IMPROVED SENSITIVITY ANALYSIS IN CIVIL ENGINEERING DESIGN USING STATISTICAL DESIGN OF EXPERIMENTS (DOE)

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Abstract: An engineering analysis or design is seldom completed without conducting a proper sensitivity analysis. Sensitivity or what-if analysis is the assessment of the consequences of changes in input factors and/or model parameters, not taking into account information on the probability of these changes. These changes may be due to uncertainty in the input parameters or unforeseen deviations from specifications. In practice, changing one factor or one parameter at a time is perhaps the most popular and well-known method for conducting sensitivity analysis. This approach is advocated and described in textbooks on engineering design as well as engineering economics. The advantage of this method is that it is easy to apply and the results seem easy to understand. However, the one-factor-at-a-time or OFAT method is known to be inefficient and in some cases lead to wrong results. In this paper, a sensitivity analysis approach based on statistical design of experiments (DOE) is introduced using examples from structural and geotechnical engineering, and from engineering economics. This paper will show that the DOE approach is easy to apply, efficient, and to interpret. The approach is in fact a combined sensitivity and scenario analysis. It will also be shown that the DOE approach provides better information for decision making than the OFAT and stochastic risk analysis approaches.

1.0 INTRODUCTION

An engineering design or analysis is seldom completed without conducting some sort of sensitivity analysis (SA). Sensitivity or what-if analysis is the assessment of the consequences of changes in inputs and model parameters, not taking into account information on the probability of these changes. In SA of computer models, it aims to identify the key parameters that affect model performance and it plays important roles in model parameterization, calibration, optimization, and uncertainty quantification (Song et al, 2015). Risk analysis on the other hand tries to assess the same effects as SA does, but takes into account the (joint) probability distribution of the input parameters. However, the reliability of the results depends strongly on the knowledge of the probability distribution of the input values. Furthermore, the impact of each parameter and their possible interactions are not directly known without further analysis. That is, a probability of failure can be obtained but not which parameters are important. In practice, changing one factor or one parameter at a time or the OFAT method is practically the standard method for conducting sensitivity analysis. This approach has been advocated and commonly described in textbooks on engineering design (e.g. Dieter, 1986). It is also the approach taken by most engineering students when asked to do a SA. The advantage of this method is that it is easy to apply and the results seem easy to understand. However, the OFAT method is known to be inefficient and ineffective and in some cases can lead to disastrous and wrong results. Only the effect of each factor or parameter can be studied but not their interactions with each other. These interactions can be more significant than the individual effects (Montgomery and Myers, 2009; Lye, 2002). That is, the effect of a parameter may depend on the level of another parameter. Another form of SA is scenario analysis. A scenario analysis uses a combination of simultaneous changes in input parameters that have been deemed to be possible scenarios in future. Usually an optimistic or best case, a pessimistic or worst case, and the base case scenario are considered. With many input parameters, the number or possible scenarios increases rapidly, and the scenarios actually used are selected somewhat arbitrarily. The resulting information is usually insufficient to meet the needs of decision makers (Van Groenendaal and Kleijnen, 1997).

In this paper, the use of statistical DOE methodology for SA will be illustrated using examples from structural and geotechnical engineering, and an example from engineering economics. The method is one of many methods for SA. Other methods of SA, particularly for computer models have been reviewed by Iman and Helton (1988) and more recently by Song et al (2015). The DOE approach is highlighted in this paper because of its ease of computation, direct interpretation, and its wide applicability for many engineering design and analysis problems. The first example considers a reinforced concrete beam design problem involving four parameters and the interest is in the moment of resistance. The second example is a slope stability problem with five parameters and the interest is in the factor of safety. The final example is in engineering economics involving seven parameters and the interest is in the net present value. The software Design-Expert 9.0 by Statease, Inc., a software for design and analysis of experiments, will be used in this paper. Other software such as Minitab® or spreadsheet like Excel® could also be used.

In the next section, the basic idea of DOE and what it can achieve will be described. This is followed by the application of factorial design and fractional factorial design methodologies for the three different examples described above. The paper ends with a discussion of further applications of DOE methodologies in civil engineering and conclusions.

