Leadership in Sustainable Infrastructure
Leadership en Infrastructures Durables

Vancouver, Canada
May 31 – June 3, 2017/ *Mai 31 – Juin 3, 2017*

A BAYESIAN MODEL FOR IMPROVED OPTIMAL BID PRICE ESTIMATION IN TRANSPORTATION PROJECTS

Abotaleb, Ibrahim S.[1], and El-adaway, Islam H.[2,3]
[1] Graduate PhD Student, Civil and Environmental Engineering, University of Tennessee-Knoxville, USA.
[2] Associate Professor, Civil and Environmental Engineering, University of Tennessee-Knoxville, USA
[3] eladaway@utk.edu

Abstract: Transportation projects represent 42% of the total public construction projects in the US. With such immense size, proper competitive bidding is a must to ensure appropriate utilization of taxpayer's money. Contractors submitting high bid prices are less likely to be awarded projects. Contrarywise, those submitting low bid prices are awarded but they become claim-oriented to recover the resulting losses; leading to quality, schedule, and cost impacts. Several models have been developed to help contractors determine balanced bid prices based on analyzing competitors' history. However, the statistical integrity of such models is compromised in cases of imperfect information and dynamic behavior; where a competitor's old bidding strategies behavior contradicts its more recent ones. This paper presents an advanced model that utilizes Bayesian statistics and decision theory for optimal bid price determination. The developed model is based on a three-stepped algorithm that enables drawings sound inferences in cases of incomplete historical data and dynamic behavior of competitors. The first step is fitting the competitors' data into appropriate Bayesian prior density functions. The second step is developing the likelihood functions through recent observation(s). The third step is developing posterior distributions from which the joint probability of winning and optimum bidding price can be calculated. The proposed model was applied to a case study to demonstrate the effects of different parameters. The research will be beneficial to the transportation infrastructure economy by ensuring that contractors submit reasonably priced bids; which will make them less susceptible to claim-oriented behavior and eventually lead to healthier contracting.

1 Introduction

The transportation infrastructure is considered one of the major sectors in public spending and one of the most significant areas in construction. In fact, the total value of transportation construction was $125.7 billion in 2014 (US DOT 2015). Moreover, currently, transportation projects constitute 11.6% of the total construction projects and 42% of the public construction projects (Census Bureau 2016). The American Society of Civil Engineers (ASCE) estimates that more than $2 trillion dollars are needed to rehabilitate and expand America's existing transportation infrastructure (ASCE 2011). With this enormous volume of the current and upcoming transportation projects, wise investment is a must. Since most of the transportation projects are publicly funded, such projects are subject to competitive bidding where the qualified contractors with the lowest bid prices is awarded the projects. This ensures that taxpayers obtain the value for money and forces the contractors to pursue value engineering and adopt technological innovations (Lingard et al 1998). The bid price B_i for each contractor i is the summation of the estimated total cost of the project C_i and the decided markup M_i (Eq. 1); where M_i is generally a percentage of the estimated total project cost that accounts for profits and unforeseen risks.

[1] $B_i = C_i(1 + M_i\%)$

Higher markup percentage means more profits for the contractor, but also means lower probability of winning the bid since other competitors will be expected to submit lower bid prices. On the other hand, lower markup increases the probability of winning the bid but minimizes the profit. In this case, the contractor is susceptible for what is called the “winner’s curse”. The winner’s curse is the situation where the bidder with the lowest bid price wins the project contract based on a submitted bid less than the true cost (Ahmed et al 2016). In such case, the winner is cursed with negative profits. Generally, contractors who win bids with very low prices become claim-oriented since day one of the project. This results in quality deterioration and financial losses to the owner, which is eventually the taxpayers in transportation projects. As such it is essential that the contractor submits a bid with a reasonable price that is not too high or not too low. Determining the optimal bid price that maximizes the probability of winning and minimizes the winner’s curse is an endeavor that has been undertaken by construction researchers over the years based on the valid assumption that analyzing the competitors’ historic bidding patterns can improve a contractor’s strategic competitiveness (Friedman 1956, Capen et al 1971).

Current statistical approaches for optimal bid price estimation require extensive historical data and complete information of the competitors’ bidding schemes. Moreover, the current models do not make the distinction between old historical data and recent historical data, but rather look at them as equally important data points. Such models do not hold in cases of competitors having dynamic bidding behavior; where they decided to shift their bidding scheme at a certain point in time. Also, they do not hold in case the available historic data is very old and not representative of the current bidding scheme. To tackle the problem of incomplete historic data and dynamic behavior of competitors, the authors opted to employ Bayesian statistics for its validated capability of dealing with uncertainties and behavioral dynamics (Bernardo 2011). The objective of this paper is to present a Bayesian-based statistical model for optimal bid price determination that is valid in cases of incomplete historical data and dynamic behavior of competitors.

2 Background

2.1 Frequentist Statistical Models Employing Decision Theory

According to decision theory, the optimum bidding markup is the one that maximizes both the probability of winning and profits. Usually, a contractor will have his cost estimate C_j for each of the previous bids j. Also, especially in transportation and other public projects, he would have the bid prices B_{ij} of each of his competitors i in such bids. He would then use Eq. 1 to estimate the markup percentage of each of the competitors. Assuming that the cost estimate is not the same for each of the competitors, the contractor can add a stochastic variable σ which represents the expected variance of cost estimate between the contractor and the competitor i in bid j. Accordingly, the contractor would represent each historic data point for each competitor as a density function instead of a discrete point. Finally, a probability distribution of markup percentages $f_i(r)$ is formed for each competitor by adding all the density functions of its corresponding data points; where, the probability $P_i(r)$ of winning competitor i at markup r is equal to the probability of competitor bidding with markup higher than r (Eq. 2).

