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FINITE ELEMENT MODEL BASED ON JERK-ENERGY FOR DAMAGE LOCALIZATION OF DYNAMIC RESPONSE

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ABSTRACT

A new method is proposed to identify the damages of locations based on two Jerk Energy (JE) methods. Proposed for test with pulse excitation, firstly a reduced discretized analytical model is derived from active vibration. Then, the experimental model is implemented based on knowledge measurement data under stationary and random excitations are developed using pseudo excitation method. To verify the analytical prediction an experimental test is used with different stiffness's levels of damage cases to verify the two JE methods to uses in real-time damage detection. Finally, the finite element model (FEM) updating method is adopted for quantifying the damages of structure from the experimental measured. Three degrees of freedom (DOF) coupled model are investigated in this paper to analyze the vibration sensor outputs.

Keywords: Degree Of Freedom; Damage Identification; Jerk Energy-based (JE); Dynamic Response; Finite Elemet Model.

1. INTRODUCTION

In our days we have a major problems in the Structure Health Monitoring (SHM) to identify and locate the damages, civil engineering has a various types of methods exposed on many types of damages. These methods are proposed to detect and identify the damages. Displacement-based methods aim to determine the dynamic properties of structures. The FEM updating method an iterative method is hardly used in civil engineering to analyze the variations of structures parameters (eigenfrequencies, modes shapes, damping ratio) (Gautier et al., 2015).

The FEM and model-free damage detection methods should be given priority to the development. It is difficult to establish this model for many problematic of certainties of material. The hybrid method should be adopted in civil engineering; the FEM is used for damage quantification based on the local methods while damage localization based on the global methods. The higher noise provides by the structures needs a method with higher noise detection. These methods will be improved to detect this noise. At the same time we calculate the data delivered. The pulse excitation is generally used for the regular test; while the methods of real time structural health are used for the random excitation (Zheng et al., 2015).

The present work it proposes to use JE with FE-based updating method, predict and quantify the damage by small number of sensors. A reduction model proposes to minimize the large number of sensor and speed up the time of computation.

The two first model-free methods are used to localize the damage based on pulse excitation, Mean Normalized Curvature Difference of Waveform Jerk Energy (MNCDWJE) and the Curvature Difference Probability Waveform Jerk Energy (CDPWJE). The two methods have been validated with pulse excitation and are required to be used in real time damage localization (An et al., 2015).

The general design of the structure results from the search for an accuracy between the various constraints imposed by the site, the technical and aesthetic considerations.

This work is of a great technicality: foundations of great depth in seismic zone, variable apron height, and various types of prestressing structure. This part describes in occurrence the anatomy of the work while focusing on the characteristics of the elements which will be essential and useful for us in modeling and significant in obtaining the results.

In this paper, we design the fully symmetric bridge. Since the displacement output along the sense direction, we assume that the central point of the coupled spring is unstable and establish the three DOFs coupled model (Gautier et al., 2015).

This article opens with motivation and objectives carried out on objective to use the JE with FE-based. Section 2 will introduce the analytical model. Section 3 will present the two jerk energy-based damage localization. Section 4 will discuss the numerical simulation and validation of the model. Last section 5 concludes the paper.

2. ANALYTICAL MODEL

2.1 Preliminary comments

$X_{k+1} = AX_k + Bu_k + V_k$

[1] $y_k = C^T.X_k + W_k$

$$X_k = \begin{pmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_{2n} \end{pmatrix}, 0 \leq i \leq 2n$$

