where L represents the longitudinal distance from the apex of the dent) and the circumferential angle,  $\theta$  with interval  $(-\pi \le \theta \le \pi)$ . The orientation of the pipe is such that the negative interval  $(-\pi \le \theta \le 0)$  is the bottom half of the pipeline which remains relatively unperturbed by the indenting force and the positive interval  $(0 < \theta \le \pi)$  represents the top half of the dent with the dent peak located at the  $\frac{\pi}{2}$  radian mark.

# 2.3 Deformation Analysis

The directional displacements are obtained from the shell theory by assuming the pipeline to be analyzed has geometric features similar to a "thin shell" as (discussed in Ventsel and Krauthammer 2001). The kinematic displacement equations governing the displacement of a typical thin shell is then imposed on the pipeline to obtain the respective directional displacements. By leveraging the flexibility of the pipeline in the circumferential direction it can be assumed that the perimeter of the pipeline before and after the dent formation remains the same. This implies that the deformation in the circumferential direction is dominated by flexural deformation and there is little or no extension in this direction. To enforce the assumption of the pipeline's circumference being inextensible, a mid-surface radius of the undeformed pipeline,  $R_{\rm me}$  that satisfies this equality is evaluated and used for all computations in the procedure presented herein. The angular distortion,  $\phi$  of material components in the deformed and undeformed configuration of the pipe can thus be evaluated such that for every reported coordinate in the deformed state, corresponding undeformed coordinates are evaluated. The circumferential deformation of the mid-surface of the pipeline can then be evaluated along the thickness of the pipe wall as in equation 6.

[6] 
$$u_{\theta} = R_m Sin[\phi] - t_v Sin[\phi - \theta_{\theta}]$$

where  $R_m$  is the radius of the mid-surface of the deformed pipeline,  $\phi$  is the angular distortion of the pipeline resulting from the deformation of the pipe's cross section relative to its initial configuration,  $t_v$  is the variable thickness of the pipeline with interval  $\left(\frac{-t}{2} < t_v < \frac{t}{2}\right)$ , t is the thickness of the pipe wall and  $\theta_\theta$  is the slope of the mid-surface of the pipeline in the circumferential axis. The circumferential slope  $\theta_\theta$  is evaluated mathematically as the change in the radius of the mid-surface of the deformed pipeline with respect to the circumference of the undeformed pipeline. The second component of equation [7] accounts for the effect of the slope of the pipe wall on the circumferential displacement along the thickness of the pipe wall. The radial displacement can also be evaluated using a similar approach. However, this component can be approximately evaluated as shown in equation 7.

[7] 
$$u_r = R_m Cos[\phi] - R_{me}$$

The longitudinal displacement  $u_z$  is assumed to vary linearly along the thickness of the pipeline. The mathematical expression of the longitudinal displacement of the pipeline is given in equation 8.

[8] 
$$u_z = t_v Sin[\varphi_z]$$

Where  $\varphi_z$  is the slope of the deformed profile in the longitudinal direction which is evaluated as the rate of change of the radial displacement with respect to the longitudinal axis.

The deformation gradient,  $\nabla U$ , which maps the vector positions of nodes from the deformed to the undeformed configurations of the pipeline can thus be calculated from the obtained directional displacements and it is represented mathematically by the matrix shown in equation 9.

$$[9] \nabla U = \begin{bmatrix} \frac{\partial u_r}{\partial r} & \frac{\partial u_r}{r \partial \theta} - \frac{u_{\theta}}{r} & \frac{\partial u_r}{\partial z} \\ \frac{\partial u_{\theta}}{\partial r} & \frac{u_r}{r} + \frac{\partial u_{\theta}}{r \partial \theta} & \frac{\partial u_{\theta}}{\partial z} \\ \frac{\partial u_z}{\partial r} & \frac{\partial u_z}{r \partial \theta} & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

# 2.4 Strain Analysis

The strain measure used in this study was the integrated strain measure, which is the output from the numerical solver used. The technique proposed allows some flexibility in the strain measure that can be employed and the Linear and the Lagrangian strain measures (which are based on the additive and the multiplicative identities of the deformation gradient respectively as described in (Adeeb 2011)) are investigated. The Linear strain measure allows for small deformations and is evaluated by linearizing the finite strain with respect to the displacement gradient. The Lagrangian strain measure, includes nonlinear terms, allows for large deformations and separates the rigid body rotation from the deformation. In the longitudinal direction, the extension of the membrane is allowed and the overall strain state is a result of both bending and stretching of the pipeline. In order to take this interaction into account, the Lagrangian strain measure is used for evaluating the associated strains developed in the longitudinal direction. The circumferential direction is assumed to be constrained to relatively small deformations and thus the Linear strain measure is used to evaluate the strains in the circumferential direction.

