ANALYTICAL MODEL TO DETERMINE LONG-TIME MULTIPLIERS FOR FULLY CRACKED AND UNCRACKED SECTIONS

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Abstract: Long-time multipliers are commonly used in design codes and specifications such as CSA A23.3 and ACI 318 to compute time-dependent deflections of reinforced concrete members. These multipliers are convenient for design due to their simplicity of application. However, as a result of their simplicity, they do not account for many factors that influence time-dependent deflections. This paper presents an analytical method based on a time-stepping procedure to trace deformations in cracked and uncracked sections under sustained loads. A parametric study is conducted to assess the sensitivity of the solution to the various parameters involved and implications for design are discussed.

1 INTRODUCTION

Current provisions for deflection control of concrete members are based on the use of a multiplier applied to calculated immediate deflection under sustained load. Calculated deflections under sustained load are then compared with specified deflection limits as a fraction of span length. Researchers and practitioners have raised concerns that the current multipliers in ACI 318 and CSA A23.3 are too small for application to deflection design of lightly reinforced concrete slabs (ACI 435, 1995). The present study was undertaken to examine time-dependent effects due to creep and shrinkage on singly reinforced concrete flexural sections. Since deflections are directly related to curvatures, the paper examines time-dependent curvature for uncracked and fully cracked sections, the two limiting cases, under a sustained applied bending moment. Results of the study indicate significant differences between response of uncracked and fully cracked sections. An approach to account for these differences is proposed as a basis for design application.

2 ANALYTICAL MODEL

A time-stepping procedure is developed to trace the stresses and strains on the cross-section under a sustained applied bending moment. A layered beam formulation is used along with the well-known creep superposition approach originally proposed by McHenry (1943). At each time-step, conditions of strain compatibility (linear strain with depth) and equilibrium are applied. Constant stress is assumed in each layer during each time increment. The formulation uses creep and shrinkage functions presented by ACI 209 (1992).

Figure 1 shows a rectangular section of an uncracked section divided into layers. The linear strain and stress diagrams under an applied bending moment are also shown. Figure 2 shows a rectangular cracked section and corresponding strain and stressed loads under an applied bending moment. The strain and stress diagrams are shown for two time-steps. As creep and shrinkage occur, the depth of the neutral axis gradually increases as strains increase. The stresses in the concrete and steel adjust to maintain equilibrium with the applied bending moment. For the cracked section, all tensile stress in the concrete is immediately transferred to the steel, while for the uncracked section, the tensile stress in the concrete is gradually transferred to the steel. Complete details of the time-stepping procedure and its implementation in MATLAB are provided in Karschner (2012).

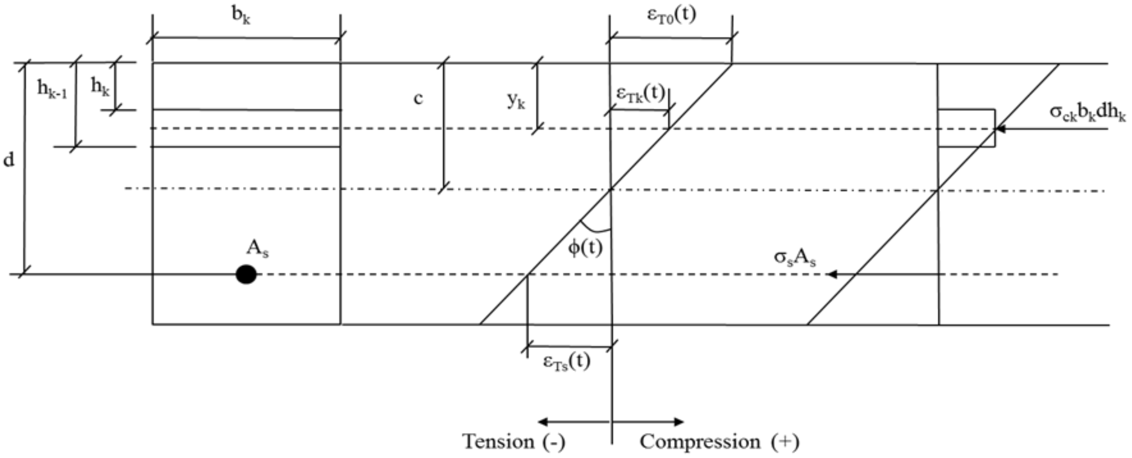


Figure 1: Diagram of uncracked reinforced concrete flexure section used for development of stress, strain, and curvature relationships.