X_k is a $2n * 1$ state vector (n means the order of the system), $t_{k+1} = t_k + \Delta t, \forall k$ (Δt means the time step);
u_k and y_k are $m * 1$ and $n * 1$ the vectors of input and output data (Gautier et al., 2015).
The matrices A, B and C are the orders respectively 2n * 2n, 2n * m and n * 2n.
(M, γ, K) : mass, stiffness and damping matrices of finite size n * n.
[2] $M\ddot{q}(t) + \gamma\dot{q}(t) + Kq(t) = B_0 f(t)$
$q(t)$, $\dot{q}(t)$ and $\ddot{q}(t)$ displacement, velocity and acceleration vector respectively with sizes n * 1 (Wang et al., 2014) (Guan et al., 2015).
$f(t)$ is a force vector with size m * 1, with m < n.
B_0 is a boolean matrix with size n * m, for localized inputs DOF (Bleicher et al., 2011).
The modal damping ratio ξ_i is calculated by the damping of structure was obtained by the free vibration structure (Zheng et al., 2015). The damping coefficient is defined as following:
$\gamma = a_1 M + a_2 K$
a_1 and a_2 are two constants, and they are determined from two different modal frequencies w_i, w_j and modal damping ratio ξ_i, ξ_j, with the expressed as following:

$$a_1 = \frac{2w_j w_i(w_j\xi_i - w_i\xi_j)}{w_j^2 - w_i^2}$$

$$a_2 = \frac{2(w_j\xi_j - w_i\xi_i)}{w_j^2 - w_i^2}$$

2.2 Identification of the matrices A, B and C

$$A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}\gamma \end{bmatrix}$$

[3] $A = e^{A_c \Delta t}$

$$B_c = \begin{bmatrix} 0 \\ M^{-1}B_0 \end{bmatrix}$$

[4] $B = (e^{A_c \Delta t} - I){A_c}^{-1}B_c$

The state vector at the discrete time t_k is expressed as following:

$X_k = [q(t_k)^T \quad \dot{q}(t_k)^T]^T$

C matrice is to be expressed as.

[5] $C = [H_d - H_a M^{-1} K \quad | \quad H_v - H_a M^{-1} \gamma]$

Where H_d, H_v and H_a are n * n Boolean matrices which are used to localize the output displacement, velocity and acceleration (Hernandez et al., 2013).

2.3 Model reduction

To rewrite the system with the larger size, the objective for the extended observability matrices Γ is to eliminate the influence of inputs and noises.

[6] $u_k = \begin{pmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+\alpha-1} \end{pmatrix}, y_k = \begin{pmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+\alpha-1} \end{pmatrix}, k = 1, \ldots, N.$

α and N are numeriques, such as $\alpha > 2n$ and $N \gg 2n$

[7] $y_k = \Gamma X_k + \theta u_k + \Xi V_k + W_k, k = 1, \ldots, N.$

V_k and W_k with size $l\alpha * 1$, vectors of stacked process and measurement noises, defined as following:

[8] $V_k = \begin{pmatrix} V_k \\ V_{k+1} \\ \vdots \\ V_{k+\alpha-1} \end{pmatrix}, W_k = \begin{pmatrix} W_k \\ W_{k+1} \\ \vdots \\ W_{k+\alpha-1} \end{pmatrix}, k = 1, \ldots, N.$

Γ is an extended matrix, whose size is $l\alpha * 2n$. This matrice is used to removing the influence of inputs and noises, and can be derived from the identification matrices A and C.

[9] $\Gamma = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{\alpha-1} \end{pmatrix}$

θ and Ξ are $l\alpha * m\alpha$, $l\alpha * 2n\alpha$block Toeplitz matrices, defined as following:

[10] $\theta = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{\alpha-2}B & CA^{\alpha-3}B & \cdots & D \end{bmatrix},$

[11] $\Xi = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ C & 0 & \cdots & 0 & 0 \\ CA & C & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{\alpha-2} & CA^{\alpha-3} & \cdots & C & 0 \end{bmatrix}$

3. TWO JERK ENERGY-BASED DAMAGE LOCALIZATION METHODS

The two JE-based MNCDWJE and CDPWJE methods are proposed for the damage detection. We present in this session the two methods.

The outputs of sensors values $y_1, y_2, \ldots, y_n$, which are accelerations, which are measured of the node k the JE-based damage feature at measured node number k is defined as following (An et al., 2015).