#### 3 RESULTS

The strains discussed in this section were obtained from the FS6 and the DS6 models which are presented in Figure 1.

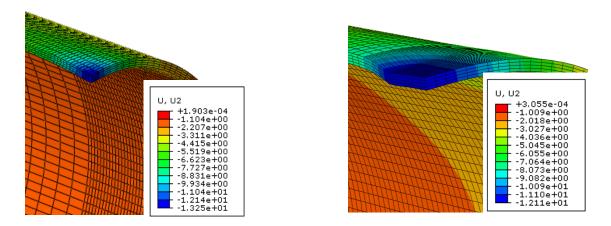


Figure 1: The contours of the vertical deformation of the numerical models DS6 (left) and FS6 (right)

From the extracted coordinates of the deformed models, strain contours which represent the strain state along the thickness of the pipe wall at the critical section were developed in the longitudinal and the circumferential axis of the models. The circumferential strain distribution predicted by the proposed technique and FEA performed on the FS6 model is presented in Figure 2.

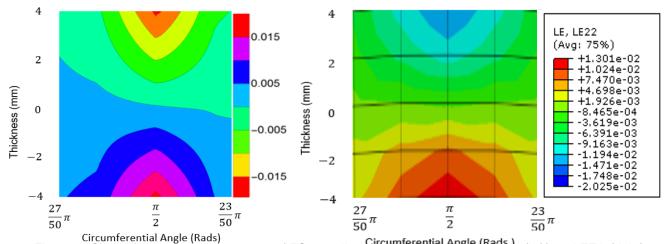


Figure 2: Circumferential strain contours of FS6 predicted by the proposed method (left) and FEA (right)

The orientation of the variable thickness of the pipe wall is such that the negative interval refers to the region between the mid-surface of the pipe wall and the internal surface of the pipe wall. The positive interval refers to the region between the mid-surface of the pipe wall and the external surface of the pipeline. The contours generated (presented in Figure 2) show that the tensile strains are developed at the internal surface of the pipe and compressive strains at the external surface of the pipe. The maximum circumferential strains developed from the nonlinear FEA are the compressive strains. The proposed technique, which uses the Linear strain measure, predicts similar strain values at the compressive zone of the pipeline with all the strain measures predicting a maximum value of about 2%. The deformations used in evaluating the Linear strain measure are obtained without the trigonometric approximations of the Euler-Bernoulli hypothesis and thus the strain distribution obtained by the procedure is nonlinear. This nonlinearity arises because the neutral axis of the pipeline does not coincide with the centroid of the pipe wall and the strain distribution at the compressive and tensile zones are not the same.

In the longitudinal direction, a good correlation is also observed in the strains predicted by the proposed technique and the results from nonlinear FEA. The Lagrangian strain used in evaluating the strain in this direction accounts for the large deformations expected along this axis leading to the observed nonlinear distribution for the strains, as the tensile and compressive strain concentrations are dissimilar. The maximum strains do not occur at the peak of the dent but are observed at the shoulder of the dent which is located at 115 mm from the center of the dent. Figure 3 presents the longitudinal strain contours generated using the technique proposed and FEA.

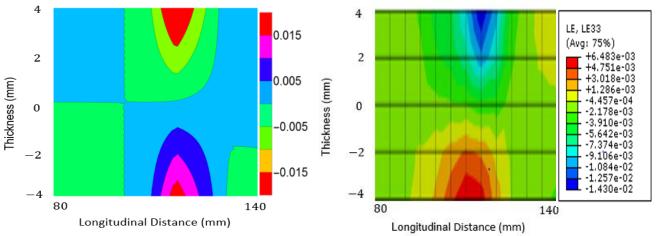


Figure 3: Longitudinal strain contours of FS6 predicted by the proposed method (left) and FEA (right)

The predictions of the Lagrangian strain measure and the FEA are similar as both models predict a maximum longitudinal tensile strain value of 1.5%.

For the DS models (i.e. the sharp dent models generated with a dome shaped indenter), a good agreement in the predicted strain distribution is also obtained. In the circumferential direction, the critical strains predicted by FEA are the compressive strains at the external surface of the pipeline and these are developed at the peak of the dent. From the strain contours of the circumferential axis in Figure 4, it can be seen that, in terms of magnitude and location, the proposed technique provides results comparable to nonlinear FEA as the proposed technique predicts a maximum compressive strain of 6% while FEA predicts a maximum compressive strain strains of 5%.