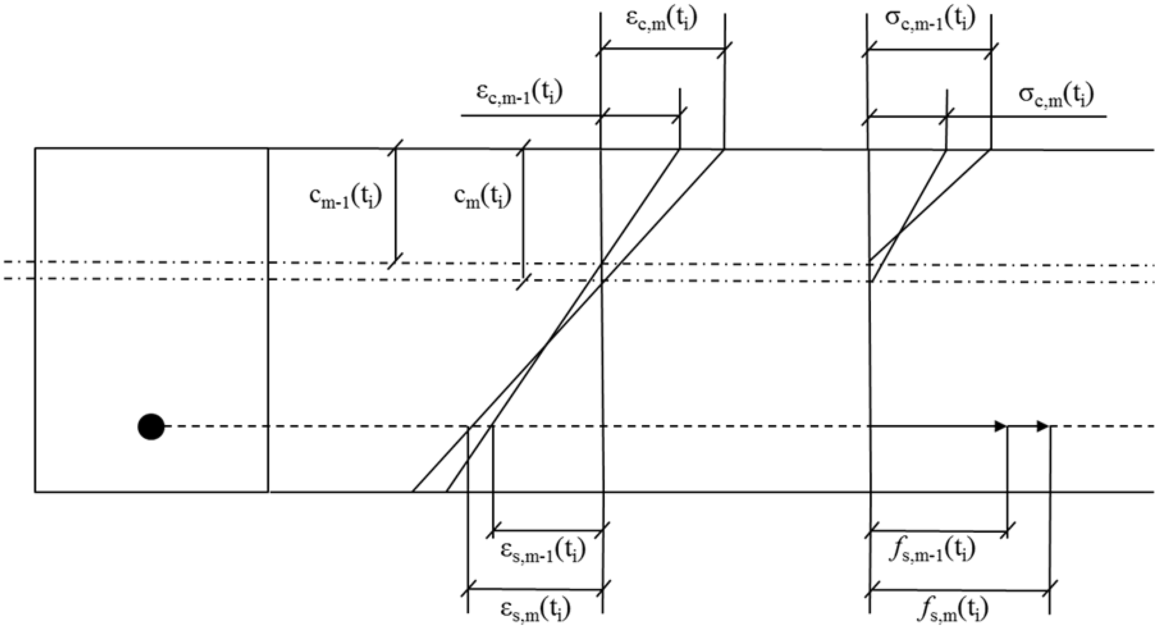
