AXIAL STRESS-STRAIN BEHAVIOR MODEL OF FRP-CONFINED CONCRETE: A STATE OF THE ART REVIEW

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Abstract: Fiber reinforced polymer (FRP) composites are often used to confine concrete for seismic strengthening of existing reinforced concrete (RC) columns and concrete filled FRP tubes as earthquake-resistant columns in new structures. For the safety and serviceability of these structural elements, it is essential to understand the behavior of FRP confinement at the element level. This is often done by observing the stress-strain behavior of FRP-confined concrete under axial compression. In the past two decades, many researchers conducted extensive experimental and analytical investigations in order to understand and predict the stress-strain behavior of FRP-confined concrete under axial compression. This paper presents an extensive review of the previously published literature on the axial stress-strain behavior of FRP-confined concrete in circular concrete sections. The reviewed models are categorized into two broad groups; design orientated models (DOMs) and analysis oriented models (AOMs). In the final part of the paper, a critical discussion is presented by comparing the constitutive models with the experimental results. The essential factors that influence the overall performance of the models are also highlighted.

1. INTRODUCTION

For the last two decades, several researchers conducted an experimental and analytical investigation to understand the stress-strain model and axial compressive behavior of FRP-confined concrete. These investigations resulted in the development of axial/monotonic stress-strain models, which are mentioned herein as a stress-strain model. Saadatmanesh et al. (1994) and Seible et al. (1995) conducted an experimental investigation on FRP confined RC columns; they directly adopted Mander et al. (1988) stress-strain model in the analysis of FRP confined concrete columns. Succeeding studies demonstrated that the direct use is incorrect, when dissimilarities in the stress-strain behavior of FRP-confined and steel-confined concrete were recognized (Mirmiran et al. 1996; Samaan et al. 1998; Miyauchi et al. 1997; Saafi et al. 1999; Spoelstra et al. 1999). Because, in Mander et al. (1988) model, a constant confining pressure was presumed, that is for steel-confined concrete, while the steel is in a plastic state, however, that is not for FRP-confined concrete. Consequently, research efforts have directed to the development of a large number of analytical stress-strain models which are specific for FRP-confined concrete under axial compression. Nevertheless, several of these models were derived based on limited experimental test results, which were derived only from the tests conducted by the original authors of the model.

This article provides a critical review of existing models of circular section of concrete confined with FRP jackets under axial compression. The constitutive models are reviewed in this article is limited to the concrete confined with FRP wrapping technique in which the fibers are oriented in the lateral direction as such wrapping technique generally used in the field for strengthening of circular section. Different models for the prediction of the ultimate stress and strain of FRP-confined concrete are reviewed, and categorized into subgroup. Critical discussion is presented in the last section of the article which affects the overall performance of the models.

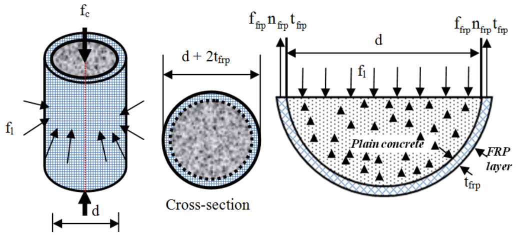
2. CONFINEMENT MECHANISM OF FRP-CONFINED CONCRETE IN CIRCULAR SECTION

When the FRP-confined circular section under axial compression stress (f'_{co}), the concrete expands laterally due to the poisons effect (Teng and Lam 2004); this expansion is restrained by the FRP confinement and therefore, the concrete is subjected to triaxial compression (Fig.1), and its axial resistance increases. In

FRP-confined circular concrete sections, the lateral confining stress (f_l) offered by the FRP composites is related to the amount and strength of FRP composites and the diameter of the confined concrete core. As shown in Fig. 1, in order to estimate the lateral stress f_l exerted by concrete confinement, a free-body diagram of a circular section is considered. The confinement applied by the FRP shell on the concrete core is passive; which means this pressure develops as a result of the lateral expansion of the concrete under axial compression. Based on static analysis, equilibrium of forces, and deformation compatibility, the confining stress (f_l) can be estimated using the Eq. (1) as a function of the ultimate tensile strain of the fibers (ε_{frp}) (Csuka and Kollar 2010).

[1] $$f_l = \frac{2E_{frp}\varepsilon_{frp}t_{frp}}{d}$$

where E_{frp}, ε_{frp}, t_{frp}, is the elastic modulus in the hoop direction, hoop strain, and thickness of FRP, respectively; and d is the diameter of the concrete section. The FRP jacket applies a uniformly distributed confining pressure after yield, and shows an elastic behavior up to rapture and therefore, exerts a constant increasing confining action (Spoelstra et al. 1999) compared to steel-confined concrete in which the confining pressure remains constant when the steel is in a plastic state (Teng and Lam 2004).



