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Vancouver, Canada

May 31 – June 3, 2017/ *Mai 31 – Juin 3, 2017*

STRUCTURAL DYNAMIC CHARACTERISTICS OF THE LONDON ANCIENT EGYPTIAN OBELISK

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Abstract: Ancient Egyptian obelisks have existed for several millennia and have withstood numerous natural disasters among which are several earthquakes. One of these obelisks is the London obelisk. This study investigates the structural dynamic characteristics of the obelisk using a two-staged approach. The first, involved a series of tests to study the mechanical properties of the Red Aswan granite of which the obelisk was made while the second incorporates the test results in modeling the obelisk using finite element software. An Eigen-value modal analysis was performed in order to generate the natural periods and corresponding modes of vibration of the structure. These natural periods were compared to the dominant periods of earthquakes in order to analyze the obelisk's response to loading and its degree of resonance. Following, a time-history analysis was performed in which the structure was subjected to an earthquake signal. The results show that the London obelisk could safely withstand seismic loads.

Keywords: - Structural Dynamics; Earthquake Engineering; Obelisks; Modal analysis; Dynamic Analysis; Granite

1. INTRODUCTION

The ancient Egyptians used numerous materials in building different structures for religious and sacral purposes such as pyramids, temples, obelisks and various forms of tomb structures. While buildings used for the dwelling of royalty and nobility were built using bricks made from the mud available by the annual flooding of the Nile, funeral and religious structures were built using more resistant material to last for eternity (Klemm and Klemm 2001). Obelisks are regarded as one of the key attractions of the ancient Egyptian civilization that are still present today. There are several obelisks today decorating major international locations like the Vatican, Lateran, Paris, London and New York, after being moved from Egypt, in addition to the Hatshepsut obelisk in the Karnak temple in Luxor, Egypt. According to (Engelbach 1923), the number of obelisks built throughout various dynasties of ancient Egypt must have been numerous. He reported that there must have been more than 50 obelisks with a length exceeding 10 meters. He attributed the decrease in number of obelisks present today to earthquakes and soil-subsidence problems.

The use of granite in construction in Ancient Egypt dated back to the reigns of kings Khufu and Menkaure as it was used in the construction of temples and in the king's chambers within pyramids (Klemm and Klemm 2008). Moreover, the majority of obelisks studied by historians and archeologists are made of red granite from quarries located to the south of Aswan in Southern Egypt. The discovery of an "unfinished obelisk" on site in that area was a clear evidence that ancient Egyptian obelisks were extracted from that quarrying area (Engelbach 1923) (Klemm and Klemm 2008) (Kelany, et al. 2009)**.** This type of granite is generally named "Red Aswan Granite". It consists of large reddish feldspar crystals in a fabric that also contains quartz, plagioclase and biotite (Klemm and Klemm 2008) (Serra, et al. 2010) (Liritzis, et al. 2008). The practical reasons for selecting this type of granite to build the obelisks is because its natural joints are far enough to enable the extraction of large structures like obelisks as a single piece of stone mass with no fissures or cracks (Engelbach 1923). It was reported that granite was quarried using stone hammers made from the large dolerite dykes found in that area. These stone hammers had sharp edges that were utilized to break away the surface of the rock mass by removing fine chips of the material through impact and knocking under the weight of the stone hammer (Klemm and Klemm 2008) (El-Sehily 2016).

Ancient Egyptian workers used the dolerite stone to make trenches around a stone mass, free from critical fissures and cracks, to free it from the parent rock. This theory for the methodology used in extracting obelisks is reinforced by observations of concave rounded indentations in the trenches along the sides of the unfinished obelisk in Aswan that are still evident today (Klemm and Klemm 2008). Several ancient Egyptian obelisks were moved to new locations around the world. The obelisk currently known as the Vatican obelisk was moved to Rome in Roman times and then moved again to its current location to the Piazza di San Pietro in 1586. Meanwhile, the Lateran obelisk, which is considered the tallest standing ancient Egyptian obelisk in the world, was erected in Piazza San Giovanni in Laterano in Rome in 1588. Another obelisk was moved to Paris in 1836 under the guidance of a French engineer named Lebas. In the second half of the 19th century, two obelisks built during Tuthmosis III reign were moved to London and New York in 1878 and 1881, respectively (Engelbach 1923).