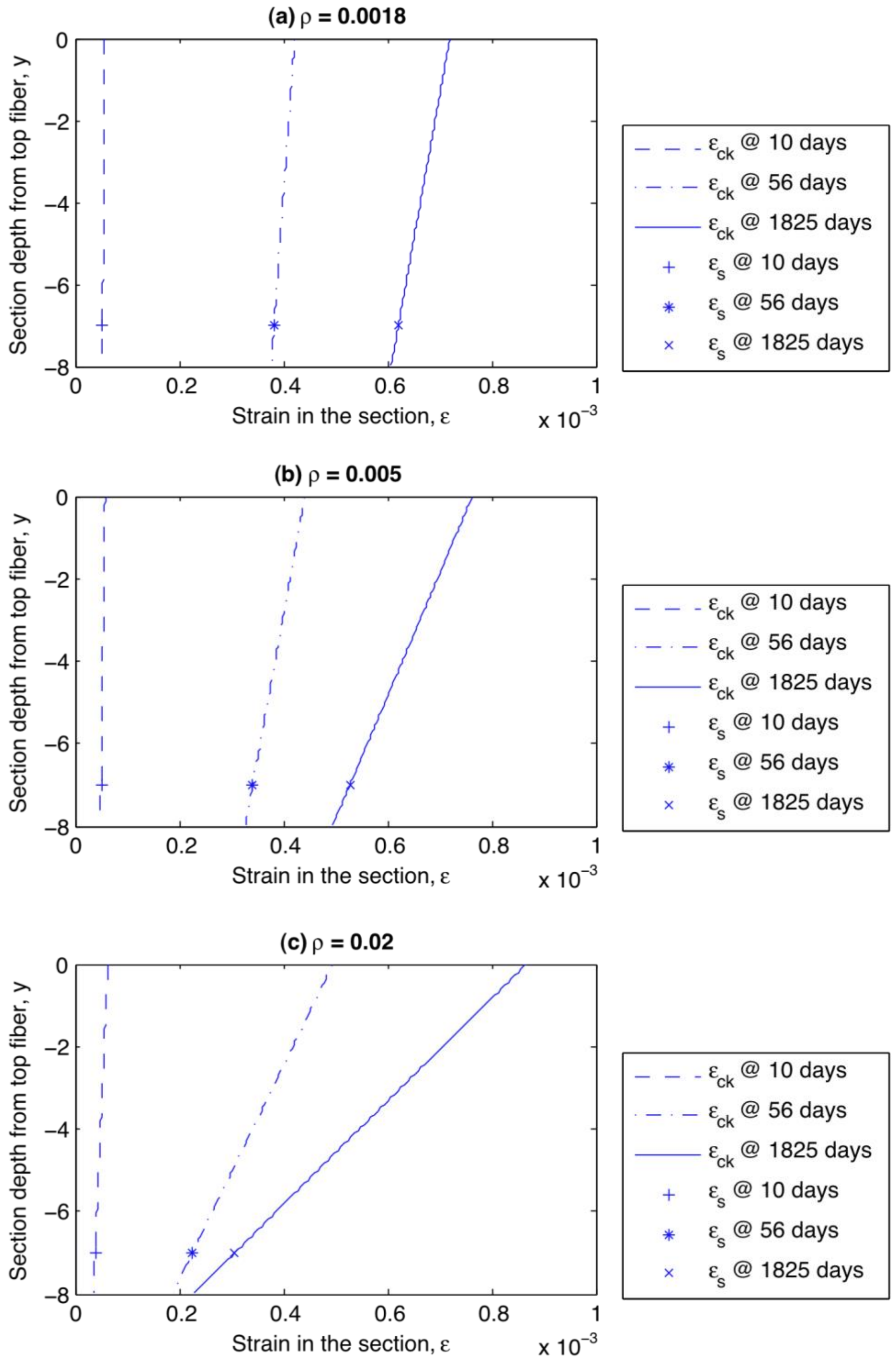