[12] $\mathrm{JE_k} = log \sum_{x=1}^{N-1} (J_x^k)^2 = log \sum_{x=1}^{N-1} \left(\frac{y_{x-1}-y_x}{\Delta t}\right)^2$

Where k presents the measured node number, Δt is the sampling interval, waveform of the jerk energy (WJE) is created through connecting JE values at every measured node.

[13] $\mathrm{C_k} = \frac{\mathrm{JE_{k-1}} - 2\mathrm{JE_k} + \mathrm{JE_{k+1}}}{\mathrm{h}^2}$

$\mathrm{C_k}$ is the curvature of sensor k, h is the distance between two neighbors sensors.

The value of difference curvature WJE before and after of damage sensor k as defined as following:

[14] $(\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs} = (\mathrm{C}_\mathrm{r}^\mathrm{k})_\mathrm{before} - (\mathrm{C}_\mathrm{s}^\mathrm{k})_\mathrm{after}$

$(\mathrm{C}_\mathrm{r}^\mathrm{k})_\mathrm{before}$ is the measure of curvature of sensor k of the $\mathrm{r^{th}}$ value before the damage.

$(\mathrm{C}_\mathrm{s}^\mathrm{k})_\mathrm{after}$ is the measure of curvature of sensor k of the $\mathrm{s^{th}}$ value after the damage.

3.1 Damage localization method MNCDWJE

The first method uses the value of the curvature $(\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}$ based on the Eq. (14) for localize the damage.

Normalize the value of the $(\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}$ for any pair is defined as following:

[15] $(\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}^* = (\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs} / \mathrm{max_k}\left[(\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}\right]$

The use of the normalized value $(\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}^*$ and the calculation of the means normalized curvature difference μ_k for the historical response is defined as following:

[16] $(\mathrm{DI_1})_\mathrm{k} = \mu_\mathrm{k} = \frac{1}{\mathrm{RS}} \sum_{\mathrm{r}=1}^{\mathrm{R}} \sum_{\mathrm{s}=1}^{\mathrm{S}} (\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}^*$

Damaged indexed = $\{(\mathrm{DI_1})_\mathrm{k} \geq \delta\}$

The value of δ is calculated by the undamaged structure.

3.2 Damage localization method CDPWJE

The second method after the normalized curvature $(\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}^*$ of the all nodes.

Determining the number of mean normalized curvature taking the value greater or equal than δ.

[17] $\Gamma_\mathrm{rs}^\mathrm{k} = \begin{cases} 1, & (\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}^* \geq \delta \\ 0, & (\mathrm{C}_\Delta^\mathrm{k})_\mathrm{rs}^* < \delta \end{cases}$

The $\mathrm{DI_2}$ damage index is based on the RS identification.

[18] $(\mathrm{DI_2})_\mathrm{k} = \frac{1}{\mathrm{RS}} \sum_{\mathrm{r}=1}^{\mathrm{R}} \sum_{\mathrm{s}=1}^{\mathrm{S}} \Gamma_\mathrm{rs}^\mathrm{k}$

Damaged indexed = $\{(\mathrm{DI_2})_\mathrm{k} \geq \delta\}$

The value of damage is greater or equal than δ where is it threshold of probability.