While extensive research was conducted on the red Aswan granite from the archeological, geological and chemical perspective (Klemm and Klemm 2001) (Kelany, et al. 2009) (Serra, et al. 2010), little information is available on some of the mechanical properties of this material such as its modulus of elasticity. Moreover, there is a scarcity in the information regarding the seismic behavior of monumental structures built using this material such as obelisks and tombs. Hence, the objective of this work is to conduct experimental testing to first determine the mechanical properties of red Aswan granite such as the unit weight, ultimate compressive strength and modulus of elasticity. Consequently, these parameters are used as input parameters to study the structural dynamic characteristics of the London obelisk through performing a free vibration analysis using a finite element model. Finally, a time-history analysis is conducted in which a real earthquake signal is applied on the structure to check its ability to withstand the earthquake vibrations.

2. MECHANICAL TESTS ON RED ASWAN GRANITE

Samples of Red Aswan granite were acquired from a quarry operating in close proximity to the “unfinished obelisk” site and sawn into eight 50 mm x 50 mm x 50 mm cubes. The unit weight was determined according to the procedures outlined in ASTM C97 (American Society for Testing Materials 2015). Moreover, the specimens were tested according to the experimental program outlined in ASTM C 170-16 for testing the compressive strength of dimension stone (American Society for Testing Materials 2016). The test was carried out using MTS 810 Material Test System equipment.

The load and the resulting displacement were instantaneously measured through an electronic data acquisition system until the specimen failed and the ultimate load at failure is recorded. The output data were used to calculate the stress and strain for each sample and to plot the stress-strain curve for each sample as shown in Figure 1. Consequently, the elastic modulus was calculated for each specimen from its stress-strain curve. The unit weight and the elastic modulus value were used as input parameters for red Aswan granite in numerical models to study the behavior of the obelisks under dynamic loading.

The results of the experimental work performed were used to represent the material properties of the red Aswan granite of which the obelisk was constructed as shown in Table 1. As the standard deviations of the material properties were not large when compared to the average values, the average values were considered to be fairly representing the general behavior of this material and hence the density was set to be 2541 kg/m^3 while the modulus of elasticity was considered to be 5425 MPa and the ultimate compressive strength was considered to be 140.6 MPa. It could be also noticed from the stress-strain diagram shown in Figure 1 that the red Aswan granite could be classified as a brittle material as it has negligible plastic strains hence having negligible ductility and considered as a linearly elastic material which agrees with the shape of the fractured surface of the failed specimen shown in Figure 2.

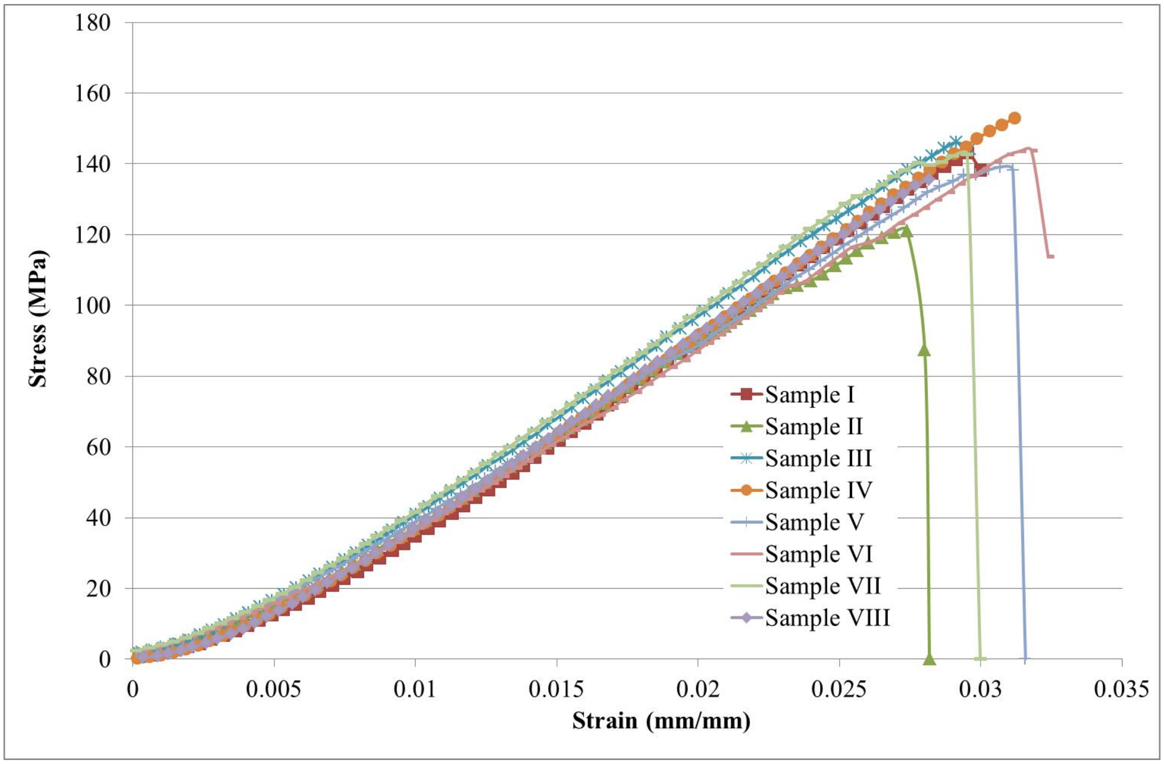

Figure 1: Stress-strain diagrams for the tested red Aswan granite samples.