2.0 DESIGN OF EXPERIMENTS (DOE)

Sensitivity analysis (SA) is basically based on the same underlying principles as experimental design and much of the terminology used in SA has originated from an experimental design or DOE setting (Campolongo and Saltelli, 2000). Experimentation in various sub-disciplines of civil engineering may be computer simulations, laboratory or field experiments. As with most engineering problems, engineers are often faced with limited time and budget. Hence efficient experimentation that gains as much information as possible is critical. In engineering, one often-used approach is the best-guess (with engineering judgment) approach. Another strategy of experimentation that is prevalent in practice is the one-factor-at-a-time or OFAT approach (also known as ceteris paribus). The OFAT method was once considered as the standard, systematic, and accepted method of scientific experimentation. Both of these methods have been shown to be inefficient and in fact can be disastrous (Czitrom, 1999; Lye, 2002; Montgomery, 2005). These methods have been outdated since in the early 1920s when Ronald A. Fisher invented methods of experimentation based on factorial designs. These approaches provide methods for selecting the combinations of factor values to be employed that will provide the most information on the input-output relationship in the presence of variation or uncertainty. These methods are based on elementary statistical principles and have wide applicability (Campolongo and Saltelli, 2000). These were further developed to include fractional factorial designs, orthogonal arrays, and response surface methodology. These statistical methods are now simply called design of experiment methods or DOE methods. Basically, DOE is a methodology for systematically applying statistics to experimentation. DOE lets experimenters develop a mathematical model that predicts how input variables interact to create output variables or responses in a process or system. The use of statistics is important in DOE but not absolutely necessary. In general, by using DOE, we can learn about the process being investigated, screen important factors, determine whether factors interact, build a mathematical model for prediction, and optimize the response(s) if required (Montgomery, 2009). In this paper, the term factors and parameters or variables will be used interchangeably.

It has been recognized that the factorial-based DOE is the correct and the most efficient method of doing multi-factored experiments; they allow a large number of factors to be investigated with few experimental runs. Two-level factorial designs in which each of the factors has exactly two levels are among the most commonly used in industrial experiments. A two-level design having k factors requires a minimum of 2^k test runs to accommodate all possible combinations of the factor levels, i.e. a full factorial. A full factorial design with five factors, for example, would require 2^5runs or 32 runs. To save on runs, only a fraction of all the possible combinations can be performed using fractional factorial designs symbolized as "2^{k-p}", where k refers to the number of factors and p is the fraction. While cutting runs saves on costs, it reduces the ability of the design to resolve all possible effects, specifically the higher order interactions. A measure of how well the design can resolve the interactions is called the resolution of the design. Ideally, the higher the resolution, the better will be the design. It is recommended that resolution V and above is best as all main effects and two-factor interactions can be estimated accurately. Nowadays, there are also novel highly efficient resolution V designs that are able to handle experiments with up to 50 factors (Oehlert and Whitcomb, 2002) with much fewer runs than the traditional fractional factorial designs. Details of factorial and fractional factorial designs can be found in Montgomery (2005), among others.

3.0 APPLICATION EXAMPLES

The first example shows the use of a two-level factorial design for a four- parameter problem. The second example uses a fractional factorial design for the five-parameter problem, and the third example shows the use of a minimum run computer generated fractional factorial design for a seven-parameter problem.

3.1 Reinforced concrete beam: sensitivity of the moment of resistance to 4 design parameters

This design problem involves a reinforced concrete beam section with a nominal size of 400 mm width by 800 mm depth. The area of steel is 3,500 mm^2 and concrete strength is 30 MPa. The clear cover for the steel is assumed to be about 40 mm. The cross-section is shown in Figure 1. It will be assumed that there are uncertainties in the width b, depth d, concrete strength f'_c, and cover c due to quality control issues at the site. According to CSA A23.3-04 (2004), the moment of resistance of an under-reinforced concrete section (Figure 1) is calculated as:

[1] $$M_r = \rho \phi_s f_y \left(1 - \frac{\rho \phi_s f_y}{2 \alpha_1 \phi_c f'_c}\right) b d^2$$

Where: ρ is the reinforcement ratio, $\rho = A_s/bd$, ϕ_c and ϕ_s are material resistance factors and are equal to 0.65 and 0.85 respectively. The yield strength if the steel is f_y and α_1 is ratio of average stress in rectangular compression block to the specified concrete strength. In order to calculate the nominal strength of the section, the material resistance factors ϕ_c and ϕ_s are set to be equal to 1.

