LEADERSHIP IN SUSTAINABLE INFRASTRUCTURE
May 31 – June 3, 2017

FINITE ELEMENT MODEL BASED ON SUBSPACE FITTING DISPLACEMENT WITH DAMAGE LOCALIZATION

Mohamed bouyahi
Communication System Laboratory Sys'Com, National Engineering School of Tunis, University Tunis El Manar, BP 37, Belvedere, 1002 Tunis, Tunisia

Tahar Ezzedine
Communication System Laboratory Sys'Com, National Engineering School of Tunis, University Tunis El Manar, BP 37, Belvedere, 1002 Tunis, Tunisia

ABSTRACT

A new method is proposed to identify locations and severities of structural damages based on the subspace fitting (SF). Firstly, a reduced discretized analytical model is derived from active displacement. Then, the experimental model is implemented based on knowledge measurement data under stationary and random excitations are developed using pseudo excitation method. To verify the analytical prediction, experiments without feedback control are conducted. Control force based on outputs of sensors. Finally, the Finite Element Model (FEM) updating method is adopted to localize the structural damage which minimizes the difference between analytical and experimental model. Three degrees of freedom (DOF) coupled model are investigated in this paper to analyze the displacement sensor outputs.

Keywords: Degree Of Freedom; Damage Identification; Damage Localization; Dynamic Response; Finite Elemet Model.

1. INTRODUCTION

Damage localization is a major problem which focuses on the Structural Health Monitoring (SHM). The control and health monitoring in civil engineering have various types of methods exposed on many types of damages. These methods are proposed to detect and quantify structure damage. Displacement-based methods aim to determine the dynamic properties of structures. The finite element (FE) model updating procedures is mostly used in civil engineering. This method estimates the parameters (Young's modulus) which are measured by experimental structure and updating of FE model. The iterative methods are used to analyze the variations of structure parameters (eigenfrequencies, modes shapes, damping ratio) (Gautier et al., 2015).

The FE model and model-free damage detection methods should be given priority to the development. Since in engineering structures are problematic for material certainties, it is difficult to establish the FE model in some structural connection parts. In this respect, hybrid damage detection should be adopted in engineering structure, the FE model is used for damage quantification based on the local methods while damage localization is used for the global methods. The response of structure provides high noise in this regard we need the methods with high noise detection. The capacity of methods to detect a higher noise needs to be improved. At the same time we minimize the location space subspace fitting method. The pulse excitation is generally used for the regular test; while the methods of real time structural health use the random excitation (Zheng et al., 2015).

The motivation of the present work is that it proposes to use SF with FE-based updating procedures, predicts and quantifies the damage by small number of sensors. A FE-based SF approach is proposed to minimize problem, a model reduction is used to speed up the computation of the SF minimization problem.

The general design of the structure results from the search for an adequacy between the various constraints imposed by the site and the technical and aesthetic considerations.

This work is of a great technicality: foundations of great depth in seismic zone, variable apron height, and various types of prestressing structure. This part describes in occurrence the anatomy of the work while focusing on the characteristics of the elements which will be essential and useful for us in modeling and significant in obtaining the results.

In this paper, we design the fully symmetric bridge. Since the displacement output along the sense direction, we assume that central point of the coupled spring is unstable and establish the three DOFs coupled model (Gautier et al., 2015).

This article opens with motivation and objectives carried out on objective to use the SF with FE-based. Section 2 will introduce the analytical model. Section 3 will discuss the numerical simulation and validation of the model. Last section 4 concludes the paper.

2. ANALYTICAL MODEL

2.1 Preliminary comments

$$X_{k+1} = AX_k + Bu_k + V_k$$

[1] $y_k = C^T.X_k + W_k$

$$X_k = \begin{pmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_{2n} \end{pmatrix}, 0 \leq i \leq 2n$$

X_k is a $2n * 1$ state vector (n means the order of the system), $t_{k+1} = t_k + \Delta t, \forall k$ (Δt means the time step);
u_k and y_k are $m * 1$ and $n * 1$ the vectors of input and output data (Gautier et al., 2015).
The matrices A, B and C are the orders respectively 2n * 2n, 2n * m and n * 2n.
(M, γ, K) : mass, stiffness and damping matrices of finite size n * n.