Figure 2: A red Aswan Granite tested specimen.

Table 1: Mechanical Properties of Red Aswan Granite.

Specimen #	Unit Weight (Kg/m^3)	Ultimate Compressive Strength (MPa)	Elastic Modulus (MPa)
I	2580.39	143.15	5687
II	2525.49	121.14	5202
III	2517.64	146.04	5608
IV	2556.86	153.02	5508
V	2529.79	139.1	5073
VI	2256.86	143.76	5137
VII	2517.64	142.68	5679
VIII	2544	135.62	5509
Average	2541.09	140.56	5425.38
Standard Deviation	22.43	9.34	249.91

3. NUMERICAL MODELLING

3.1. Finite Element Model

The London obelisk was modeled on the finite element software SAP2000 (Computers and Sturctures Inc. 2016) in which three-dimensional eight-node solid elements were used as shown in Figure 3. This choice of solid elements targeted representing the mass distribution and stiffness within the obelisk structure in the most accurate way as if such a structure was modeled using one dimensional or two dimensional elements there would have been a high loss of accuracy within at least one dimension however the three dimensional elements guarantee the highest possible accuracy in results.

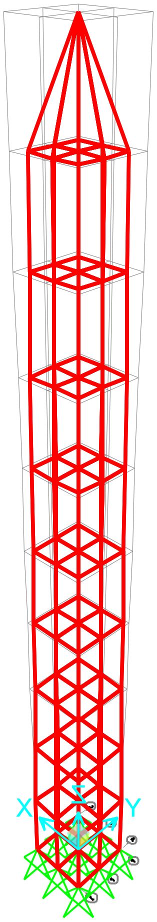

Figure 3: The three dimensional finite element model of the London obelisk.

The modeled obelisk had a total height of 20.88 m, a squared cross-section of 2.38 m x 2.38 m and tapered until having a 1.65 m x 1.65 m cross-section at the base of the pyramidal located at the top of the obelisk at a point 19.23 m above the obelisk base (Engelbach 1923).

3.2. Modal Analysis

The modal analysis was performed in order to determine the modes of vibration of the obelisk and the corresponding natural period of each. The results of the modal analysis shown in Table **2** are ordered in a descending order with the first mode corresponding to the largest natural period and the largest modal participation factor of 57% as a translation in the y direction and 0% in the other directions as shown in Table **2**. This first mode is the principal translational mode in the y-direction as shown in Figure 4.

Also, it could be noticed that the second mode of vibration has exactly the same natural period of vibration however the mode itself is changed as the modal participation factor is 57% as a translation in the x direction and 0% as a translation in the other directions. This is attributed to the fact that the cross-section is squared hence the first two translational modes are identical however they are acting perpendicular to each other while having the same period as the stiffness and mass in the x direction are exactly equal to those in the y direction hence it is expected to have the principal translational modes in each direction occurring at the same natural period. This could also be seen occurring for modes three and four, six and seven, nine and ten as each of these modes has exactly the same natural period of vibration with another mode with the same values of modal participation factors in the x direction in one mode equal to those in the y direction in the other mode.

Table 2: The natural periods and the modal participation factors.

		Modal Participation Factors		
Mode #	Period (s)	Translation in X	Translation in Y	Translation in Z
1	0.592885	0%	57%	0%
2	0.592885	57%	0%	0%
3	0.124909	1%	21%	0%
4	0.124909	21%	1%	0%
5	0.077982	0%	0%	0%
6	0.050550	9%	0%	0%
7	0.050550	0%	9%	0%
8	0.046188	0%	0%	80%
9	0.033548	1%	4%	0%
10	0.028123	4%	1%	0%
11	0.028123	0%	0%	0%
12	0.021079	2%	1%	0%

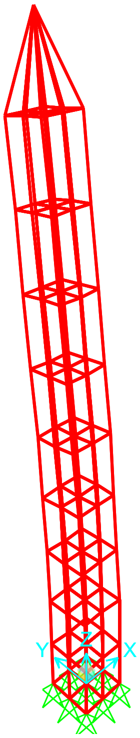

Figure 4: The first mode of vibration

On the other hand, it could be noticed that the fifth and eleventh modes have no translational components as they are both twisting modes as shown in Figure 5. However, such modes are not considered to be of a concern as the structure has a center of rigidity which is coinciding with its center of mass causing no eccentricity and hence no expected excitation of these twisting modes.