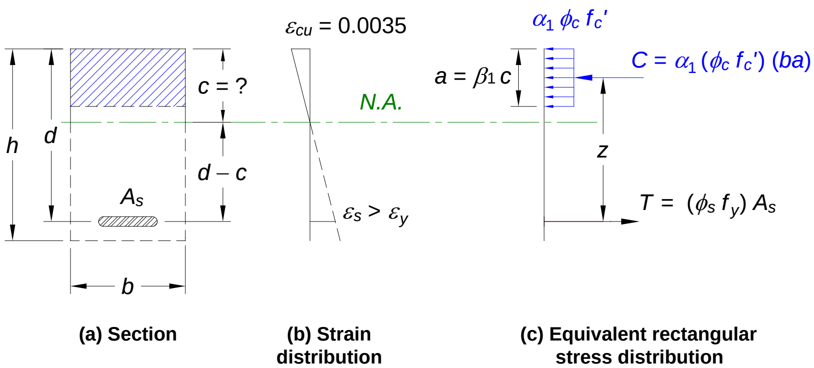

Figure 1: Cross-section of the RC beam

The ranges of the four design parameters are given in Table 1. The area of steel is assumed to be fixed. The sensitivity of the calculated moment of resistance (M_r) of the beam due to these uncertainties is desired.

Table 1: Parameters and ranges for RC beam problem

Parameter	Units	Name	Low	High
b	mm	Width	380	420
d	mm	Depth	760	840
f'_c	MPa	Concrete strength	27	33
c	mm	Cover	30	50

A simple 2-level factorial design will be used for this four-parameter problem. A 2^4 design requires 16 run combinations – that is all combinations of the 4 parameters. The combinations and the resulting moment of resistance (in kN-m) are shown in Table 2.

An effective way to determine the significance of the effect of each parameter on the response (M_r) is by means of a half-normal probability plot (Montgomery, 2009). Effects that are random are expected to be normally distributed and hence will plot along a straight line on a probability plot. Non-random effects will fall off the straight line and are considered to be significant effects. In our case, these will be the most sensitive parameters. The mechanics of effects calculations and doing the plot are well covered in Montgomery (2009) among others. Figure 2 shows the half-normal plot of the 15 effects calculated from the 16-run combination.

Table 2: Run combinations from a 2^4 design for the concrete beam problem

Run	B	d	f'_c	c	M_r
1	380	840	27	30	1058
2	380	760	33	30	966
3	420	760	33	50	976
4	380	840	33	50	1078
5	380	760	27	50	946
6	380	840	27	50	1058
7	420	840	27	30	1069
8	420	840	33	30	1088
9	380	760	33	50	966
10	420	840	27	50	1069
11	420	840	33	50	1088
12	380	760	27	30	946
13	420	760	27	30	957
14	420	760	33	30	976
15	380	840	33	30	1078
16	420	760	27	50	957