Figure 2: Schematic of concrete creep effect on cracked reinforced concrete section.

3 PARAMETRIC STUDY

The algorithm developed in this study was used to study the effects of several parameters on the time-dependent increase in curvature due to creep and shrinkage under sustained moment. In all cases, shrinkage was assumed to be uniform across the cross-section. Figure 3 shows the effect of reinforcement ratio on strain distribution at increasing times under shrinkage alone.



Note: t_0 = 7 days; f'_c = 4000 psi; C_u = 2.35

Figure 3: Effects of shrinkage warping on the long-term curvature of the uncracked reinforced concrete flexure section: (a) ρ = 0.0018; (b) ρ = 0.005; (c) ρ = 0.02.

Due to the eccentricity of the reinforcement, shrinkage warping occurs. As the reinforcement ratio increases, the strain in the reinforcement decreases, but the curvature due to warping increases.

To determine a long-time multiplier based on curvature, the multiplier λ_Δ was taken as the difference between the curvature at 5 years minus the initial curvature divided by the initial curvature. The effect of reinforcement ratio on the long-time multiplier for uncracked and cracked sections is shown in Fig. 4. Multipliers are plotted for three cases, creep only, shrinkage only, and total (creep plus shrinkage). Results are presented for initial loading at seven days and twenty-eight days. For the uncracked case, the results show that the total multiplier decreases with increasing reinforcement ratio, while for the cracked section, the multiplier increases slightly with increasing reinforcement ratio. In all cases, the multiplier for the cracked case is approximately 1. For comparison, the ACI 318 (and CSA A23.3) multiplier of 2 is also shown. For uncracked sections the total multiplier is larger than 2 for all reinforcement ratios. Of course, the initial curvature for the uncracked case will be significantly smaller than the curvature for the fully cracked case. Deflections are usually a design issue for members that are cracked; in which case, the actual multiplier would be expected to be somewhere between the two limits (uncracked and fully cracked). In this case, the initial deflection would be calculated using an effective stiffness between the uncracked and fully cracked cases.

A similar comparison is shown in Fig. 5 where the effect of age at initial loading is shown for various reinforcement ratios. The creep correction factor for age at loading given by ACI 209 was used in defining the creep characteristics for the concrete. As before, the total multiplier for the uncracked section is greater than the current coded value, while for the fully cracked case, the multiplier is less than the current code value. The results suggest that long-time multipliers for lightly reinforced sections (such as slabs) should be higher than multipliers for more-heavily-reinforced members. Also, a correction factor for members loaded at early age should be provided.

Additional results for the effects of variation in concrete compressive strength, creep coefficient, and ultimate shrinkage strain can be found in Karschner (2012).

4 IMPLICATIONS FOR DESIGN

Based on the results of the parametric study, the following expressions are suggested for long-time multipliers for uncracked and cracked sections in terms of reinforcement ratio.