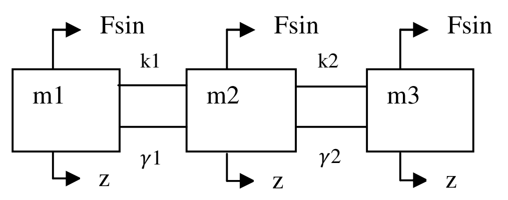

Fig 1: Two DOFs model of ideal bridge.

$$M = \begin{bmatrix} \text{m1} & 0 & 0 \\ 0 & \text{m2} & 0 \\ 0 & 0 & \text{m3} \end{bmatrix}$$

$$k = \begin{bmatrix} \text{k1} & -\text{k1} & 0 \\ -\text{k1} & 0 & \text{k2} \\ 0 & -\text{k2} & \text{k2} \end{bmatrix}$$

$$y = \begin{bmatrix} a_1\mathrm{m1} + a_2\mathrm{k1} & -a_2\mathrm{k1} & 0 \\ -a_2\mathrm{k1} & a_1\mathrm{m2} & a_2\mathrm{k2} \\ 0 & -a_2\mathrm{k2} & a_1\mathrm{m3} + a_2\mathrm{k2} \end{bmatrix}$$

[2] $M\ddot{q}(t) + \gamma\dot{q}(t) + Kq(t) = B_0 f(t)$

$q(t)$, $\dot{q}(t)$ and $\ddot{q}(t)$ displacement, velocity and acceleration vector respectively with sizes n * 1 (Wang et al., 2014) (Guan et al., 2015).

$f(t)$ is a force vector with size m * 1, with m < n.

B_0 is a boolean matrix with size n * m, for localized inputs DOF (Bleicher et al., 2011).

The modal damping ratio ξ_i is calculated by the damping of structure was obtained by the free vibration structure (Zheng et al., 2015). The damping coefficient is defined as following:

$\gamma = a_1 M + a_2 K$

a_1 and a_2 are two constants, and they are determined from two different modal frequencies w_i, w_j and modal damping ratio ξ_i, ξ_j, with the expressed as following:

$$a_1 = \frac{2w_j w_i (w_j \xi_i - w_i \xi_j)}{w_j^2 - w_i^2}$$

$$a_2 = \frac{2(w_j \xi_j - w_i \xi_i)}{w_j^2 - w_i^2}$$

$u(t) = q(i)\sin(iwt)$

$\dot{u}(t) = iwq(i)sin\,(iwt)$

$\ddot{u}(t) = -w^2 q(i)\sin(iwt)$

$u(t)$, $\dot{u}(t)$ and $\ddot{u}(t)$ mode shape vectors displacement, velocity and acceleration respectively with sizes n * 1.

2.2 Identification of the matrices A, B and C

$$A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}\gamma \end{bmatrix}$$

[3] $A = e^{A_c \Delta t}$

$$B_c = \begin{bmatrix} 0 \\ M^{-1}B_0 \end{bmatrix}$$

[4] $B = (e^{A_c \Delta t} - I){A_c}^{-1}B_c$

The state vector at the discrete time t_k is expressed as following:

$$X_k = [q(t_k)^T \quad \dot{q}(t_k)^T]^T$$

C matrice is to be expressed as.

[5] $C = [H_d - H_a M^{-1}K \quad | \quad H_v - H_a M^{-1}\gamma]$

WhereH_d, $H_v$and H_a are n * n Boolean matrices which are used to localize the output displacement, velocity and acceleration (Hernandez et al., 2013).

2.3 Subspace estimate

To rewrite the system with the larger size, the objective for the extended observability matrices Γ is to eliminate the influence of inputs and noises.

[6] $u_k = \begin{pmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+\alpha-1} \end{pmatrix}, y_k = \begin{pmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+\alpha-1} \end{pmatrix}, k = 1, \dots, N.$

α and N are numeriques, such as A $> 2n$ and $N \gg 2n$

[7] $y_k = \Gamma X_k + \theta u_k + \Xi V_k + W_k, k = 1, \dots, N.$

V_k and W_k with size $l\alpha * 1$, vectors of stacked process and measurement noises, defined as following:

[8] $V_k = \begin{pmatrix} V_k \\ V_{k+1} \\ \vdots \\ V_{k+\alpha-1} \end{pmatrix}, W_k = \begin{pmatrix} W_k \\ W_{k+1} \\ \vdots \\ W_{k+\alpha-1} \end{pmatrix}, k = 1, \dots, N.$

Γ is an extended matrix, whose size is $l\alpha * 2n$. This matrice is used to removing the influence of inputs and noises, and can be derived from the identification matrices A and C.