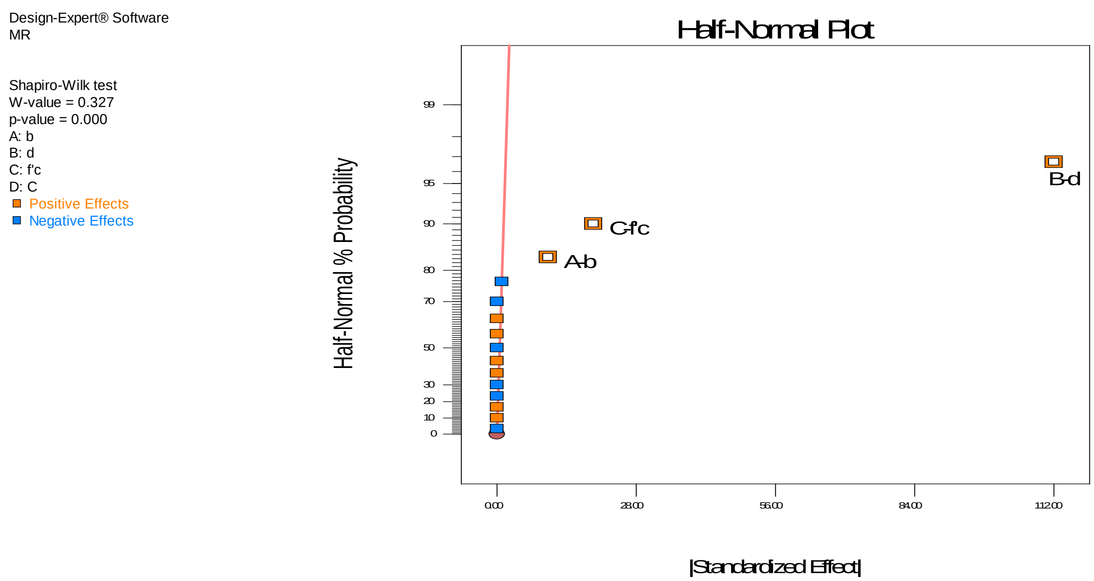

Figure 2: Half-normal plot of effects for the concrete beam problem.

Figure 2 shows that only the depth (d) is the most significant or sensitive parameter followed by the concrete strength (f'_c), and then the width (b). Surprisingly, the M_r is not sensitive to the cover (c) for the ranges considered. A prediction equation in terms of the significant effects (parameters) can also be developed using regression analysis. The prediction can be written in terms of coded or actual factors.

Coded factors means that the parameter used has been scaled to -1 to +1, while the actual factors will be in terms of the actual units used. For interpretation of the coefficients, use of coded factors is preferred of their direct correspondence to the sensitivity of the parameter. For the concrete beam problem, the moment of resistance can be predicted within the range of the parameters used using:

[2] $M_r = 1017.32 + 5.13\ A + 56.0\ B + 9.70\ C$

Where: A = coded factor of depth d, B = coded factor of concrete strength f'_c, and C = coded factor of width b. The adjusted R^2 of the regression equation is 0.999 which is an excellent fit. The regression equation can also be written in terms of the actual factors as:

[3] $M_r = -302.34 + 0.257\ b + 1.400\ d + 3.233\ f'_c$

Where: b, d, and f'_c have been defined earlier.

As can be seen, Table 2 and equation [2] or [3] can be used to determine the combination that would give the best and worst case scenario. Hence using DOE is doing sensitive and scenario analyses in one go.

3.2 Slope stability problem: sensitivity of the factor of safety to 5 soil parameters

Next a 5-parameter design problem is considered. A half-fraction resolution V factorial design with 16 run combinations will be used. The geometry of the slope used in this example is shown in Figure 3. A *D*-m high river bank slope is chosen which has an angle of β° with the horizontal. The ground surface is considered flat. It is assumed that the slope has a uniform layer of soil above the strong base layer. Typical soil properties such as unit weight (γ), cohesion (*c*) and angle of internal friction (*ϕ*) is considered. The groundwater table is shown by the dashed line. For simplicity the ground water table is kept constant at EFKH throughout this study. The failure of this slope could occur under different conditions because of the variation of slope geometry (*D*, β) and soil parameters (γ, *c*, *ϕ*). In this study, to check the effect of these five parameters (*D*, β, γ, *c*, *ϕ*) on slope stability, combinations of different factors are considered using the low and high value of each factor as stated in Table 3. In other words, this study investigates the sensitivity of the factors that have the greatest effect on slope failure, and whether is there any interaction among these factors. Morgenstern-Price (limit equilibrium) method (Das, 2010) available in Slope/W (2007) software by GEOSLOPE International Ltd is used to determine the Factor of Safety (*FS*) of the slope.

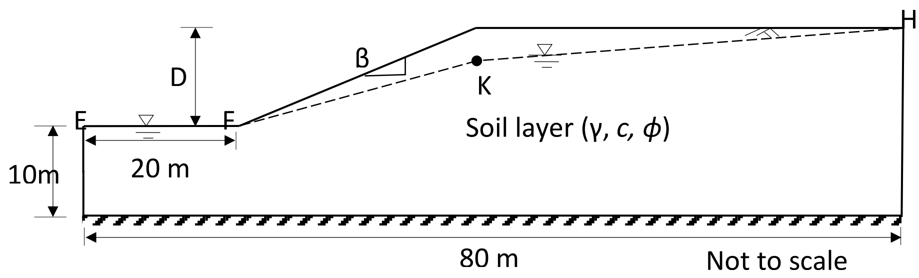

Figure 3: Geometry of the slope used in example problem

Table 3: Parameters and ranges for slope stability analysis

Parameter	Units	Name	Low value	High value
c	kPa	Cohesion	30	40
ϕ	degrees	Friction angle	18	28
γ	kN/m^3	Unit weight	15	20
D	m	Height of slope	10	15
β	degrees	Slope angle	24	32