[2] $P_i(r) = \int_r^{\infty} f_i(r)\, dr$

If the contractor is competing against more than one competitor, he would obtain the probability of winning each competitor separately and then use Friedman’s formula (Friedman 1956) or Gates’ formula (Gates 1967) to obtain the probability of winning all competitors. Friedman’s formula views the competitors’ bids as independent while Gates’ formula views them as dependent. Both formulas are widely accepted in the literature (Crowly 2000). Generally, Friedman’s formula results in a bid price with lower markup percentage than Gates’ formula. Accordingly, Friedman’s formula helps the contractor win more projects than Gate’s formula. However, due to the low markup, Friedman’s formula does not provide the contractor with high long-term profits as those of Gates’ formula (Benjamin and Meador 1979). Friedman’s formula is shown in Eq. 3 and Gates formula is shown in Eq. 4. According to decision theory, the optimal markup is the one at which a certain utility function is maximized. The utility function in this case is the expected profit; where the expected profit at any markup percentage is the multiplication of the probability of winning and the profit as shown in Eq. 5.

[3] $P_{win}(r) = \prod_{i=1}^{n} P_i(r)$

[4] $P_{win}(r) = \left[\sum_{i=1}^{n}\left(\frac{1-P_i(r)}{P_i(r)}\right) + 1\right]^{-1}$

[5] $EP(r) = r.P_{win}(r)$

Significant works employing decision theoretic concepts in conventional statistical models are those of Yuan (2012), Yuan (2011), Skitmore et al (2007), Touran (2003), Lo and Lam (2001), Ranasinghe (2000), Skitmore and Pemberton (1994), Skitmore (1991), Carr (1982), Winkler and Brooks (1980), Dixie (1974), and Rosenshine (1972).

2.2 Bayesian Statistics

The two major domains in mathematical statistics are conventional (or frequentist), and Bayesian. Bayesian statistics provide a more complete paradigm for both statistical inference and decision making under uncertainty (Bernardo 2011). Additionally, it makes it possible to incorporate scientific hypothesis, or educated 'beliefs', in the analysis by the means of the prior distributions when the available data is not sufficient to produce sound frequentist statistical inferences (Cowles 2012). According to Bayes' equation (Eq. 6), there are two sources of information about the unknown parameters of interest; the prior distribution and the likelihood function.

[6] $p(\theta|D) \propto f(D|\theta)\pi(\theta)$

The prior distribution $\pi(\theta)$ represents the original prior data based on the available information while the likelihood function $f(D|\theta)$ represents the observed behavior of uncertainty. Based on those two sources of information, the posterior distribution $p(\theta|D)$ is calculated according to Eq. 6. All statistical inferences are eventually gathered from the posterior distribution. The proportionality symbol $\propto$ means that the right hand side of the equation has to be normalized so that its integration over its support is equal to 1. The posterior function $p(\theta|D)$ provides a weighted compromise between the prior information and the likelihood data in statistically sound environment (Cowles 2012).

3 Methodology

The Bayesian framework is what signifies the developed statistical model. The incorporation of Bayesian statistics is elucidated in the treatment of historical bidding data of competitors. In frequentist statistics, all historical data points are treated equally with similar weights and fitted in density functions. On the other hand, the developed Bayesian model makes the distinction between old historical data and the more recent historical data. In other words, the old historical data are represented in the prior distribution $\pi(\theta)$. The more recent observations are represented in the likelihood function $f(D|\theta)$; where such observations are considered a good representative of the competitors' current behavior. Accordingly, the model is sensitive to both the less recent and the more recent observations with stronger emphasis on the more recent ones because a competitor is more likely to continue its recent bidding pattern than to return to an older one. The developed algorithm can be divided into three stages: The first stage is fitting the competitors' data into appropriate Bayesian prior density functions. The second stage is developing the likelihood functions through the most recent historic observation(s). The third stage is developing the posterior distributions from which the joint probability of winning and the expected profit can be calculated. Each stage has various steps. The following paragraphs provide explanations for such steps.

3.1 Step 1: Preliminary Distribution Density Function (PDDF)

There is stochastic variability between the cost estimate of the contractor and the cost estimate of each competitor in each single bid. This stochastic variability is referred to as σ_{ij}. For example, if $\sigma_{ij} = 1$, then the firm expects that his cost estimate his competitor's cost estimate in bid j lies within a standard deviation of 1% of the cost estimate of the firm. In the developed model, a preliminary distribution density function (PDDF) is calculated for each competitor in accordance to Eq. 7.