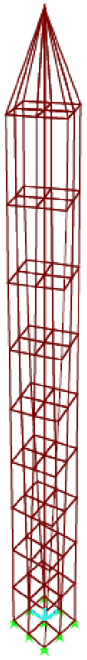

Figure 5: Fifth mode of vibration

It could be also noticed that for eleven of the twelve modes, the vertical modal participation is 0% while it is 80% for mode number eight which has modal participation factors of 0% in the two horizontal directions as it is the principle mode in the vertical direction as shown in Figure 6.

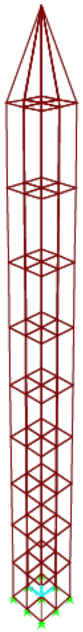

Figure 6: Eighth mode of vibration

Additionally, and as shown in Table **2**, the period for the first two modes of vibration was 0.59 s which is within the range of the dominant periods of typical earthquakes and typical boundary winds which exceed 0.5 seconds (Tedesco, McDougal and Ross 1999). That implies that the London obelisk is expected to resonantly respond due to earthquakes and winds and will mainly behave in a dynamic manner when subjected to such dynamic loads. Hence, it is necessary to perform a dynamic analysis when structurally analyzing this obelisk as a static analysis is not expected to produce accurate results.

3.3. Time-History Analysis

Following on the modal analysis, a time-history analysis was performed in order to determine the time-varying response of the structure while subjected to a real earthquake using the Newmark direct integration method. The earthquake chosen was the 1940 El Centro earthquake that happened in California, USA with perceived intensity of X on the Mercalli intensity scale (Wikimedia Foundation, Inc. 2016). This earthquake is significantly stronger than any recorded earthquake that has ever been recorded in Egypt in which the obelisk is located. Hence, the signal of the earthquake event was scaled down to match the peak ground acceleration of Luxor, Egypt (where the London obelisk was located for most of its life), which is 0.125g according to the Egyptian loading code (Housing and Building National Research Center 2008).

The Newmark direct integration method used applies the concept of proportional damping in which the coefficients α and β are multiplied by the stiffness and mass matrices. These two coefficients were calculated based on a conservatively assumed damping of 1%. This percentage of damping is based on conservatively assuming that the granite has a similar performance as concrete which is typically considered to have a damping even larger than this percentage (Tedesco, McDougal and Ross 1999). Another factor taken into account is the time step size as according to (Bathe 2006) the solution will not converge if the time step exceeds T/π and it could only produce accurate results if it is less than or equal to $T/10$ where T is the smaller of the natural period of the highest mode of interest and the dominant period of loading.

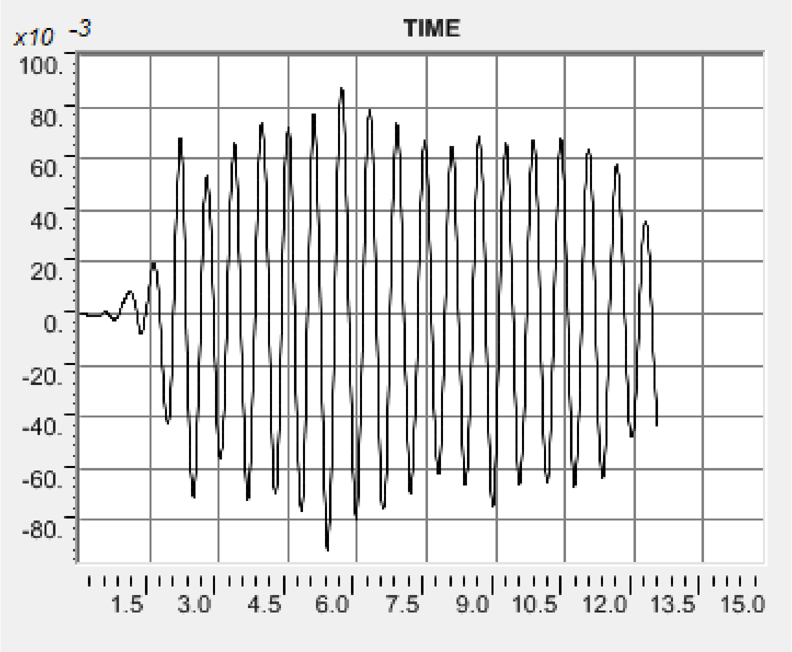

Figure 7: Variation of the horizontal displacement of the uppermost point with time.