The Morgenstern-Price (M-P) method is a general method of slices to determine the Factor of Safety (*FS*) developed on the basis of limit equilibrium method. This method considers both shear and normal interslice forces, and satisfies both moment and force equilibrium assuming various user-specified inter-slice force functions such as constant, half-sine, trapezoidal function (Slope/W, 2007). The 16-run combinations from a Resolution V 2^{5-1} fractional factorial design is given in Table 4.

The corresponding response, the *FS*, is also shown in Table 4. For a 16-run Resolution V fractional factorial design, 15 effects can be estimated without ambiguity. Significant effects can be picked out using the half-normal plot as discussed in the first example or can be formally tested using analysis of variance or ANOVA. From the half-normal plot shown in Figure 4, it is clear that the parameters that the *FS* is most sensitive to is the soil friction angle ϕ and depth *D*, followed by the cohesion *c*, then unit weight γ and slope β. For the range of parameters used, there is no interaction among the five parameters.

Table 4: Run combinations of a 2^{5-1} design for the slope stability problem.

Run	*c*	ϕ	γ	*D*	β	*FS*
1	30	18	15	10	24	2.019
2	40	18	15	10	32	2.349
3	30	28	15	10	32	2.296
4	40	28	15	10	24	2.915
5	30	18	20	10	32	1.681
6	40	18	20	10	24	2.182
7	30	28	20	10	24	2.351
8	40	28	20	10	32	2.442
9	30	18	15	15	32	1.492
10	40	18	15	15	24	1.912
11	30	28	15	15	24	2.067
12	40	28	15	15	32	2.178
13	30	18	20	15	24	1.516
14	40	18	20	15	32	1.562
15	30	28	20	15	32	1.803
16	40	28	20	15	24	2.257

A prediction equation can be obtained for the *FS* as a function of the five significant effects in terms of coded factors:

[4] $FS = 2.06 + 0.16\,A + 0.22\,B – 0.09\,C - 0.22\,D – 0.088\,E.$

In [4], A is the coded factor of cohesion, B is the coded factor of ϕ, C is the coded factor of unit weight, D is the coded factor of depth, and E is the coded factor of the slope angle. A coded factor has a range of -1 to +1.

The prediction equation developed fulfilled the assumptions of regression with an adjusted R^2 of 0.967 which is a reasonably good fit. In other words, the fairly complex calculations of the *FS* can be simply reduced to equation [4] which shows the sensitivity of each parameter on the *FS* directly. From Table 4 and equation [4], one can easily see the most critical combinations that lead to lower *FS*. Clearly, to increase *FS*, one must reduce the uncertainty of ϕ and *D* in particular.