[7] $pd_i(\mathrm{r}) \propto \sum_{j=1}^{n_i}\left[N\left(r\middle|\frac{B_{ij}\times 100}{C_j}-100,\sigma_{ij}\right)\left(1-\emptyset_{ij}\right)+\Gamma\left(r|\alpha_{ij},\beta_{ij}\right)\left(\emptyset_{ij}\right)\right]$

In the PDDF function (Eq. 7), r signifies the markup; where $r \in (0,\infty)$. Also, n_i is the total number of available bid value data points for competitor i. The term $N(r|x,y)$ refers to the normal distribution probability density function with mean x and standard deviation y on the support of r, and the term $\Gamma(r|x,y)$ refers to the gamma distribution probability density function with shape x and rate y. The term σ_{ij} reflects the contractor's perception of variance between its cost estimate for bid j and the cost estimate of its competitor i for bid j. As shown in Eq. 7, for each competitor, each historic bid provides a normally distributed density with mean equals to the estimated markup of the competitor and a standard deviation of σ_{ij}; where the value of σ_{ij} is project-dependent and not fixed for all projects or all competitors.

The term $\emptyset_{ij}$ is a binary variable; where $\emptyset_{ij}=0$ if $B_{ij}/C_i \geq 1$ and $\emptyset_{ij}=1$ if $B_{ij}/C_i < 1$. This means that as long as the competitor's bid price is higher than the contractor's cost estimate in a certain bid, the normal distribution part of the function will be active. If the competitor's bid price is higher than the contractor's cost estimate, then the gamma distribution part will be active. Having B_{ij}/C_i that is less than 1 is very uncommon. It might result from either 1) the contractor having an unreasonably high cost estimate, or 2) the competitor bidding for the project with negative markup. The gamma term represents what is called the educated belief; where the α_{ij} and β_{ij} parameters are inputted based on the contractor's belief of the competitor's markup density in such cases. Both the α and β parameters are non-zero non-negative parameters that collectively control the skewness, location of peak, and width of peak of the Gamma probability density function. The Gamma distribution is selected to represent the educated belief in this case because it is a shape shifter that can assume a range of shapes, and because its support extends in the positive real number range only from 0 to ∞. This ensures that the $pd_i(\mathrm{r})$ function does not have values in the negative r region. A final note about the PDDF is that the historical bids that are used in it are those which precede the "latest common bids". The latest common bids are the less recent bids; further definition is provided in step 5.

3.2 Step 2: Sampling from the PDDF

In this step, for each competitor, independent and identically distributed *iid* random variables are sampled from its preliminary density function using Markov Chain Monte Carlo Metropolis-Hastings (MCMC M-H) algorithm. MCMC M-H is a technique of generating random variables following any desired probability density function (PDF), named the target distribution f, throughout simulated draws from an easy-to-sample PDF $q(y|x)$, named the proposal distribution (Robert and Casella 2009). The use of MCMC M-H is essential in this step because the resulting PDDF does not follow any traditional parametric PDF, but rather a non-parametric one.

There are several mathematical requirements for a successful sampling using the M-H algorithm, however, these requirements are minimized if the Markov transition kernel is generated using a random walk following a symmetric distribution. A random walk means that each generated point is a function of the preceding generated point. The random walk allows for local exploration of random variables all over the support of the target function whilst maintaining the ergodic properties of the chain. To construct a Markov transition kernel $X_0,\ldots,X_T$ via the Metropolis-Hastings algorithm; given an initial random variable X_t, generate $Y_t = X_t + \varepsilon_t$ then obtain X_{t+1} (Eq. 8 and 9); where ε_t is a random perturbation with a normal distribution of mean = 0 and standard deviation = 2. So for each simulation, a random variable ε_t is generated from g and added to the preceding X_t.

[8] $X_{t+1}=\begin{cases} Y_t & \text{with probability } \rho(x_t,Y_t) \\ X_t & \text{with probability } 1-\rho(x_t,Y_t) \end{cases}$

[9] $\rho(x,y)=\min\left\{\frac{f(y)}{f(x)},1\right\}$

The resulting Markov kernel will form the prior distributions. Since, computing powers of standard computers are very powerful, the number of simulations S for each competitor is suggested to be 10,000;

where the first 1,000 simulations are not taken into consideration for they are considered the chain's burn-ins. The burn-in is for the purpose of insuring, in theory, the Markov chain is intrinsically equivalent to a standard *iid* simulation from f. The generated Markov chain for each competitor, which are the output of this step, are referred to as $[X_{1,001}, X_{1,002} \dots, X_{10,000}]_i$. The numbering starts from 1,001 because the first 1,000 points are eliminated for the chain's burn, and ended at 10,000 because the total number of simulations (draws) is 10,000. The initial random variable X_0 is proposed to be a number within the range of the markup percentages. The target function is pd_i; which is obtained from step 1. The flow chart of the MCMC M-H algorithm is shown in the right part of Figure 1.

3.3 Step 3: Finding Best Fit Probability Distributions for the Resulting Markov Chains

The Markov chains obtained from the previous step are fitted in density functions that will be used as prior distributions. To do so, each of the resulting variables $[X_{1,001}, X_{1,002} \dots, X_{10,000}]_i$ is named M_{ij} where M represents the markup percentage, i represents the number of competitors and j represents the number of generated random variables in the Markov chain of each competitor. Selecting the best fitting probability distribution from a pre-defined family of distributions requires a systematic procedure. This procedure starts with plotting the histogram of M_{ij} of each competitor. If the histogram has a single peak, then plot the Cullen-Frey Graph; which is a graph that displays which parametric distribution best fits the dataset (Cullen, and Frey 1999). To ensure that the selected distribution is a good fit for the data, perform the one-sample Kolmogorov-Smirnov (K-S) test; which is a statistical nonparametric test that is used to compare a sample distribution with a reference probability distribution. The null hypothesis H_0 is that the used probability distribution fits the data. The null hypothesis is rejected if the p-value is less than the desired significance level α. The most commonly used number for α in these cases is 0.05 (Nuzzo 2014). Accordingly, if the selected probability distribution had a p-value higher than 0.05, then we conclude that it fits the data.

