Calibration of an MPS model by simulating a pool-and-weir fishway

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Abstract: Pool-and-weir fishway is an important type of fishway to aid migrate fish ascending upstream through obstructions in rivers or streams. It consists of a set of fishway pools separated with weirs. Water flows through this type of fishway either by plunging into the fishway pool or streaming above weirs. In this study, a pool-and-weir fishway flow is modelled by using a mesh-free method, moving particle semi-implicit method (MPS). This method is widely used to simulate flows in hydraulic engineering such as dam-breaking flow, open-channel flow, hydraulic jump and weir flows. Due to the Lagrangian nature of the method, it is good at handling large deformation and free surface problems since moving particles are used instead of fixed meshes in the traditional Eulerian methods. This flow in fishway is successfully reproduced by MPS. The water surface profiles are compared with experimental measurements, and good agreement is achieved.

Keywords: pool-and-weir fishway, Moving particle semi-implicit method, mesh-free, flow structure

1 Introduction

Moving particle semi-implicit method, (MPS), is a mesh-free method, proposed ten years ago (Koshizuka and Oka 1996, Koshizuka et al. 1998). It is developed in the framework of Lagrangian method by tracing each particle's movement. Movable particles are used to represent fluid, and it is unnecessary to construct fixed meshes in this method. Accordingly, drawbacks of using fixed meshes are no longer associated with MPS. On the other hand, due to its mesh-free nature without constrains of fixed meshes, it is good at handling problems with interface and big deformations of fluid flow problems. Therefore, it has been widely employed to simulate engineering problems, and successful applications have been reported in hydraulic engineering (e.g., Nazari et al. 2012, Shakibaeinia and Jin 2010), nuclear engineering (e.g, Koshizuka and Oka 1996), and coastal engineering (e.g., Khayyer and Gotoh 2009), etc.

However, as a Lagrangian method, MPS suffers from computing efficiency due to the necessity of tracing each fluid particle's movement. In order to improve efficiency, an equation of state is used to obtain pressure field in mesh-free methods to replace the Poisson equation which requires solving a large matrix (Monaghan 1994). Therefore, the fluid is assumed to be weakly compressible when the equation of state is used for pressure calculation.

In this study, MPS with the weakly-compressibility treatment is employed to simulate a fishway flow. In fact, fishway is widely designed to aid migrant fish ascending upstream through obstructions in rivers or streams (DVWK 2002). It is a very important hydraulic structure. There are many types of fishways, such as vertical slot fishway, baffle fishway, and pool-and-weir fishway. Pool-and-weir fishway is proved to be effective and successful in engineering experience when favorable flow structure is formed. Water flows through it by plunging over weirs and has great mixture in the fishway pools by the form of jets and eddies. In this study, the flow pattern in the pool-and-weir fishway is modelled by the MPS method, and the free-surface profiles are compared with an experimental measurement.

2 Numerical method

2.1 Governing equations

The governing equations for the viscous fluid include the continuity and momentum equation, and the form of them in the Lagrangian description is expressed as:

$$[1]\quad \begin{array}{l} \frac{D\rho}{Dt} = 0 \\ \frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + g \end{array}$$

where ρ is density, **u** is the velocity vector, υ is the viscosity, g is the gravity.

2.2 MPS operators

In MPS, the fluid in the domain is represented by a set of separately movable particles. These particles are distributed in an average distance with each other, namely particle distance DL. Although particles can move without any restriction seemingly, they are intangibly connected by a kernel or weighting function. This weighting function determines a circle which is considered as an interacted area for a target particle. That means that when calculating the flow properties of a particle, only particles located in the circle defined by the weighting function are able to contribute flow values of that calculated particle, and the effects from other particles beyond the circle are assumed negligible. The weighting function used in this study is expressed in Eq.2 and its shape is illustrated in Figure 1.

$$[2]\quad W(r_{ij}) = \begin{cases} (1 - r_{ij}/r_e)^3 & r_{ij} < r_e \\ 0 & r_{ij} \geq r_e \end{cases}$$

where r_{ij} is the distance between the particle i and particle j, i is the target particle and j is one of the neighboring particles in the interaction circle, r_e is the radius of the interaction circle, and r_e=3.5DL is adopted in this study.