[9] $\Gamma = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{\alpha-1} \end{pmatrix}$

θ and Ξ are $l\alpha * m\alpha$, $l\alpha * 2n\alpha$ block Toeplitz matrices, defined as following:

[10] $\theta = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{\alpha-2}B & CA^{\alpha-3}B & \cdots & D \end{bmatrix},$

[11] $\Xi = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ C & 0 & \cdots & 0 & 0 \\ CA & C & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{\alpha-2} & CA^{\alpha-3} & \cdots & C & 0 \end{bmatrix}$

2.4 Model reduction

"Eq. 12" refers the solution of the following eigenproblem:

[12] $K^h u_j^h = \left(w_j^h\right)^2 M^h u_j^h$

Where $\{w_j^h\}$ are to be understood as the eigenpulsations of the displacement structure is expressed as following:

$w^h = \sqrt{\frac{K^h}{M^h}}$

The reduced mode present by the vector of modal displacements q^husing a reduced set of mode shape vectors $\{u_j^h\}_{j=1\ldots r} \subset \{u_j^h\}_{j=1\ldots n^h}$where $r \ll n^h$.

The reduced extended observability matrix $\tilde{\Gamma}^h$ has a size of $l\alpha * 2r$

$\tilde{\Gamma}^h = \begin{pmatrix} \tilde{C}^h \\ \tilde{C}^h \tilde{A}^h \\ \vdots \\ \tilde{C}^h (\tilde{A}^h)^{\alpha-1} \end{pmatrix}$

Where $\tilde{u}^h$ is expressed as $\tilde{u}^h = [\tilde{u}_1^h \ldots \tilde{u}_r^h]$ and $\tilde{q}^h(t) = [\tilde{q}_1^h(t) \ldots \tilde{q}_r^h(t)]$.

The second order differential equation:

[13] $M^h \ddot{q}^h(t) + \gamma^h \dot{q}^h(t) + K^h q^h(t) = B_0{}^h f(t)$

On the reduced basis $\{\tilde{u}_j^h\}_j$. This equation multiplying by $(\tilde{u}^h)^T$, T mean the transpose of the mode shape vectors, $(\tilde{u}^h)^T M^h \tilde{u}^h = I$, $(\tilde{u}^h)^T K^h \tilde{u}^h = diag\{(\widetilde{W}_j^h)^2\}_j$ and $(\tilde{u}^h)^T \gamma^h \tilde{u}^h = diag\{2\tilde{\xi}_j^h \widetilde{W}_j^h\}_j$ assumed to be diagonal.

The "Eq. 13" is replaced by "Eq. 14" as follows:

[14] $\ddot{q}^h(t) + diag\{2\tilde{\xi}_j^h \widetilde{W}_j^h\}_j \dot{q}^h(t) + diag\{(\widetilde{W}_j^h)^2\}_j q^h(t) = (\tilde{u}^h)^T B_0{}^h f^h(t)$

The experimental identification of the matrices $\tilde{A}^h, \tilde{B}^h, \tilde{C}^h$ and $\widetilde{D}^h$are expressed as.

$\tilde{A}^h = e^{\tilde{A}_c^h \Delta t}$

[15] $\tilde{A}_c^h = \begin{bmatrix} 0 & I \\ -diag\left\{\left(\widetilde{W}_j^h\right)^2\right\}_j & -diag\{2\tilde{\xi}_j^h \widetilde{W}_j^h\}_j \end{bmatrix}$

$\tilde{B}^h = \left(e^{\tilde{A}_c^h \Delta t} - I\right)(\tilde{A}_c^h)^{-1} \tilde{B}_c^h$

[16] $\tilde{B}_c^h = \begin{bmatrix} 0 \\ (\tilde{u}^h)^T B_0^h \end{bmatrix}$

[17] $\tilde{C}^h = \left[H_d^h \tilde{u}^h - H_a^h \tilde{u}^h diag\left\{ \left(\widetilde{\mathcal{W}}_j^h \right)^2 \right\}_j \quad | \quad H_v^h \tilde{u}^h - H_a^h \tilde{u}^h diag\left\{ 2\tilde{\xi}_j^h \widetilde{\mathcal{W}}_j^h \right\}_j \right]$