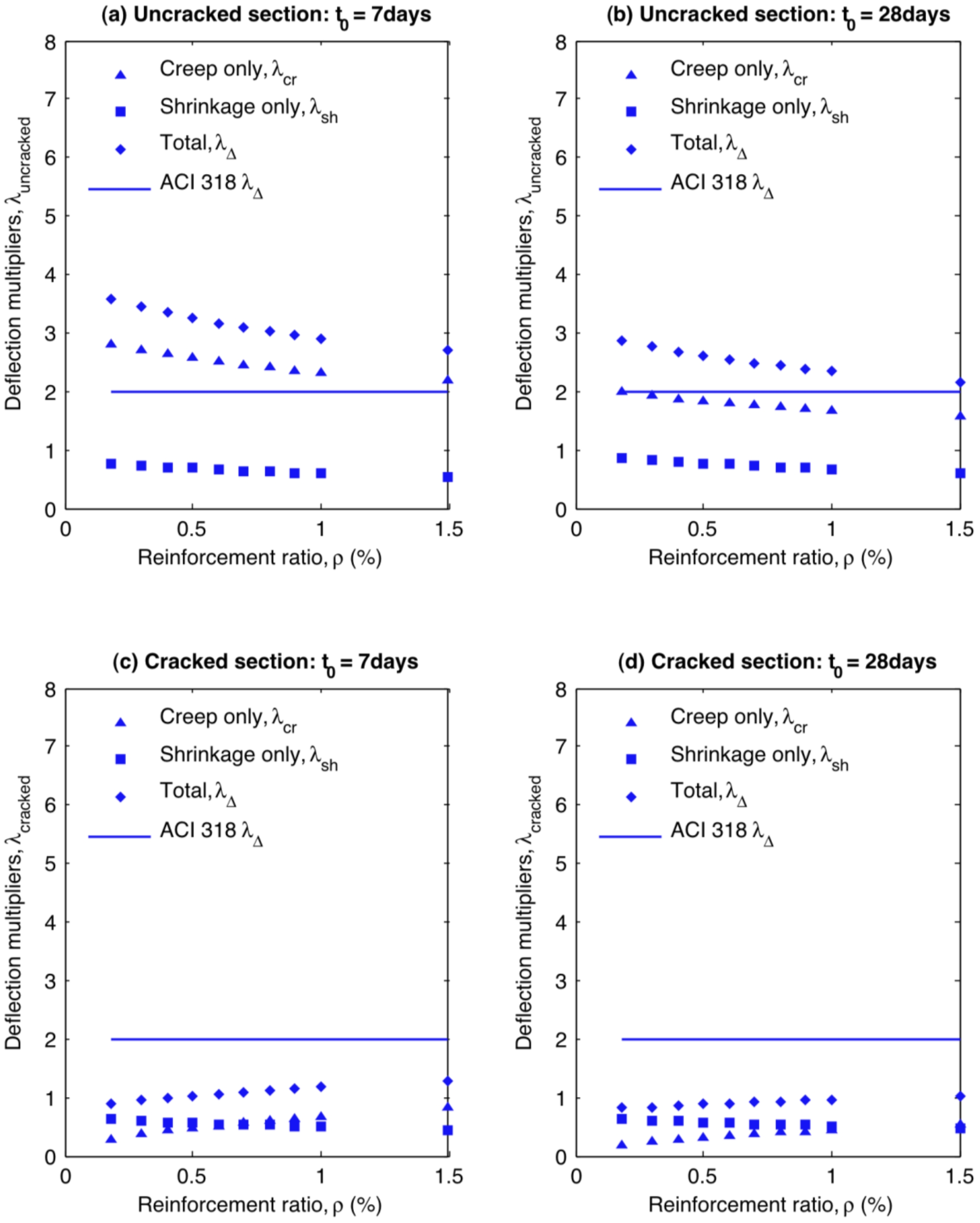
[1] $$\lambda_{ucr}^{\rho} = 2.75 + \frac{1}{-1.32}\left(\rho - 1.5\right) \geq 2.0$$

[2] $$\lambda_{cr}^{\rho} = 1.0$$

Since these expressions represent upper and lower limits for members that are cracked under service loads, an approach similar to that used for the effective moment of inertia of a member could be considered such as Eq. [3]:

$$\lambda_{eff} = \lambda_{ucr}^{\rho}\left(\frac{M_{cr}}{M_a}\right)^m + \lambda_{cr}^{\rho}\left(1 - \left(\frac{M_{cr}}{M_a}\right)^m\right)$$

[3]