Most parametric probability distributions have one significant peak. If the histogram of M_{ij} has multiple significant peaks, then the Cullen-Frey Graph and the K-S test will not be of any good. In such case, logspline fitting is recommended. The logspline fitting forms a function from a space of cubic splines that a finite number of pre-specified knots and are linear in tails (Kooperberg and Stone 1991). The K-S test, Cullen-Frey Graph, and Logspline fitting can be easily performed using statistical software packages.

3.4 Step 4: Forming the Prior Functions

The prior distribution $\pi(\theta)_i$ for each competitor is the fitted distribution resulting from the previous step. Such prior distribution shall be normalized with a positive support. In cases of incomplete data where the contractor has little accurately recorded data of previous bids, the concept of educated belief should be utilized. As such, the contractor would use a prior distribution based on his expert judgement or on the concept of the average bidder. The concept of average bidder is proposed by Friedman (1956) and acknowledged by other researchers later (Capen et al 1971). The average bidder is a composite of all bidders that the contractor has faced in the past. If the user has very limited information about the competitor's history and intends to form its corresponding prior distribution without sufficient information to form a good educated belief, the authors encourage him to use a probability distribution with parameters that result in a minimally informative prior. This is a traditional practice in Bayesian statistics (Bennet et al 1996). A minimally informative prior is a one with an insignificant peak; such as a normal distribution with a large standard deviation, a Gama distribution with a small rate, or even a uniform distribution. The advantage of Bayesian statistics in this regard is that it allows for such incorporation without jeopardizing the statistical integrity of the solution. Despite that, still having complete and accurate data is much better and much credible than having incomplete data.

3.5 Step 5: Forming the Likelihood Functions

The latest common bids are those that are very recent and share commonality where they represent the current bidding strategies of the competitor. Such commonality can be in the recency; meaning that they are made within a short period of time. It can also be in factors affecting the bidding behavior such as the management team, project type, location, cost range, risk, and so on. No matter what the commonality is, the recency commonality has to exist. Such commonalities strengthen the statistical significance given the conditional properties of the Bayesian statistics. A likelihood function $f(D|\theta)_i$ for each competitor i is calculated only from the latest common bids (Eq. 10).

[10] $f(D|\theta)_i \propto \sum_{k=1}^{K} N\left(r\middle|\frac{Z_{ik}\times 100}{C_k} - 100, \sigma_{ik}\right)$

where, $N(r|x,y)$ is the normal distribution PDF with mean x and standard deviation y, K is the number of latest common bids, Z_{ik} is the bid value of competitor i in bid k, C_k is the contractor's cost estimate of project k, and σ_{ik} represents the stochastic uncertainty of the difference between the firm's cost estimate for bid k and the cost estimate of its competitor i for the same bid.

3.6 Step 6: Obtaining the Posterior Distributions and Deciding on the Optimum Markup

After obtaining the prior and likelihood function for each competitor, the posterior distribution $p(\theta|D)_i$ is calculated and normalized in accordance to Eq. 6. All of the preceding statistic inferences are made from the posterior distributions. In this step, the Bayesian concepts and the decision theoretic concepts meet. After calculating the posterior distribution for each competitor, set $f_i(r) = p(\theta|D)_i$, then use Eq. 2 to calculate the probability of winning each of the competitors. Then use Friedman or Gates's formulas to obtain the probability of winning all competitors. Both Friedman and Gates' formulas are valid and each firm has the freedom to choose which ever suits its view of the bidding process. In the case study, the authors used both formulas and it was shown that there were no significant variances in the resulting optimal markups. The final step is to use Eq. 5 is used to calculate the expected profit $EP(r)$ at each value of markup r. The optimum markup is r where $EP(r)$ is maximized.

Figure 1 shows a flowchart demonstrating the heuristic of the of the developed Bayesian model.