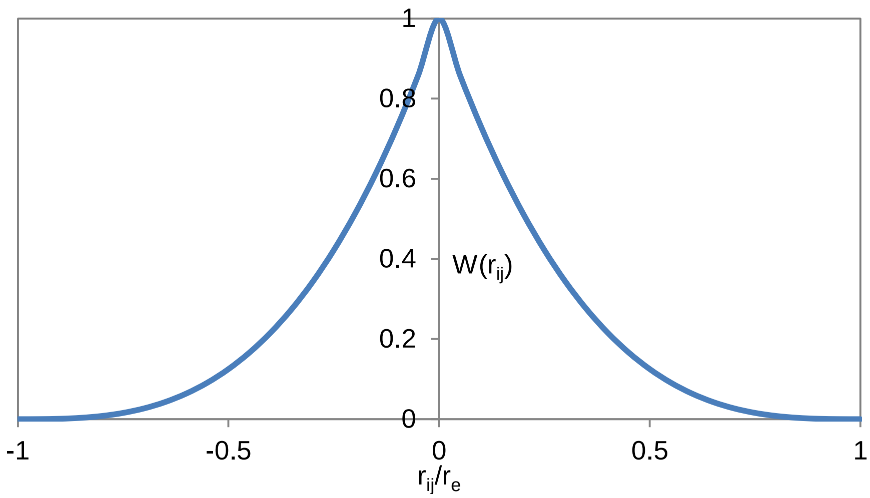

Figure 1: The shape of the weighting function

The particle number density for a target particle i in MPS is defined as the summation of values of the kernel function for neighboring particles j in the interaction circle. The particle number density can be expressed as:

$$[3]\quad \langle n\rangle_i = \sum_{j\neq i} W(r_{ij})$$

To approximate the first and second order differentials in the governing equations, the gradient, divergence, and Laplacian models are developed. One of gradient models commonly used in MPS is (e.g., Toyota et al 2005):

[4] $$\langle \nabla \phi \rangle_i = \frac{D_m}{n_0} \sum_{j \neq i} \frac{\phi_j + \phi_i}{r_{ij}^2} \mathbf{r_{ij}} W(r_{ij})$$

where D_m is the dimension of space, e.g., D_m=1 for one-dimensional problem, 2 and 3 for two- and three-dimensional problem, respectively. n_0 is the average particle number density in the calculation, $\mathbf{r_{ij}}$ is particle position vector between particle i and j.

The Laplacian model used in this study, is proposed by Koshizuka et al (1998) and expressed as:

[5] $$\langle \nabla^2 \phi \rangle_i = \frac{2D_m}{\lambda n_0} \sum_{j \neq i} (\phi_j - \phi_i) W(r_{ij})$$

The coefficient λ is defined as:

[6] $$\lambda = \frac{\int_V W(r) r dV}{\int_V W(r) dV}$$

2.3 Numerical scheme

The predictor-and-corrector numerical scheme is employed in MPS to solve the governing equations. In the predictor step, the viscous and external force terms are solved to obtain a temporal velocity field, and particles are temporally moved to new positions.

[7] $$\begin{aligned} \mathbf{u}^* &= \mathbf{u}^n + (\nu \nabla^2 \mathbf{u} + g)\Delta t \\ \mathbf{r}^* &= \mathbf{u}^* \Delta t \end{aligned}$$

According to the new particle positions, temporal particle number density <n>* can be calculated. In this study, the fluid is assumed to be weakly compressible, which allows the pressure field to be computed based on the equation of state. This treatment improves computing efficiency by avoiding solving the Poisson equation. The equation of state is expressed as:

[8] $$p_i = \frac{\rho C_0^2}{7}\left(\left(\frac{\langle n^* \rangle_i}{n_0}\right)^7 - 1\right)$$

where C_0 is an equivalent sound speed. To keep the fluid compressible less than 1%, C_0 has to be 10 times maximum velocity in the simulation.