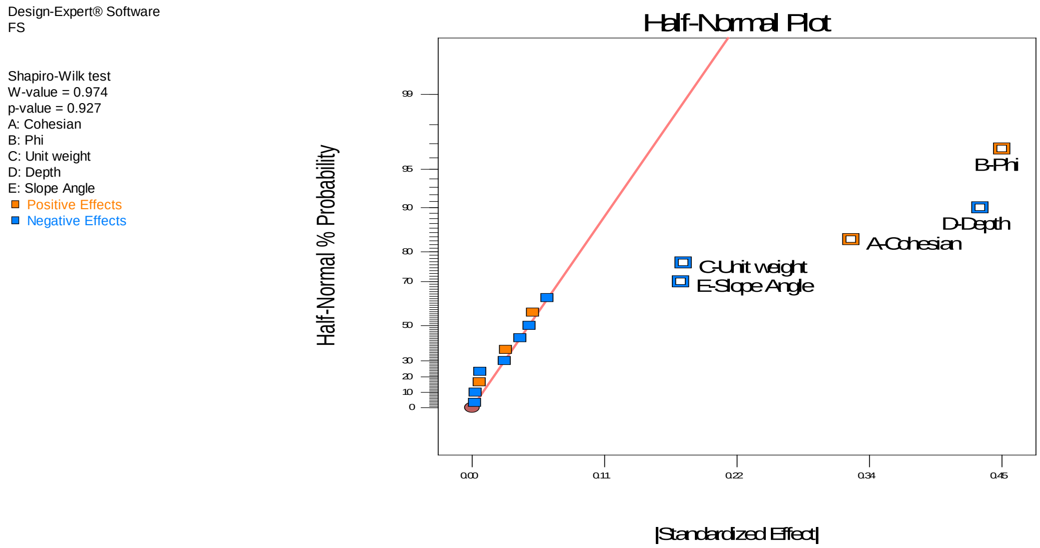

Figure 4: Half-normal plot of effects for the slope stability problem.

3.3 Engineering economic analysis – sensitivity of the NPV to 7 parameters

According to Van Groenendaal and Kleijnen (1997) less information is required when using fractional factorial designs for sensitivity analysis in an economic analysis. The sensitivity analysis is more robust, and leads to results that better satisfy the information needs of decision makers compared to Monte Carlo risk analysis. In this example, the sensitivity of seven economic parameters on the NPV or net present value, a commonly used criterion for large investment projects. Consider a corporation that is considering replacing its current steam plant with a 5-megawatt cogeneration plant that will produce both steam and electric power for operations. The new plant will use waste wood as a source of fuel, which will eliminate the need to purchase electric power from the public utility. The estimated initial cost (I) for the equipment and installation is $3M, but there is some uncertainty about this estimate. The plant is expected to last 20 years with no salvage value. In addition to the initial cost, the equipment will require an overhaul with an estimated cost (OC) of $35,000 at the end of the 4th, 8th, 12th, and 16th year. In addition, the cooling tower will also need an overhaul at the end of the 10th year at an estimated cost (CT) of $17,000. Operation and maintenance cost for the new plant is expected to be higher than the current system and this incremental cost (OM) has been estimated at $65,000 annually. The incremental annual cost for the wood fuel (W) is $375,000 but this will obviate the need to purchase 40 MW-hours of electricity per year at about $0.025 per kW-h (EC). The discount rate (i) is estimated to be about 12%. The NPV is thus a function of seven economic parameters. That is:

[5] NPV = f(I, OC, CT, OM, W, EC, i)

Where: NPV = net present value, I = initial cost, OC = overhaul cost, CT = cooling tower overhaul cost, OM = operation and maintenance cost, W = wood fuel cost, EC = electricity cost, and i = discount rate. The NPV is calculated using:

[6] NPV = -I – (OM + W + EC *4MW) (P/A, i, 20) – CT(P/F,i,10) – OC [P/F,i,4) + (P/F,i,8) + (P/F,i,12) + (P/F,i,16)]

Where: (P/A,i,n) and (P/F,i,n) are standard compound interest factors with discount rate of i and n compounding periods. The minimum run resolution V design with 30 run combinations is shown in Table 5 together with the NPV for each combination.

The half normal plot of the significant effects of each factor or parameter and their interaction is shown in Figure5. It can be seen that the most significant factor affecting NPV is the cost of electricity (EC). A higher cost of electricity translate to more savings and hence a higher NPV. The discount rate is the next most significant factor. It has a negative effect on the NPV. A higher discount rate would lead to a reduction in the NPV. This is followed by the initial cost, wood fuel cost, and there is also an interaction effect between the cost of electricity and discount rate. This means that the effect of the discount rate on the NPV depends on the level of the electricity cost. Figure 6 shows this interaction effect. NPV is not sensitive at all to operation and maintenance, and plant overhaul costs for the ranges used.

Table 5: Run combinations of minimum run resolution V design with 7 parameters.