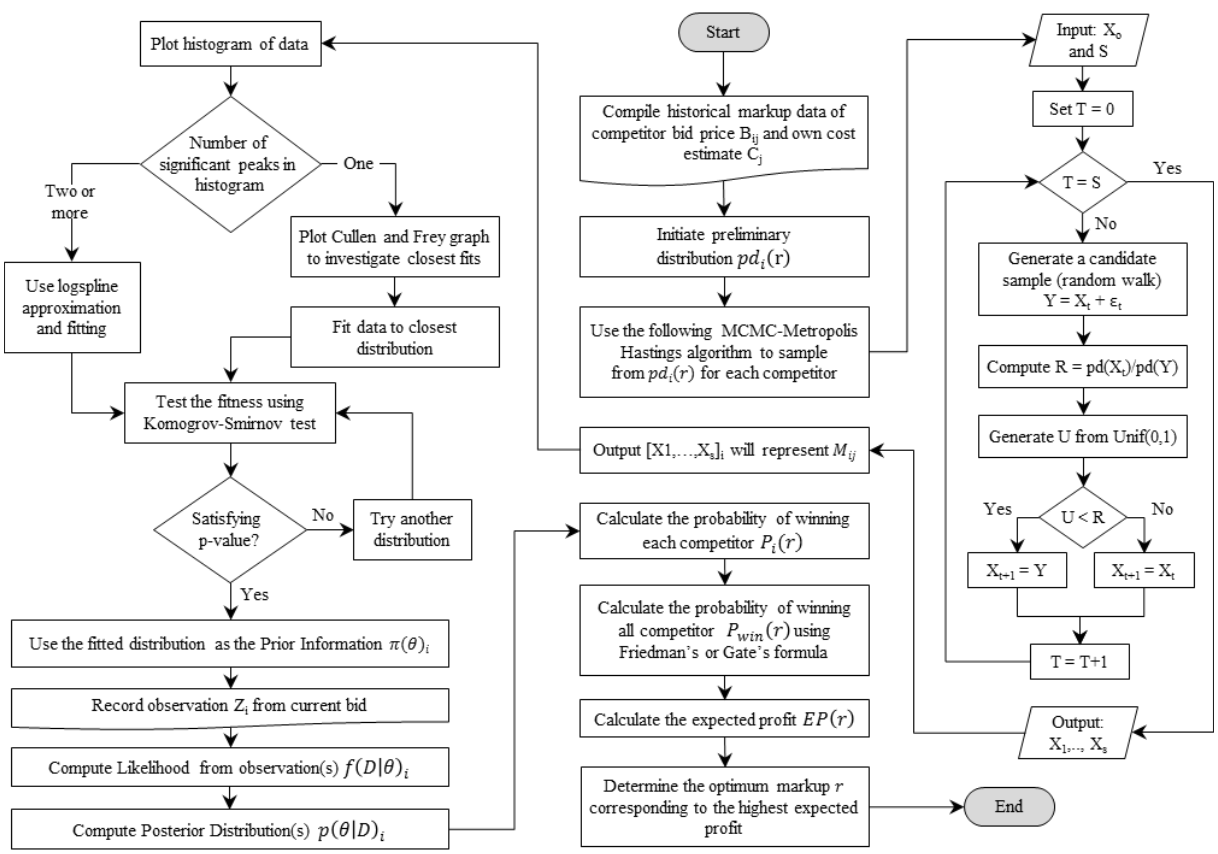

Figure 1: Methodology of the Developed Model

4 Case Study

The developed model is tested on a case study from the literature. The historical data of bid prices were obtained from Skitmore-Pemberton (1994). The results were compared to those of models developed by Skitmore-Pemberton (1994) and Yuan (2011). The comparisons are not made to display the superiority of the proposed model; but rather to demonstrate its use, observe the differences between its results and those of other models to deduct behavior patterns, and illustrate the effects of different parameters.

A total of four competitors were studied and analyzed. The available data were of real bids entered by a construction firm, where it recorded its own cost estimates C_j and the bid prices of each of its competitors B_{ij}. The 4 competitors were coded 1, 55, 134, and 221 by the firm. Since the data did not specify which bids are to be considered as the latest common bids, the authors simulated two different scenarios. In scenario 1, the latest observation for each competitor is considered as the latest common bid; which is the bid that forms the likelihood function. In scenario 2, the latest two observations are considered the latest common bids. The used value of σ_{ij} is 2% for all competitors. Moreover, since the stochastic variability between the firm's cost estimate and the competitors' cost estimate is not known, several values of σ_{ik} were simulated. The values used for σ_{ik} were 2%, 3%, and 4%. With regards to the Gamma distribution parameters (α_{ij} and β_{ij}), the following numbers were used in cases where the firm's cost estimate exceeded the competitor's bid price: For competitor 1, $\alpha_{ij} = 3$ and $\beta_{ij} = 1.3$; for competitor 55, $\alpha_{ij} = 3$ and $\beta_{ij} = 2$; and for competitor 134, $\alpha_{ij} = 5$ and $\beta_{ij} = 2$. α_{ij} and β_{ij} were not required for competitor 221 because all of its bid prices exceed the firm's cost estimates. In reality, when the proposed model is used by a firm, the assumptions will be minimal because the user firm will have all information.

4.1 Results and Discussion

The 6 steps of the model were followed. The PDDF of each competitor was calculated in accordance to Eq. 7. The sampling was made using the MCMC M-H algorithm, where the acceptance rate of the Markov kernel ranged from 77% to 88%; which is a good acceptance rate because it is higher than 25%. Figure 2 shows the convergence of the Markov chains and the resulting histograms. After the sampling, prior distributions were fitted to the resulting Markov chains and the fits were tested using the K-S test. The resulting prior function for competitors 55 and 221 follow the Weibull distribution with shape = 1.503 and scale = 4.22, and shape = 2.106 and scale = 7.181 respectively. For competitor 134, the logspline fitting was used to obtain its prior distribution because the histogram resulting from its Markov chain had more than one significant peak. For competitor 1, the available data points show that his bidding prices were lower than the firm's cost estimate. Accordingly, the corresponding prior distribution was based on an assumption that such competitor is a risk taker and that his markup values are extremely low. As such, the prior distribution of competitor 1 follows the Gamma distribution with shape = 3 and rate = 1.3.