In the corrector step, the pressure gradient is computed for correction of the temporal velocity field, and particles are accordingly moved to new positions. Therefore, velocity and position fields at current time step are obtained.

[9] $$\begin{aligned} \mathbf{u}^{n+1} &= \mathbf{u}^* - \Delta t \frac{1}{\rho} \nabla p \\ \mathbf{r}^{n+1} &= \Delta t \mathbf{u}^{n+1} \end{aligned}$$

3 Boundary conditions

Boundary conditions employed for treatments in this study are the solid boundary condition, inflow and outflow condition, and free-surface condition.

Solid particles are used to represent the solid boundary and they are evenly discretized as a distance DL (in Figure 2), meanwhile the solid boundary is technically extended outward by using several layers of ghost particles beyond the solid particles because these ghost particles can properly fill the empty area of the interaction circle when calculating a target particle close to the boundary as illustrated in Figure 2. The no-slip condition is assigned to the solid and ghost particles.

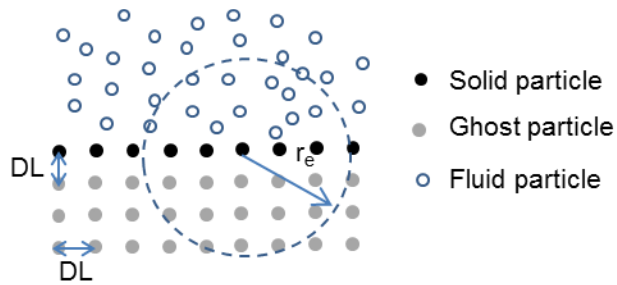

Figure 2: Solid boundary treatment

A particle whose particle number density $\langle n \rangle_i$ is less than a threshold is regarded as the representation of free surface, therefore, the free surface particles can be identified as by the following condition:

[10] $n^* \leq \beta n_0$

where β=0.97 is used in this study.

At the inlet, particles are periodically injected into the domain according to the discharge, and inlet velocity is assigned to the injected particles. Meanwhile, particles reaching the outlet are prescribed as pressure based on the static water in the downstream. These particles are removed from the domain after they pass the end of the simulation domain.

4 Pool-and-weir fishway flow simulation

In this study, a pool-and-weir fishway flow is modelled by using this meth-free method. The investigated fishway consists of five fishway pools separated by six vertical weirs (Figure 3a). The weirs are very sharp, whose thickness is dx=0.08 m. The height of weirs is Ly=0.24 m, and length of each fishway pool is Ly=0.48 m, shown in Figure 3b. The slope of fishway bed is 1/12. The discharge to the fishway is 0.08 m^3/s.

To model this flow, all the boundary condition treatments including inflow and outflow condition explained at section 3 are implemented in the simulation. Moreover, a turbulence model, derived from LES, is incorporated into MPS to model the turbulence characteristics. This turbulence model has been successfully applied to simulate jet flow and open channel flows in MPS (e.g., Gotoh et al 2001). In the simulation, two particle distances DL=0.01 m and 0.005 m are considered, and numerical results are compared, shown in Figure 4. When using DL=0.01 m, there are only several particles over the weir to represent the jet flow (Figure 4a), so the jet becomes very weak in the fishway pool due to lack of sufficient fluid particles for the jet. However, after using DL=0.005 m, the fluid particles for the jet doubles, making the jet flow in the fishway pool is quite obvious, shown in Figure 4b.

Initially, the five fishway pools are empty and particles are only set up before the first weir in Figure 3a. After t=0.0 s, particles start to move, flow over the weir, and plunge into the fishway pool. Meanwhile, fluid particles are gradually and periodically pouring into the fishway through the inflow boundary condition. In this way, all the fishway pools are gradually becoming full successively. This water filling process to the fishway is illustrated in Figure 5. The flow behaves violently in this process, especially impacting the fishway bed. Splashes of water particles are observed in this process (Figure 5a and 5c). Once the fishway pool becomes full, water only flows over weirs, and behaves like streaming flow, shown in Figure 5e.