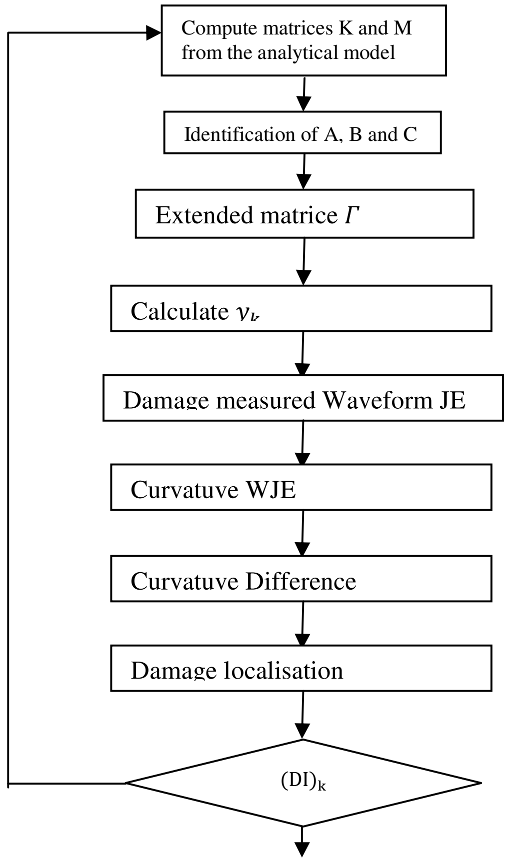

Fig 1: Flowchart of identification procedure.

This flowchart is begins by identifying the matrices A, B and C. When the model is based on multiple series of sensors it can use the extended matrices Γ. We calculate the output of sensors for detection and localize the damage (Mousavi et al., 2016).

First we begin with waveform JE, then we calculate the curvature based on the neighbors waveform. Next we examine the difference between before and after curvature. Finally we calculate the damage index in order to determine if the structure is damaged or not damaged.

4. NUMERICAL SIMULATION

To verify the performance of the two jerk energy-based methods, the 4-story shear structure is selected as research object (Zheng et al., 2015) (Gautier et al., 2015).

$$M = \begin{bmatrix} m1 & 0 & 0 \\ 0 & m2 & 0 \\ 0 & 0 & m3 \end{bmatrix}$$

$$k = \begin{bmatrix} k1 & -k1 & 0 \\ -k1 & 0 & k2 \\ 0 & -k2 & k2 \end{bmatrix}$$

$$y = \begin{bmatrix} a_1 m1 + a_2 k1 & -a_2 k1 & 0 \\ -a_2 k1 & a_1 m2 & a_2 k2 \\ 0 & -a_2 k2 & a_1 m3 + a_2 k2 \end{bmatrix}$$

Where the value of lamped masses m_1, m_2, m_3; stiffness of the 2 stories are K_1, K_2 respectively; 3 damping constants y_1, y_2, y_3; these parameters are based on the random excitation.
$m_1 = m_2 = m_3 = 1.0 * 10^5 Kg$, $K_1 = K_2 = 2.0 * 10^8 N/m^3$.

The module young's 200 Gpa $= 200 * 10^9$ pa, Air session $A = 0.0249 * 0.0053 = 13.197 * 10^{-5} m^2$, $L = 1$ m, the proper's pulsations is used are $\sqrt{5}$ and $\sqrt{15}$, $a_1 = 1.1381$, $a_2 = 7.4714 * 10^{-5}$ and damping ratio: $\xi_i = \xi_j = 0.25$ (An et al., 2015) (Lu et al., 2010).

The next Fig represents the model of our application, the blue points represent the vibrations sensors, the green rectangular represent the stiffness matrices and the red points represent the wireless and controller node, these nodes can be transmit the data to the server when we able to use them later.

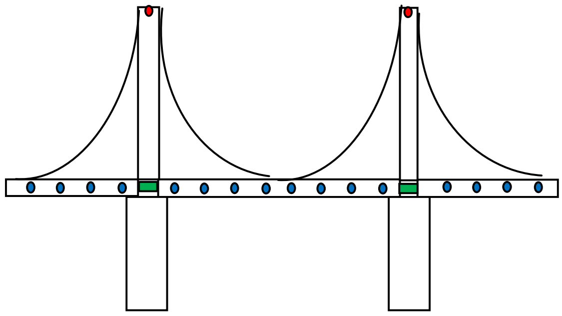

Fig 2: The model of bridge structure.

4.1 Update of finite element method

The finite element method is the output of bridge can be reduced in each iteration because the stiffness matrices reduced to with the value 5% in each iteration.