The results of the time-history dynamic analysis were plotted twice; once in the time domain in which the horizontal displacement of the uppermost point was plotted versus time as shown in Figure 7 and the other time as a response spectrum in which the spectral horizontal displacement is drawn versus the period of vibration as shown in Figure 8. The maximum horizontal displacement shown in Figure 7 was 91.5 mm which is 1/228 of the height of the obelisk showing that this obelisk could safely withstand such an earthquake in terms of the deflection criteria. Additionally, the maximum stress due to the seismic load was 0.52 MPa which is significantly less than the ultimate strength of the red Aswan granite which was 140.1 MPa showing that the structure passes the strength criteria.

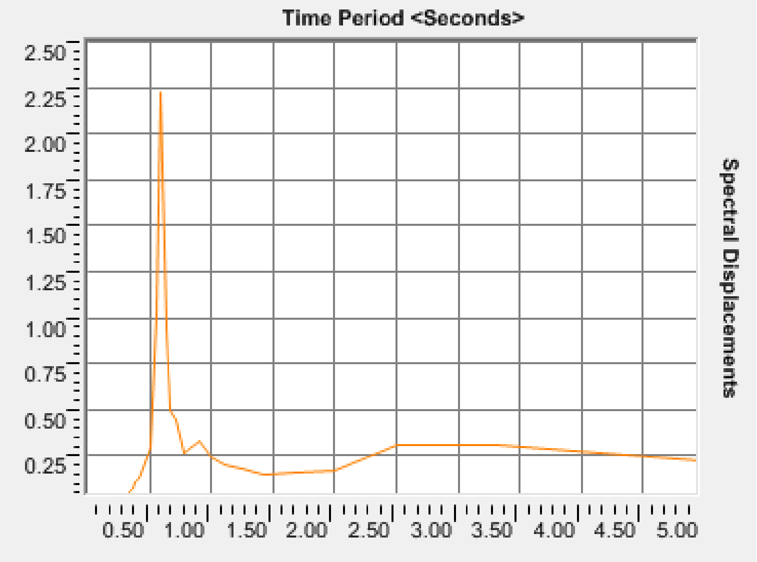

Figure 8: Response spectrum of the spectral horizontal displacement of the uppermost point with the period of vibration.

On the other hand, when examining the response spectrum shown in Figure 8 it could be obviously seen that the maximum horizontal spectral displacement occurs at a period that is matching the principal natural period of 0.59 s reported in Table **2** and the area beneath this region is significantly large indicating a significantly large resonant component of vibration. Hence, the 91.5 mm drift is mainly a result of the first mode of vibration that was surprisingly within the deflection criteria explaining how this obelisk existed to date and raising a question mark on how did the ancient Egyptians design such a structure thousands of years before the invention of structural dynamics?

4. CONCLUSIONS AND RECOMMENDATIONS

The following conclusions could be drawn from the performed study:

- The red Aswan granite has an average density of 2541 kg/m^3, an ultimate compressive strength of 140.6 MPa and a modulus of elasticity of 5425.4 MPa.
- The natural periods for the twelve modes of vibrations of the studied obelisk ranged between 0.59 seconds and 0.02 seconds.
- The principal natural periods of vibration for the various modes were within the range of the dominant periods of the earthquake ground motions implying that the resonant component of vibration is expected to be significant.
- According to the results produced by the time-history analysis, the maximum spectral displacement occurred at a period equal to the principal natural period of the structure confirming the occurrence of resonance due to the earthquake signal.
- The maximum stresses are significantly less than the ultimate strength of the red Aswan granite proving that the structure could safely withstand the earthquake load.
- According to the results produced by the time-history analysis, the horizontal drift at the top of the obelisk was 1/228 of the obelisk height which is within the safe limits of vibrations explaining how that obelisk existed for thousands of years.

Based on the performed study, the author provides the following recommendations for future research:

- A set of wind tunnel tests or alternatively computational fluid dynamic (CFD) models are recommended to be performed in order to produce the distribution of wind pressure on the surfaces of the obelisks in order to dynamically analyze the obelisks under synoptic wind load.
- More efforts are needed from the sides of Egyptologists in order to find out how the ancient Egyptians knew that the geometry of the obelisks together with the red Aswan granite as a material will produce structures that could with stand earthquakes for thousands of years before the invention of the science of structural dynamics.

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