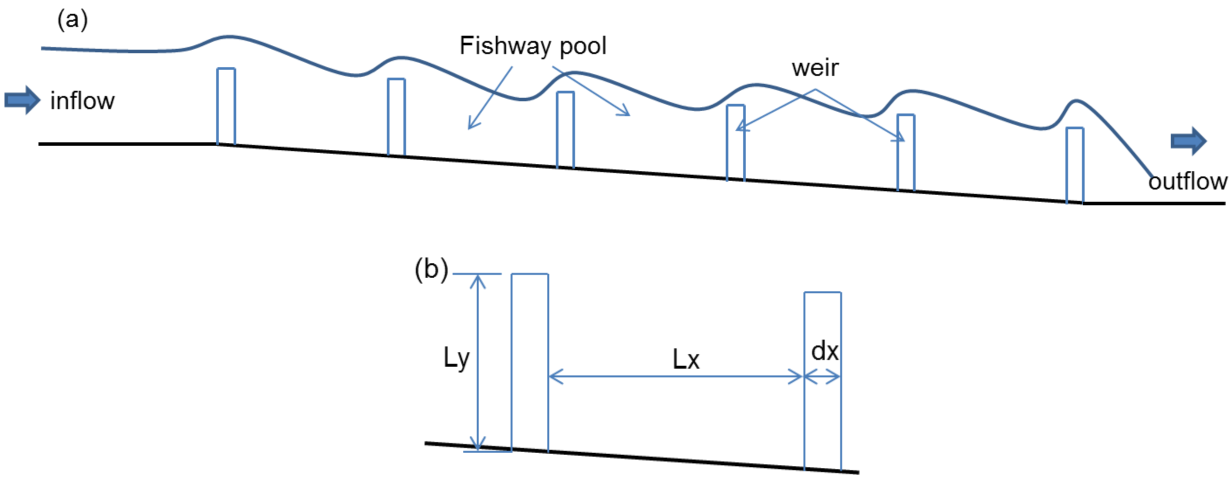

Figure 3: Pool-and-weir fishway

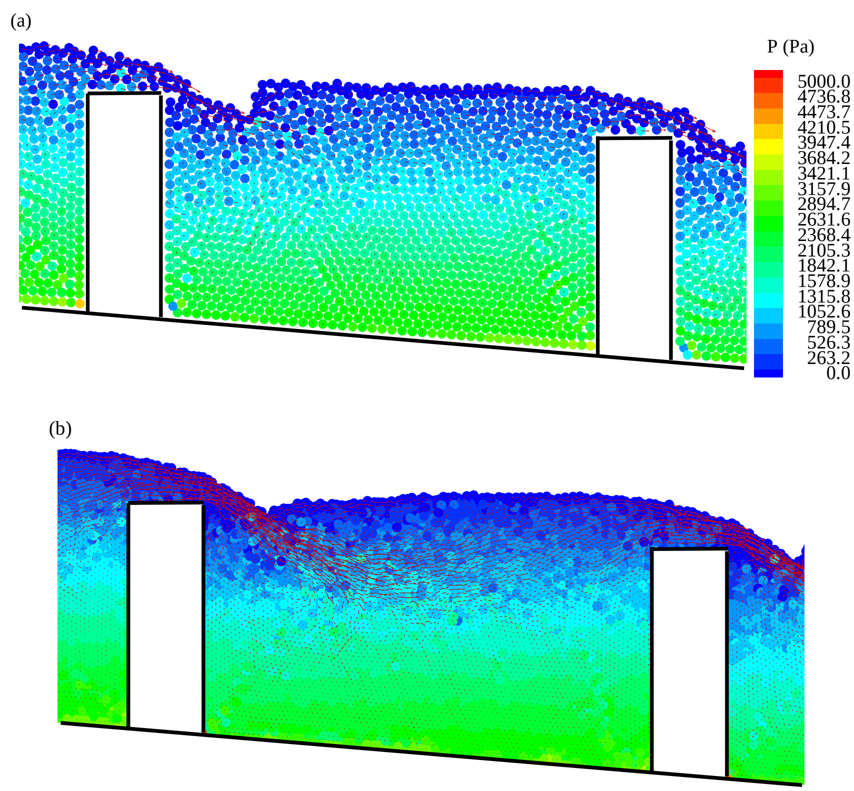

Figure 4: Flow patterns with two different particle distances, (a) DL=0.01 m and (b) DL=0.005 m

The steady state of the fishway flow is shown in Figure 4. Figure 4b clearly shows the jet and some eddies in the fishway pool. The jet flow is obviously strong after it steps into the fishway pool. But it becomes much weaker since the jet sufficiently mixes with water in the pool, causing energy dissipation. On the other hand, the large eddy below the jet flow is not clearly reproduced in this study, partly because some terms in the governing equations, such as gradient terms, are not perfect to simulate this kind of eddy.