This algorithm procedure uses more CPU times, to solve this minimization problem we use the mode of reduction (Mousavi et al., 2016). The modes which expose by higher dynamic behaviour of the structure. The mode is H_j of each mode j is to be expressed as following:

$$H_j = \frac{\|B_{c|j}^h\|_2 \|C_{|j}^h\|_2}{2\sqrt{\xi_j^h * \mathcal{W}_j^h}} \; j = 1, \dots, n^h$$

Where $B_{c|j}^h$ and $C_{|j}^h$ are expressed from the jth block row/column components of B_c^h and C^h, i.e.:

[18] $B_{c|j}^h = \begin{bmatrix} 0 \\ u_j^T B_0^h \end{bmatrix} j = 1, \dots, n^h$

[19] $C_{|j}^h = \left[(H_d)^h u_j - (H_a)^h u_j w_j^2 \quad | \quad (H_v)^h u_j - (H_a)^h u_j 2\xi_j w_j \right] j = 1, \dots, n^h$

[20] $E = argmin \left\| vec\{ (I - \tilde{\Gamma}^h \tilde{\Gamma}^{h^+}) \Gamma \} \right\|_2^2$

The Fig. 2 presents the procedure of the identification technical.

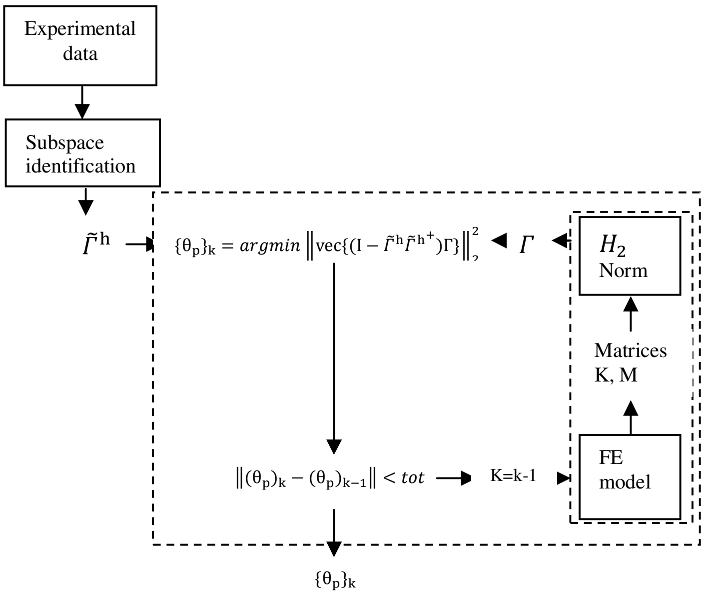

Fig 2: Flowchart of identification procedure.

2.5 Damage localization

To localize and quantify the damage in the structure the FE based subspace method is applied. The damages are modeled in terms of element stiffness matrices, K_e^h that are used for bluilding the FE model of the structure perturbation $K_e^h = [1 - \beta_e^h]K_e^h$, where $\beta_e^h(0 \leq \beta_e^h \leq 1$ is a scalar parameter mean the damage severity.

The technical procedure of damage localization is composed by many steps. Step 1, the FE mesh of the structure is defined by (Ω^h) is partitioned by two parts $(\Omega_1^h)_1$ and $(\Omega_2^h)_1$. The two parts $(\Omega_1^h)_1$ and $(\Omega_2^h)_1$ remeshed another to construct two stiffness matrices. The two matrices $(K^h)_1^1$ and $(K^h)_1^2$are calculated by $[1-(\beta^h)_1^1](K^h)_1^1$, $[1-(\beta^h)_1^2](K^h)_1^2$.

The severity parameters can be identified as following:

[21] $\{(\beta^h)_1^1, (\beta^h)_1^2\} = argmin \left\| vec\{\left(I - \tilde{\Gamma}^h \tilde{\Gamma}^{h^+}\right)\Gamma\}\right\| {}_2^2$

For any $p = 1{,}2$, if $(\beta^h)_1^p \geq \epsilon$ (ϵ is a small tolerance threshold), the subdomain $(\Omega_p^h)_1$ is considered as damaged.

Step 2, takes the subdomain $(\Omega_p^h)_1$remeshed another to construct two stiffness matrices. And repeat the calculation of severity parameter to identify the damage.

3. VALIDATION OF THE FE-BASED SF APPROACH

To verify the performance of the method subspace fitting, 3-story shear structures is selected as research object (Zheng et al., 2015) (Gautier et al., 2015).