Run	Initial Cost	OM	WC	EC	i	CT	OC	NPV
1	$2,700,000	$58,500	$337,500	$0.03	10%	$15,300	$38,500	$4,074,104
2	$2,700,000	$71,500	$337,500	$0.02	14%	$15,300	$31,500	-$154,585
3	$2,700,000	$71,500	$412,500	$0.03	10%	$15,300	$38,500	$3,324,910
4	$2,700,000	$58,500	$412,500	$0.02	10%	$15,300	$38,500	$30,161
5	$3,300,000	$71,500	$412,500	$0.03	14%	$18,700	$38,500	$1,388,103
6	$2,700,000	$58,500	$337,500	$0.02	14%	$18,700	$31,500	-$69,402
7	$3,300,000	$71,500	$337,500	$0.02	10%	$18,700	$31,500	-$31,508
8	$2,700,000	$71,500	$337,500	$0.03	14%	$18,700	$31,500	$2,493,750
9	$3,300,000	$58,500	$412,500	$0.02	14%	$15,300	$31,500	-$1,165,219
10	$3,300,000	$71,500	$337,500	$0.03	10%	$18,700	$38,500	$3,362,117
11	$2,700,000	$58,500	$337,500	$0.03	10%	$18,700	$31,500	$4,084,593
12	$2,700,000	$58,500	$337,500	$0.03	14%	$18,700	$38,500	$2,570,939
13	$2,700,000	$58,500	$412,500	$0.03	14%	$15,300	$31,500	$2,084,033
14	$3,300,000	$58,500	$337,500	$0.03	14%	$15,300	$31,500	$1,980,768
15	$3,300,000	$71,500	$337,500	$0.03	10%	$15,300	$31,500	$3,375,228
16	$2,700,000	$71,500	$412,500	$0.02	10%	$15,300	$31,500	-$68,715
17	$3,300,000	$71,500	$337,500	$0.02	14%	$18,700	$38,500	-$764,414
18	$3,300,000	$71,500	$412,500	$0.02	10%	$18,700	$38,500	-$681,826
19	$2,700,000	$71,500	$412,500	$0.02	14%	$18,700	$31,500	-$652,237
20	$3,300,000	$71,500	$412,500	$0.03	14%	$15,300	$31,500	$1,397,932
21	$3,300,000	$58,500	$337,500	$0.02	14%	$15,300	$38,500	-$677,396
22	$3,300,000	$58,500	$337,500	$0.02	10%	$18,700	$38,500	$67,368
23	$3,300,000	$58,500	$412,500	$0.03	14%	$18,700	$31,500	$1,483,116
24	$2,700,000	$58,500	$412,500	$0.02	10%	$18,700	$31,500	$40,651
25	$3,300,000	$58,500	$412,500	$0.03	10%	$15,300	$31,500	$2,847,387
26	$2,700,000	$71,500	$412,500	$0.02	14%	$15,300	$38,500	-$660,232
27	$3,300,000	$71,500	$337,500	$0.02	10%	$15,300	$38,500	-$41,998
28	$2,700,000	$58,500	$412,500	$0.03	10%	$18,700	$38,500	$3,434,276
29	$3,300,000	$58,500	$412,500	$0.03	14%	$15,300	$38,500	$1,475,121
30	$2,700,000	$71,500	$337,500	$0.02	10%	$18,700	$38,500	$556,691