As commonly known, mesh-free methods are good to identify the free water surface. The surface profiles simulated in MPS by using two different particle distances DL=0.01 m and 0.005 m are extracted and plotted in Figure 6. They are also compared with the experimental measurements (Onitsuka et al 2007). From the comparison, numerical results show good agreement with experimental data although some differences at 0.2<x/Lx<0.4 are observed. For the results simulated from the two particle distances DL=0.01 m and 0.005 m, there is small difference between them.

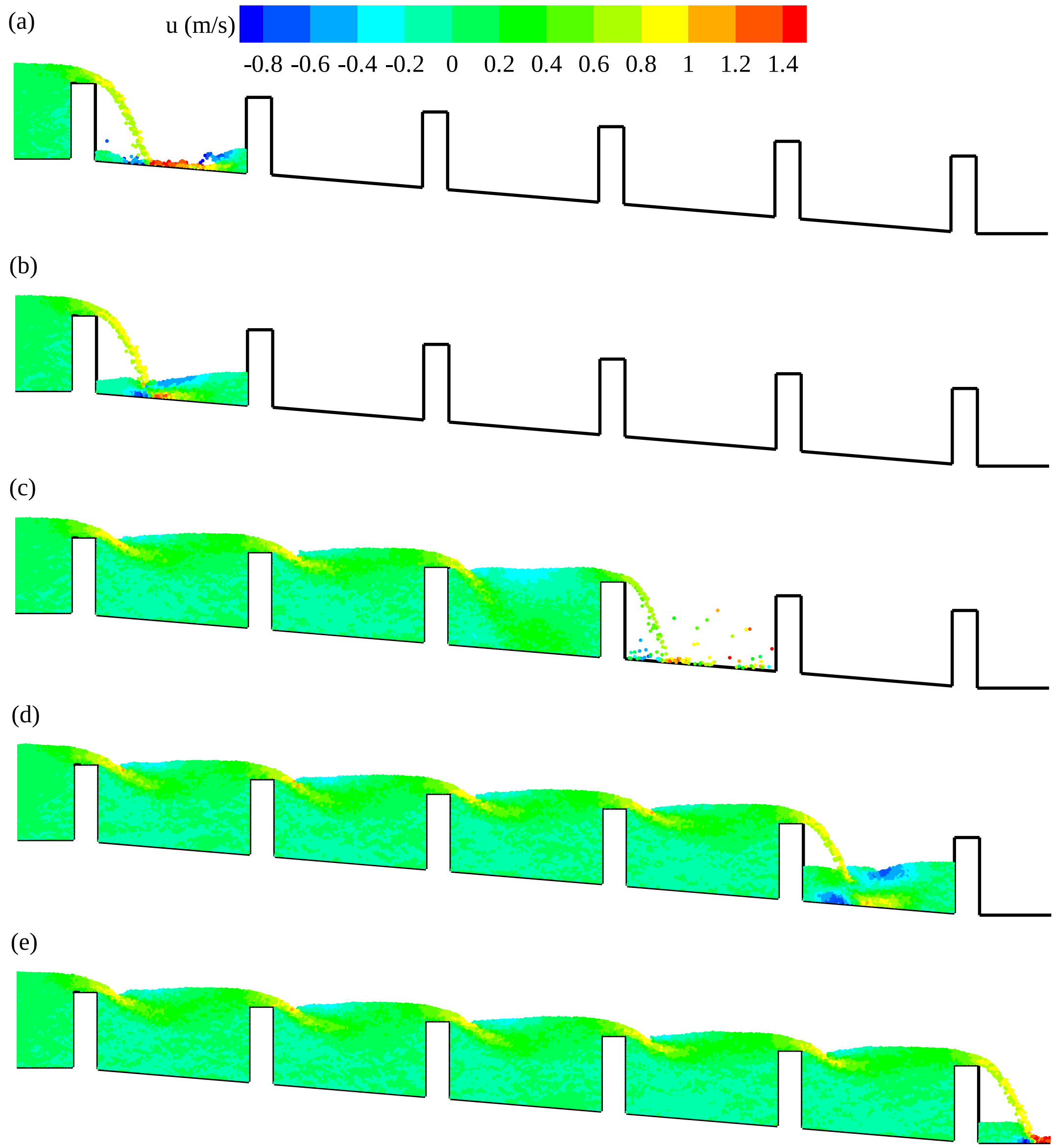

Figure 5: Filling process of water to the fishway (a) t=1.0 s, (b) t=2.0 s, (c) t=20.0 s, (d) t=30.0 s, and (e) t=40.0 s

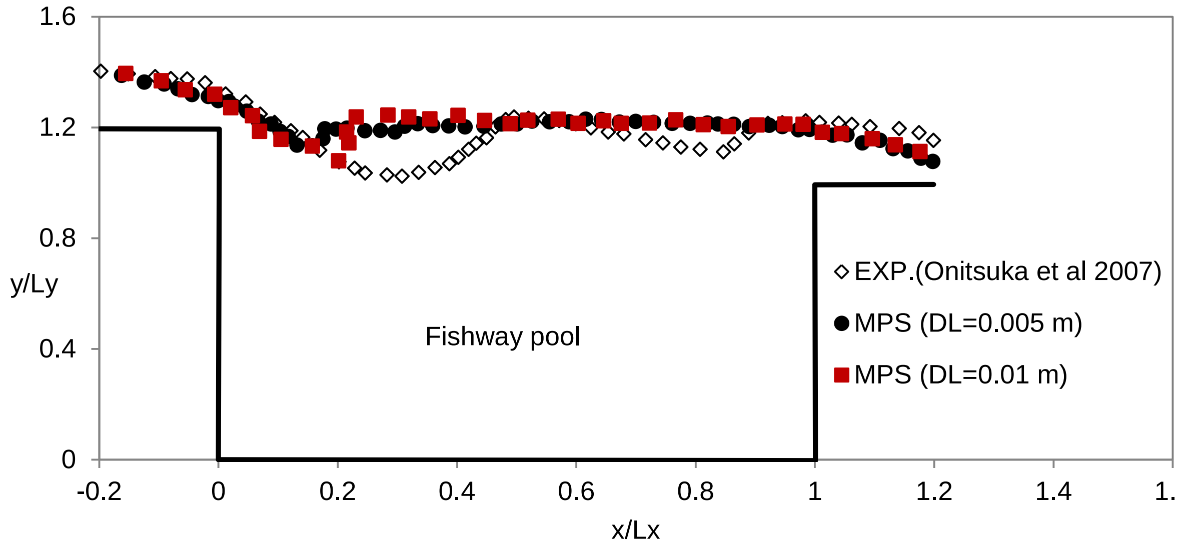

Figure 6: Comparisons of free surface profiles

Conclusions

In this paper, a pool-and-weir fishway flow is modeled by using a mesh-free method, Moving particle semi-implicit method (MPS). The modeled fishway consists of five fishway pools, separated by six sharp weirs. The fishway pools are initially empty, and fluid particles gradually fill the fishway pools through injecting particles at the inlet. In the initial stage, the flow passes the fishway violently when filling the fishway pools. After all the pools are full, the flow becomes considerably mild. Water flows above weirs, behaving like streaming flow. Two particle distances DL=0.01 m and 0.005 m are used in the simulation. Large particle distance cannot sufficiently represent the jet flow over the fishway weir. Using the finer particle distance in MPS, a clear jet and mixture in the fishway pool are obviously reproduced. The extracted water surface profiles are compared with experimental measurements, and good agreement is observed.

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