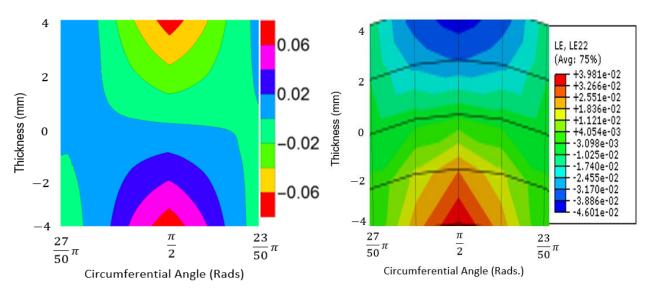


Figure 4: Circumferential strain contours of DS6 predicted by the proposed method (left) and FEA (right)

The critical strains in the longitudinal direction as predicted by FEA and the proposed technique are the tensile strains at the external surface of the dent. Strain concentrations are also predicted at the peak of the dent with a maximum value of 4% predicted by both strain models. The obtained strain contours are shown in Figure 5.

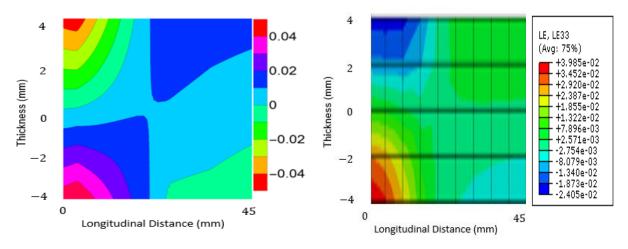


Figure 5: Longitudinal strain contours of DS6 predicted by the proposed method (left) and FEA (right)

The maximum equivalent strains are combined using the codified equations presented in equations 4 and 5 and the obtained values are compared to the plastic equivalent strains predicted by FEA and the ASME B31.8 strain limit. The variation in the obtained results is shown in Figure 6.

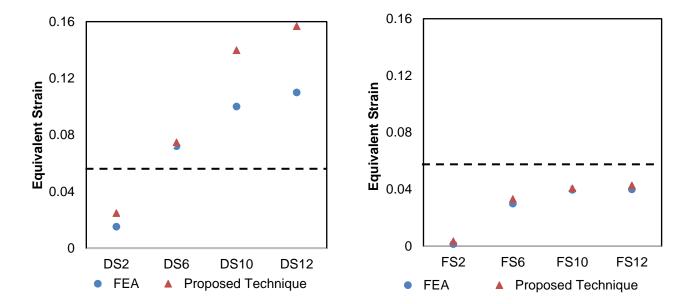


Figure 6: Equivalent Strains from the DS models (left) and FS Models (right)

From the plots above it can be deduced that the geometry of the dent may be a key factor in determining the severity of the dent. This analysis shows some discrepancy between the recommendations from the ASME B31.8 criteria and the CSA Z662 depth-based codes as the DS12 and FS12 models violate both the permissible depth criterion (as stipulated in the CSA Z662 codes) but while the DS12 model develops strain values above the limit in the ASME B31.8 codes, the FS12 model develops strains within the acceptable limit of 6% (represented by the horizontal line in the plots in Figure 6).

The proposed technique is also observed to lose accuracy (relative to the FEA) as the strain magnitude increases, however, this does not necessarily affect the reliability applicability of the method as the proposed technique predicts conservative strain values which would serve as an indication that more detailed analysis might be required.

### 4 DISCUSSIONS

The discrepancies observed in the magnitude of peak strains predicted by the numerical models and the proposed mathematical model are due to the fact that the mathematical models are formulated such that the normals to the mid-surface remain straight while the numerical model developed with FEA allows for the deformation of the normals to its mid-surface. This additional degree of freedom of the numerical model thus allows for a more accurate prediction of strains, however, the penalty for the increased accuracy is the increased computation time for the dent analysis. The implementation of the proposed procedure significantly reduces the computation time required for the in-depth analysis of thin-walled structures as the strains are predicted instantaneously with the input of the coordinates of the dent profile. The equations used to evaluate the equivalent strains in this study are not accurate as they are based on the existing strain based models which assume that the strain components are aligned in the principal axis. The proposed technique can be extended to obtain the shear strain components which would account for scenarios when the dent features are not aligned in the principal directions.

#### 5 CONCLUSION

The proposed kinematic model for the evaluation of strains from the deformed pipeline is a good alternative to FEA for the analysis of dents in pipelines. The procedure introduced allows for detailed strain analysis of dented pipelines as the different strain components associated with the deformation can be evaluated

solely on the geometric properties of the deformed pipe. Further work to investigate the sensitivity of the methodology by varying the wall thickness of the pipe, dent shape and diameter of the pipe would be beneficial in investigating the suitability of this procedure for a host of practical scenarios. The analytical approach presented in this study for predicting the strains in thin walled structures would seem useful for reducing the computation time required to analyze the strain state of dented pipelines.

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