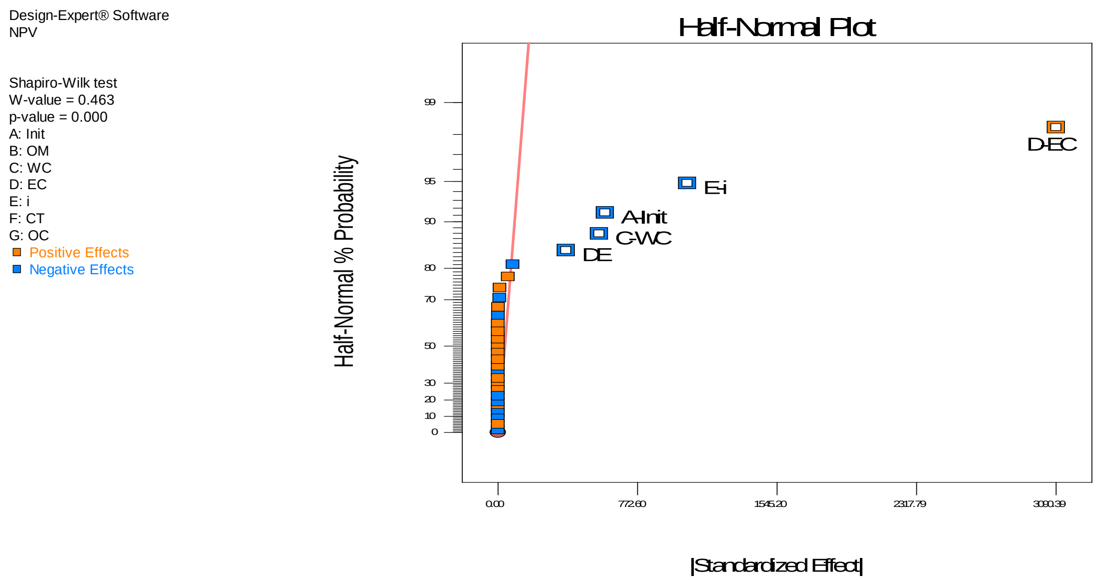

Figure 5: Half normal plot of effects for the engineering economics problem

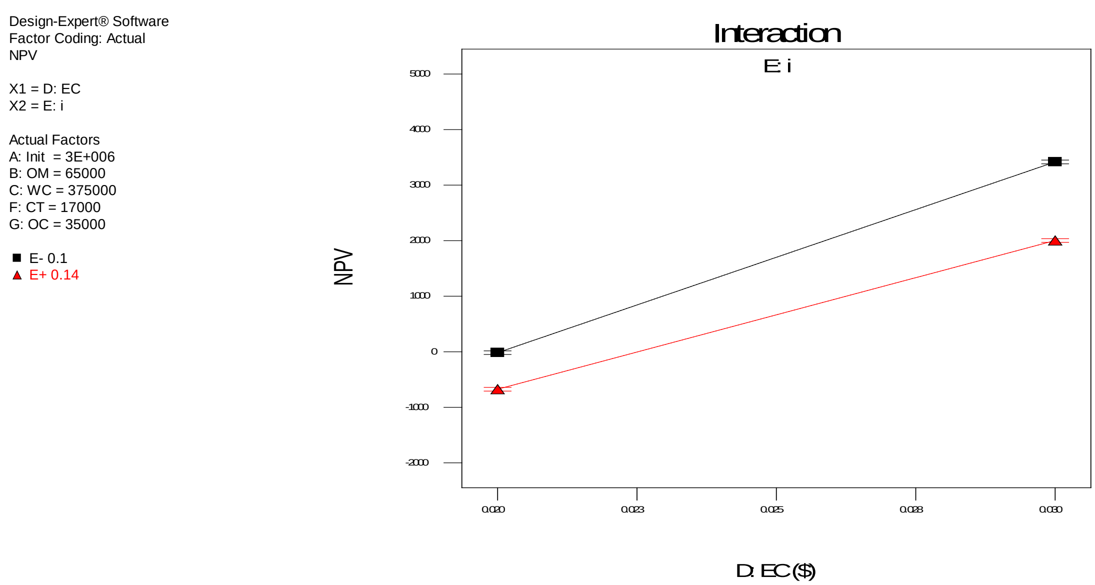

Figure 6: Interaction plot of discount rate (i) and electricity cost (EC).

From the above analysis, a regression equation relating the NPV to the significant effects in terms of coded factors is:

[7] $NPV = 1182.73 - 296.59\ A - 280.17\ C + 1527.96\ D - 518.19\ E - 189.10\ DE$

Where: NPV is the net present value, A is the coded factor of the initial investment, C is coded factor of the wood fuel cost, D is the coded factor of the electricity cost, E is the coded factor of the discount rate, and DE is the interaction effect between the cost of electricity and the discount rate. The adjusted R^2 of the regression equation is 0.999. Note that the interaction of the two parameters would not be estimable if one has used a one factor at a time approach.

4.0 CONCLUSIONS

This paper has used three examples from civil engineering to illustrate how the simple DOE approach using two-level factorial and fractional factorial designs can be used for conducting sensitivity analysis. The approach is quite straightforward and the results obtained are directly interpretable. The importance of each parameter and their possible interactions on the response are all quantified and a prediction equation is obtained as a by-product of the analysis. The approach is in fact a combined sensitivity and scenario analysis. The method also does not need excessive information or computation time compared to Monte Carlo methods or methods that require calculus. While the DOE approach is easy to apply and has many advantages, it is unfortunate that most engineers are still not familiar with them and they still rely on the one factor at a time approach. Lye (2012) has argued that with the current engineering curriculum in Canadian schools, if DOE is not taught, then it would be impossible to meet the new CEAB graduate attributes requirements.

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