Where the value of lamped masses m_1, m_2, m_3; stiffness of the 2 stories are K_1, K_2,respectively; 3 damping constants y_1, y_2, y_3; these parameters are based on the random excitation.

$m_1 = m_2 = m_3 = 1.0 * 10^5 Kg, K_1 = K_2 = 2.0 * 10^8 N/m^3.$

The module young's 200 $Gpa = 200 * 10^9\ pa$, Air session $A = 0.0249 * 0.0053 = 13.197 * 10^{-5} m^2$, L = 1 m, the proper's pulsations is used are $\sqrt{5}$ and $\sqrt{15}$, $a_1 = 1.1381$, $a_2 = 7.4714 * 10^{-5}$and damping ratio: $\xi_i = \xi_j = 0.25$ (An et al., 2015) (Lu et al., 2010).

The next Fig represents the model of our application, the blue points represent the vibrations sensors, the green rectangular represent the stiffness matrices and the red points represent the wireless and controller node, these nodes can be transmit the data to the server when we able to use them later.

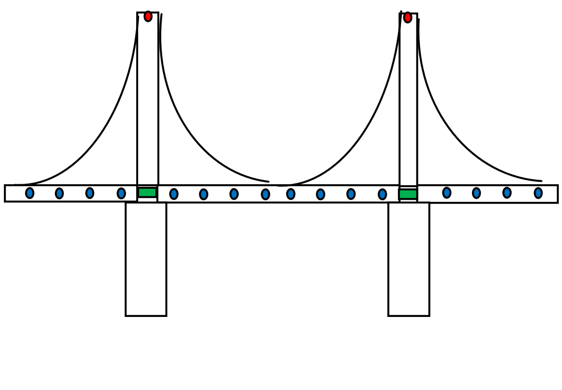

Fig 3: The model of bridge structure.

3.1 Update of finite element method

The finite element method is the output of bridge can be reduced in each iteration because the stiffness matrice reduced to with the value 5% in each iteration.

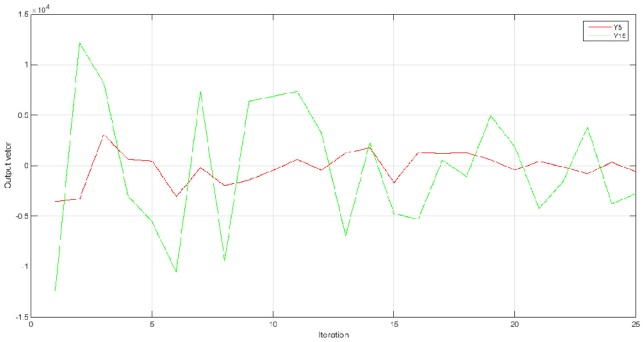

Fig 4: Output vector of FEM.

3.2 Acceleration sensors

In this Fig we show the acceleration of vibrations sensors in two proper's pulsations $\sqrt{\mathbf{5}}$ and $\sqrt{\mathbf{15}}$ by 25 iterations (Zheng et al. 2015).

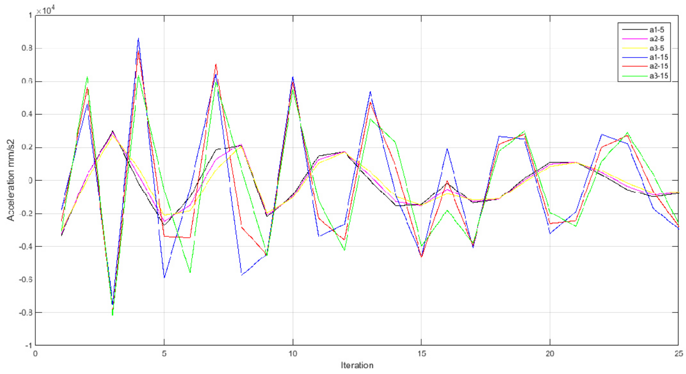

Fig 5: Acceleration of sensors.

3.3 Effect of measurement noise

$\tilde{a} = a + n = a + \mu\sigma_a r$

We simulate the error of the measuring nodes for detecting the rang of error without two proper pulsations $\sqrt{5}$ and $\sqrt{15}$., $\tilde{a}$ and a mean the accelerations with and without noise, n is the noise; σ_a is standard deviation of the accelerations is based on the accelerations stories; μ is noise level; r is a vector of random distributed numbers (An et al., 2015).