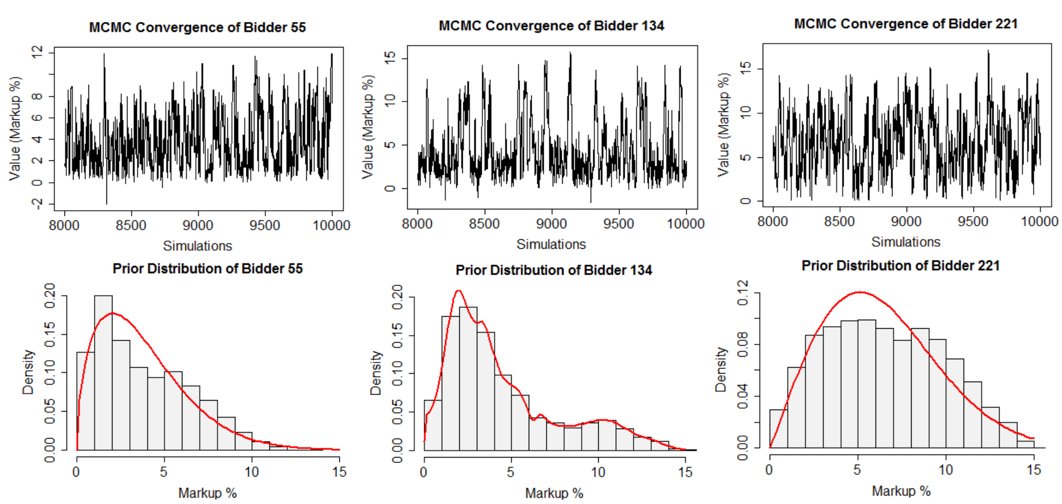

Figure 2: The MCMC Convergence of Competitors and the Resulting Prior Distributions

The resulting optimum markup in the two scenarios are shown in Figure 3. The optimum markup and probability of winning were calculated for bidding against 6 combinations of the competitors. The combinations were selected this way to provide a comparison between the proposed model and the models of Skitmore-Pemberton and Yuan; given that they used the same combinations except for the last one. By comparing the results of Yuan's model and those of Skitmore-Pemberton, it can be observed that Yuan's optimum markup is much lower than Skitmore-Pemberton's. The later takes the cost estimation error into consideration in an amplified way leading to higher markup values. However, Yuan's markup values are close to those resulting from the developed model. What is for sure is that the models of Yuan and Skitmore-Pemberton do not take the dynamic behavior of competitors into consideration in the same sense as the developed model does.

Scenario 1 – Developed Model												
Winning the following bidders	Combined probability based on Friedman's equation						Combined probability based on Gates' equation					
	$\sigma = 2$		$\sigma = 3$		$\sigma = 4$		$\sigma = 2$		$\sigma = 3$		$\sigma = 4$	
	M%	P_{win}	M%	P_{win}	M%	P_{win}	M%	P_{win}	M%	P_{win}	M%	P_{win}
1 + 55 + 134	1.20	62.4%	1.24	61.4%	1.26	59.9%	1.33	59.5%	1.41	57.7%	1.46	55.7%
1 + 55	1.57	64.5%	1.55	62.5%	1.51	61.4%	1.58	64.4%	1.61	61.4%	1.62	59.3%
1 + 134	1.21	62.1%	1.29	61.4%	1.35	61.3%	1.33	59.7%	1.43	58.6%	1.5	58.2%
55 + 134	1.67	62.4%	1.81	61.0%	1.9	59.7%	1.69	61.9%	1.90	59.9%	2.12	56.4%
55 + 221	2.41	66.7%	2.55	63.4%	2.58	60.4%	2.47	65.9%	2.78	61.0%	2.9	57.1%
All	1.12	62.5%	1.17	61.5%	1.2	60.6%	1.28	58.6%	1.36	57.1%	1.41	56.1%

Scenario 2 – Developed Model												
Winning the following bidders	Combined probability based on Friedman's equation						Combined probability based on Gates' equation					
	$\sigma = 2$		$\sigma = 3$		$\sigma = 4$		$\sigma = 2$		$\sigma = 3$		$\sigma = 4$	
	M%	P_{win}	M%	P_{win}	M%	P_{win}	M%	P_{win}	M%	P_{win}	M%	P_{win}
1 + 55 + 134	1.32	63.7%	1.32	62.0%	1.32	60.7%	1.45	60.7%	1.49	58.4%	1.51	55.9%
1 + 55	1.53	65.0%	1.51	63.5%	1.58	61.3%	1.56	64.3%	1.60	60.6%	1.61	58.7%
1 + 134	1.35	63.6%	1.41	62.2%	1.45	61.6%	1.47	60.1%	1.53	59.9%	1.57	59.4%
55 + 134	2.19	62.4%	2.16	60.7%	2.2	58.3%	2.32	60.4%	2.41	57.4%	2.57	53.6%
55 + 221	2.57	68.2%	2.57	63.1%	2.57	59.9%	2.72	66.4%	2.84	60.1%	2.89	56.5%
All	1.25	64.0%	1.26	62.0%	1.25	60.5	1.4	60.6%	1.44	58.2%	1.47	56.5%