(a) Uncracked section: t_0 = 7days

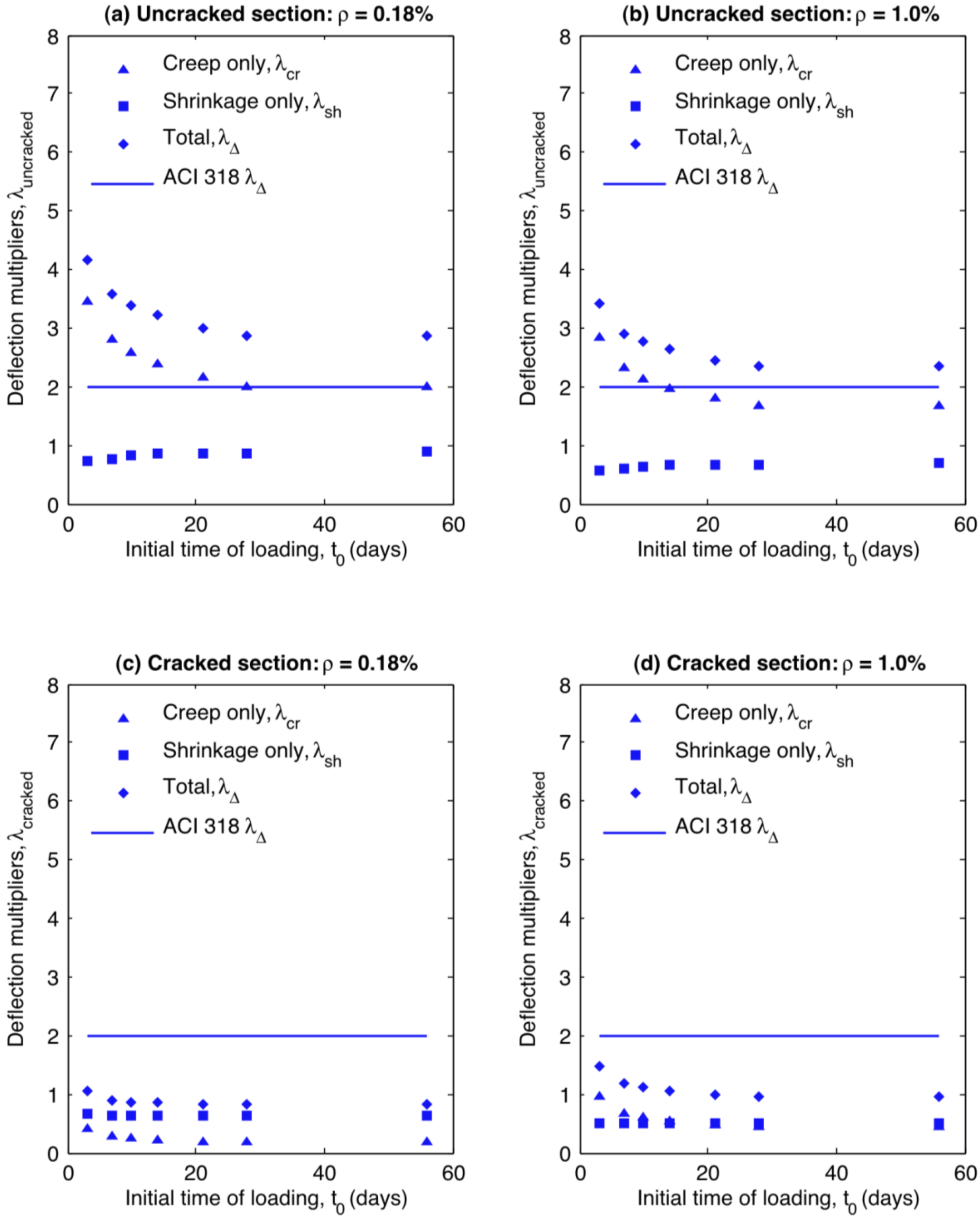
(b) Uncracked section: t_0 = 28days

(c) Cracked section: t_0 = 7days

(d) Cracked section: t_0 = 28days

Note: $f'_c = 4000$ psi; $C_u = 2.35$

Figure 4: Long-term deflection multipliers vs. reinforcement ratio:
(a) Uncracked section: t_0 = 7 days; (b) Uncracked section: t_0 = 28 days;
(c) Cracked section: t_0 = 7 days; (d) Cracked section: t_0 = 28 days



Note: f'_c = 4000 psi; C_u = 2.35

Figure 5: Long-term deflection multipliers vs. initial time of loading:
(a) Uncracked section: ρ = 0.18%; (b) Uncracked section: ρ = 1.0%;
(c) Cracked section: ρ = 0.18%; (d) Cracked section: ρ = 1.0%.

Further research is needed to determine whether an expression such as Eq. [3] would be applicable for design use.

5 CONCLUSION

An analytical procedure has been developed to evaluate the time-dependent response of reinforced concrete flexural members under sustained moment at the member section level. Uncracked and fully cracked member sections have been examined. Studies have shown that the time-dependent response is affected by reinforcement ratio and age at initial loading. A general approach has been suggested by the author to account for the effect of reinforcement ratio on long-time multipliers for calculating time-dependent deflection. More work is needed to evaluate the use of this approach for design.

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