$\sigma_a = \sqrt{\frac{(a_1-\bar{a})^2+(a_2-\bar{a})^2+\cdots+(a_p-\bar{a})^2}{N}}$, where acceleration in times $a_i 1 \le i \le n$ and $\bar{a} = \frac{1}{N}\sum_{i=1}^{N} a_i$

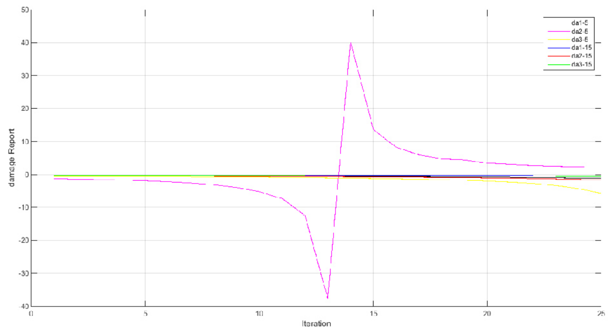

Fig 6: Damage Report.

Fig 6. shows the cases of damages for validating the two jerk energy-based methods. The two proper pulsations are used $\sqrt{5}$ and $\sqrt{15}$ on random excitation. We calculate the story damage based on the noise levels 20% of the acceleration signal added to the original acceleration.

3.4 Young's Modulus

After 25 iterations the Young Module passed from 200 $Gpa = 200 * 10^9$ to 200 $Gpa = 200 * 10^9 \mp 0.02 Gpa$.

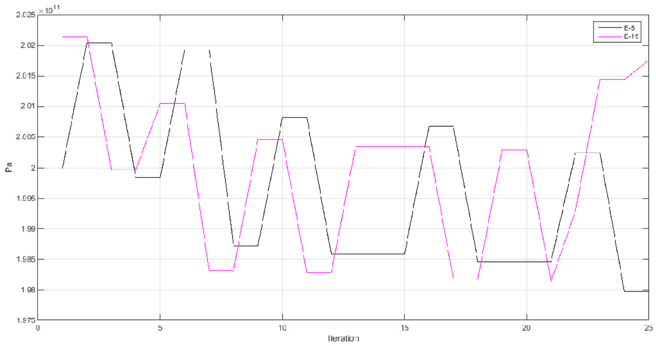

Fig 7: Young's Modulus

4. CONCLUSION

The minimization approach has been developed for structure identification and localization damage. Updating method based on minimization problem is elaborated for identifying the displacement of structure. A reduction model is used to minimize the time of CPU. The iterative domain has been proposed to localize the damages in the structures. This model has proved its single and multiple localizations.

references

An, Y, Spencer, B.F, and Ou, J. 2015. Real-time fast damage detection of shear structures with random base excitation, Measurement, 74: 92-102.

Bleicher, A, Schlaich, M, Fujino, Y, and Schauer, T. 2011. Model-based design and experimental validation of active vibration control for a stress ribbon bridge using pneumatic muscle actuators, Engineering Structures, 33: 2237–2247.

Gautier, G, Mencik, J.M, and Serra, R. 2015. A finite element-based subspace fitting approach for structure identification and localization, Mechanical Systems and Signal Processing, 58-59: 143-159.

Guan, Y, Gao, S, Lui, H, and Niu, S. 2015. Acceleration sensitivity of tuning fork gyroscopes: theorical model, simulation and experimental verification, Microsystem Technologies, 21: 1313-1323.

Hernandez, E.M and Bernal, D. 2013. Iterative finite element model updating in the time domain, Mechanical Systems and Signal Processing, 34: 39-46.

Lu, Z.R, Huang, M, Chen, W.H, Liu, J.K, and Ni, Y.Q. 2010. Assessment of Local Damages in Box-girder Bridges Using Measured Dynamic Responses by Passing Vehicle, Prognostics and System Health Management Conference, IEEE, Macau, 7 p.

Mousavi, M, and Gandomi, A.H. 2016. A hybrid damage detection method using dynamic-reduction transformation matrix and modal force error, Engineering Structures, 111: 425-434.

Wang, X, Yang, C, Wang, L, and Qiu, Z. 2014. Probabilistic damage identification of structures with uncertainty based on dynamic responses, Acta Mechanica Solida Sinica, 27: 172-180.

Zheng, Z.D, Lu, Z.R, Chen, W.H, and Liu, J.K. 2015. Structural damage identification based on power spectral density sensitivity analysis of dynamic responses, Computers and Structures, 146: 176-184.