Comparison to Other Models				
Winning the following bidders	Skitmore-Pemberton's Model (1994)		Yuan's Model (2011)	
	M%	P_{win}	M%	P_{win}
1 + 55 + 134	13.47	N/A	2.79	N/A
1 + 55	10.98	N/A	3.69	N/A
1 + 134	12.06	N/A	3.74	N/A
55 + 134	7.70	N/A	3.29	N/A
55 + 221	6.62	N/A	--	--

Figure 3: Optimum Markup Determination (M) and Corresponding Probability of Winning (P_{win})

When it comes to the developed model, it obtained markup values that are even less than those of Yuan's model. Such low markups are a direct result of the made assumption that competitor 1 is a risk taker; where he always bids in low markups. It can be observed that, in all of the combinations that involve competitor 1, the optimum markup never exceeded 1.62%. However, for the combinations that did not involve competitor 1, the optimum markup reached 2.90%. Generally, such low markups are also a result of the developed model's inclusion of the dynamic behavior; which makes it more accurate than previous models. Accordingly, the more recent observations have a greater impact on the resulting optimum markup. In the case study, the recent observations of competitors had low bid ratios.

In this case study the results were not very sensitive to σ, meaning that the firm does not have to exert effort and time to estimate the value of σ. However, with more certainty of the stochastic variance of cost estimating among competitors, the probability of winning increases. More certainty means a lower value of σ. For example, in the first scenario, the probability of winning competitors 55 and 221 at $\sigma = 2$ is 66.7%. With less certainty, where $\sigma = 4$, the probability of winning the same contractors is 57.1%. It shall be stated that no matter how high the probability of winning, winning is not guaranteed. Neither this model nor any other markup decision model guarantees winning bids. Competitors might take irrational decisions leading to unexpected results. Figure 4 shows the expected profits and probability of winning all competitors in scenario 1 as a demonstration.

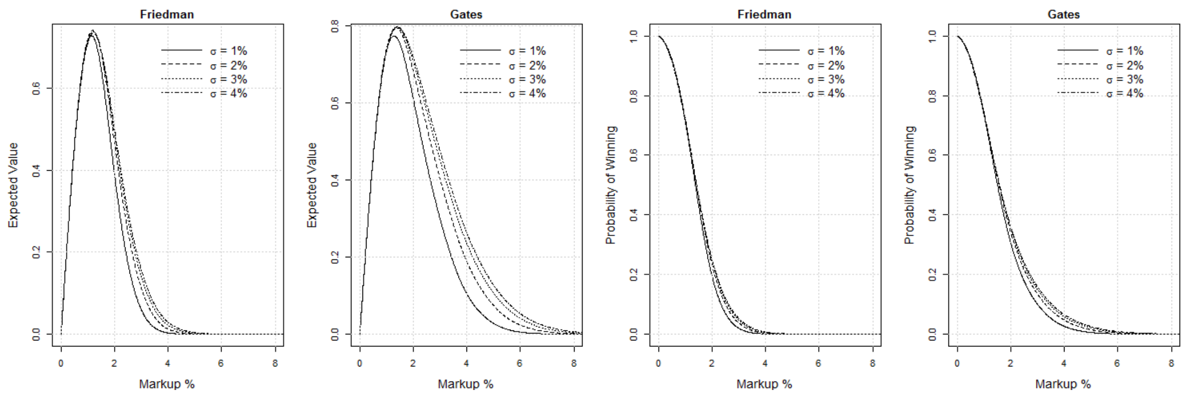

Figure 4: Expected Profit and Probability of Winning All Competitors (Scenario 1)

Finally, to be able to use the developed model in its full capacity, certain information of the historical bids of competitors have to be gathered more than just their bid prices. Such information includes the dates of the historical bids, the estimated stochastic variance of project cost between the firm and each of the competitor, and information on which bids to be considered the latest common bids as defined in this paper. In the case study, such additional information was not available so the model was run based on reasonable assumptions to demonstrate the different steps and parameters. However, in reality, the assumptions will be minimal and there will be only one value for markup and one for probability of winning, leaving no room for errors. Also, a firm that opts to use the proposed model can do its own sensitivity analysis and scenarios.

5 CONCLUSION

The volume of transportation projects in the US is massive. Most of such projects undergo competitive bidding where the qualified contractors with the lowest bid prices are awarded the projects. Contractors submitting unreasonably low bid prices end up acquiring a claim-oriented behavior to recover their losses. Such behavior results in disputes related to quality degradation, schedule overruns, and cost overruns. As such, a healthy bidding environment will ensure disputes are minimal and costs are reasonable. Several models have been developed to help contractors determine their bid prices based on statistical analysis of competitors' history; however, such models do not consider cases of imperfect information and dynamic behavior of competitors; where a competitor's old behavior contradicts its more recent one. This paper presents a Bayesian-based statistical model for optimal bid price determination that is valid in cases of incomplete historical data and dynamic behavior of competitors.