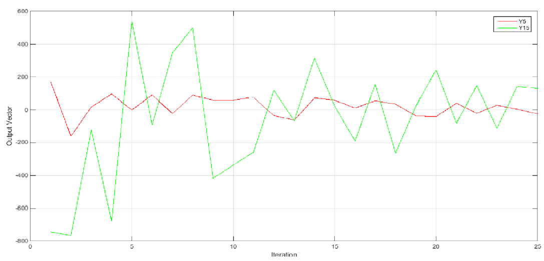

Fig 3: Output vetor of FEM.

4.2 Acceleration sensors

In this Fig we show the acceleration of vibrations sensors by 25 iterations (Zheng et al. 2015).

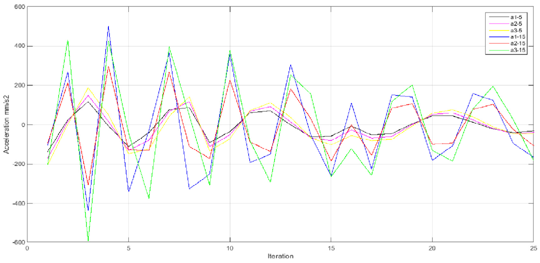

Fig 4: Acceleration of sensors.

4.3 Effect of measurement noise

$\tilde{a} = a + n = a + \mu\sigma_a r$

We simulate the error of the measuring nodes for detecting the rang of error without two proper pulsations $\sqrt{5}$ and $\sqrt{15}$., $\tilde{a}$ and a mean the accelerations with and without noise, n is the noise; σ_a is standard deviation of the accelerations is based on the accelerations stories; μ is noise level; r is a vector of random distributed numbers (An et al., 2015).

$\sigma_a = \sqrt{\frac{(a_1-\bar{a})^2+(a_2-\bar{a})^2+\cdots+(a_p-\bar{a})^2}{N}}$, where acceleration in times $a_i 1 \leq i \leq n$ and $\bar{a} = \frac{1}{N}\sum_{i=1}^{N} a_i$

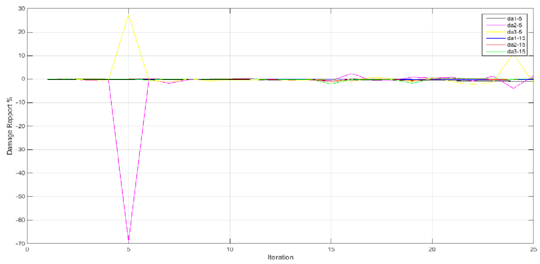

Fig 5: Damage Report.

This Fig. shows the cases of damages for validating the two jerk energy-based methods. The two proper pulsations are used $\sqrt{5}$ and $\sqrt{15}$ on random excitation. We calculate the story damage based on the noise levels 20% of the acceleration signal added to the original acceleration.

4.4 Damage cases and determination of the threshold δ

The threshold δ is a value greater than mean normalized curvature difference. The value of μ is fixed after some undamaged cases with different noise levels by 0.4.

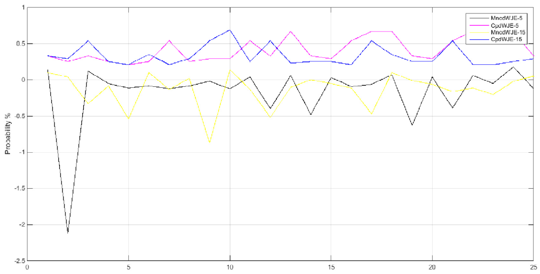

Fig 6: Damage Index stories damage cases.

5. CONCLUSION

The localization damage based on the MNCDWJE or CDPWJE has been developed for structure identification and localization damage. Updating method based on MNCDWJE or CDPWJE is elaborated for identifying accelerations of structure. To minimize the time of CPU it is required to use the reduction model and to localize the damages in the structure. In this regard iteration domain has been proposed. This model has been provided in damage localization and is required for the real time damage detection.

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