The authors presented the detailed algorithm of the model and employed it in a case study; where the results were compared to two other models from the literature review on the same case study. The purpose of the case study was to demonstrate the use of the model and illustrate the effect of different parameters on the resulting optimum markup and probability of winning. The results show that the more recent bidding strategies of competitors play a significant role in predicting the future ones. The results also show that as the variability of cost estimates between a firm and its competitors increase, the optimum markup for that firm attain lower probability of winning. The model's multi-stage approach accurately simulates the competitors' dynamic behavior. Moreover, its ability to produce sound statistical inferences in cases of incomplete information is abstracted from the widely credible Bayesian statistical concepts. One important assumption, which is used in most models in the literature, is that the number of competitors competing in the given bid is known. The research will be beneficial to the transportation infrastructure economy by ensuring that contractors submit bids with reasonable prices; which will make them less susceptible to claim-oriented behavior and eventually lead to healthier contracting environments. The authors recommend applying the model to more case studies to be able to have better understanding of its behavior, advantages, and limitations. Moreover, extended research should attempt to combine non-cooperative game theory with Bayesian statistics to provide a more holistic and comprehensive understanding of the bidding process in transportation projects.

6 References

Ahmed, M., El-Adaway, I., Coatney, K., and Eid, M. (2015). “Construction bidding and the winner’s curse: Game theory approach.” *J. Constr. Eng. Manage.*, **142**(2), 04015076.

Benjamin, N. B., and Meador, R. C. (1979). “Comparison of Friedman and Gates competitive bidding models.” *J. Constr. Div.*, **105**(1), 25–40.

Bennett, J. E., Racine-Poon, A., & Wakefield, J. C. (1996). *MCMC for nonlinear hierarchical models* (pp. 339-357). London, UK: Chapman and Hall.

Bernardo, J. M. (2011). “Bayesian statistics.” *International encyclopedia of statistical science*, Springer, Berlin, 107–133.

Capen, E. C., Clapp, R. V., and Campbell, W. M. (1971). “Competitive bidding in high-risk situations.” *J. Pet. Technol.*, **23**(06), 641–653.

Carr, R. I. (1982). “General bidding model.” *J. Constr. Div.*, **108**(4), 639–650.

Census Bureau. (2016). *Construction Spending*. United States Department of Commerce.

Cowles, M. (2012). What Is Bayesian Statistics? *Springer Texts in Statistics Applied Bayesian Statistics*, 1–11

Crowley, L. (2000). “Friedman and gates: Another look.” *J. Constr. Eng. Manage.*, **126**(4), 306–312.

Cullen, A. C., and Frey, H. C. (1999). *Probabilistic techniques in exposure assessment: A handbook for dealing with variability and uncertainty in models and inputs*, Plenum, New York.

Dixie, J. M. (1974). “Bidding models: The final resolution of a controversy.” *J. Constr. Div*., **100**(3), 265–271.

Friedman, L. (1956). “A competitive-bidding strategy.” *Oper. Res.*, **4**(1), 104–112.

Gates, M. (1967). “Bidding strategies and probabilities.” *J. Constr. Eng. Div.*, **93**(1), 75–110.

Hanssmann, F., and Rivett, B. H. P. (1959). “Competitive bidding.” *Oper. Res.*, **10**(1), 49–55.

Ioannou, P., and Awwad, R. (2010). “Below-average bidding method.” *J. Constr. Eng. Manage.*, **136**(9), 936–946.

Kooperberg, C., and Stone, C. J. (1991). “A study of logspline density estimation.” *Comput. Stat. Data Anal.*, **12**(3), 327–347.

Lingard, H., Hughes, W., and Chinyio, E. (1998). “The impact of contractor selection method on transaction costs: A review.” *J. Constr. Procurement*, **4**(2), 89–102.

Lo, H. P., and Lam, M. L. (2001a). “A bidding strategy using multivariate distribution.” J*. Financial Manage. Prop. Constr.*, **6**(3), 155–165.

Nuzzo, R. (2014). “Scientific method: Statistical errors.” *Nature*, **506**(7487), 150–152.

Ranasinghe, M. (2000). “Impact of correlation and induced correlation on the estimation of project cost of buildings.” *Constr. Manage. Econ.*, **18**(4), 395–406.

Robert, C., and Casella, G. (2009). *Introducing Monte Carlo methods with R*, Springer-Verlag, New York.

Rosenshine, M. (1972). “Bidding models: Resolution of a controversy.” *J. Constr. Div.*, **98**(1), 143–148.

Skitmore, M. (1991). “The construction contract bidder homogeneity assumption: An empirical test.” *Constr. Manage. Econ.*, **9**(5), 403–429.

Skitmore, M., and Pemberton, J. (1994). “A multivariate approach to construction contract bidding mark-up strategies.” *J. Oper. Res. Soc.*, **45**(11), 1263–1272.

Skitmore, R., Pettitt, A., and McVinish, R. (2007). “Gates’ bidding model.” J. Constr. Eng. Manage., **133**(11), 855–863.

The American Society of Civil Engineers ASCE (2011). *Failure to Act: The Economic Impact of Current Investment Trends in Surface Transportation Infrastructure*.

Touran, A. (2003). “Probabilistic model for cost contingency.” *J. Constr. Eng. Manage.*, **129**(3), 280–284.

United States Department of Transportation. (2015). *Freight Facts and Figures 2015*. Bureau of Transportation Statistics.

Winkler, R. L., and Brooks, D. G. (1980). “Competitive bidding with dependent value estimates.” *Oper. Res.*, **28**(3-part-i), 603–613.

Yuan, X. X. (2011). “A correlated bidding model for markup size decisions.” *Constr. Manage. Econ.*, **29**(11), 1101–1119.

Yuan, X. X. (2012). “Bayesian method for the correlated competitive bidding model.” *Constr. Manage. Econ.*, **30**(6), 477–491.