Cylinder fully wrapped with FRP

Cross-section

Free body diagram of FRP-confined concrete

Figure 1: Confinement mechanism of FRP-confined concrete in circular section

The confinement action applied by the FRP composite depends on the lateral expansion of concrete under axial compression, which is affected by the confining pressure. As the axial compressive stress increases, the corresponding lateral strain increases and the confining device develop a tensile hoop stress balanced by a uniform radial pressure, which reacts against the concrete lateral expansion (De Lorenzis et al. 2001, and Teng et al. 2002). When the circumferential strain of FRP reaches its ultimate strain and the fibers rupture leading to brittle failure of the circular section. This lateral confining pressure applied to the concrete by the FRP composite at ultimate stress (f_{lu}) can be estimated using Eq. (2) as a function of the ultimate tensile strain of fibers (ε_{frp}). However, several researchers, for instance, Xiao and Wu (2000); Pessiki et al. (2001); Harries and Carey (2002); Lam and Teng (2003); De Lorenzis (2003); Lam and Teng (2004); Theriault et al. (2004); Matthys et al. (2006); and, Ozbakkaloglu and Oehlers (2008) demonstrated that the ultimate strain assessed on the FRP composites at the time of FRP hoop rupture ($\varepsilon_{h,rup}$) is lower compared to an ultimate tensile strain of the fibers ε_{frp} or FRP material.

[2] $$f_l = \frac{2E_{frp}t_{frp}\varepsilon_{fu}}{d} = \frac{2t_{frp}f_{frp}}{d} = \frac{f_{frp}\rho_{frp}}{2}$$

where f_l is the lateral confining pressure, E_{frp}, ε_{fu}, f_{frp} and t_{frp} is the elastic modulus, ultimate tensile strain, ultimate tensile strength, and the total thickness of FRP composites, respectively. d is the diameter of the concrete cylinder, and ρ_{frp} is the volumetric ratio of FRP to concrete. The FRP volumetric ratio can be estimated using the Eq. (3) for fully wrapped circular section (Fig. 1) (Xiao and Wu 2001). Pessiki et al. (2001) introduced the strain reduction factor (k_ε) Eq. (4) to create the relationship amongst the hoop rupture strain of FRP composites ($\varepsilon_{h,rup}$) to the ultimate tensile strain of the material (ε_{frp}). Later on Lam and Teng (2003) found that the actual confining pressure of Eq. (5), by substituting material ultimate tensile strain (ε_{frp}) with the hoop rupture strain of the FRP shell ($\varepsilon_{h,rup}$) in Eq. (2).

[3] $$\rho_{frp} = \frac{\pi d t_{frp}}{\pi d^2/4} = \frac{4t_{frp}}{d}$$

[4] $$\varepsilon_{h,rup} = k_{\varepsilon,frp}\varepsilon_{frp}$$

[5] $$f_{lu,a} = \frac{2E_{frp}t_{frp}\varepsilon_{h,rup}}{d}$$

3. STRESS-STRAIN BEHAVIOR OF FRP-CONFINED CONCRETE

FRP-confinement have been widely used in the construction industry as confining materials for concrete columns to improve both strength and ductility (Wu et al. 2006). As the design of such confinements need a perfect stress-strain model for FRP-confined concrete. In this article an extensive literature review has been carried (until the end of year 2013) on the stress-strain behavior of FRP-confined concrete of circular section. The existing stress-strain models can be divided into two groups (Teng and Lam 2004) such as (a) the analysis-oriented models (AOMs) and (b) the design-oriented models (DOMs) which have unlike the principle approach.

The AOMs models are explicitly considered the interaction between the FRP jacket and concrete core (Teng et al. 2007). In the AMOs the estimation method adopts compatibility between the lateral strain ε_l of actively-confined concrete with a constant confining pressure f_l are similar which is provided by the jacket. To predict the stress-strain curve, the incremental iterative numerical techniques have been often used to solve the static equilibrium and radial deformation compatibility considerations. In AOMs the curves with unlike active confinement levels develop a passive confinement curve.

In DOMs models FRP confinement is considered as “single composite materials” reflecting the confinement behavior based on calibrated data; indicating that active or passive confinement is already taken into account. The active or passive confinement can be represented through two sections stress-strain relation (in some models bilinear) both axial and lateral strain (Marques and Chastre 2012). Thus, these models are simple and appropriate to apply in design, although in some cases the proposed expressions are laborious (Marques and Chastre 2012). The later models considered the FRP shell and the concrete section independently. These models predict the behavior of FRP-confined concrete using an explicit account of the interaction amongst the FRP shell and confined concrete core through radial displacement compatibility and equilibrium equations (Jiang and Teng 2007, Teng et al. 2007, Samaan et al. 1998, Toutanji 1999, and Saafi et al. 1999). Toutanji’s (1999) model revised by Matthys et al. (2006) and, Chastre and Silva’s (2010) for FRP-confined concrete columns based on two parts confinement model. Mirmiran and Shahawy (1997a, 1997b) and, Spoelstra and Monti (1999), proposed the group of confinement models using Mander et al. (1988) model.

Teng et al. (2007) stated that the incremental iterative numerical techniques is the key elements which determines the accuracy of prediction of the active-confinement model and the lateral-to-axial strain relationship. The performance of the active-confinement model depends on the peak axial stress and the corresponding axial strain; and the stress-strain equation. In this article, the existing models are reviewed in terms of three key elements such as the stress-strain equation and the peak axial stress-strain point of the active confinement model and the lateral to the axial strain relationship.

3.1. Peak Axial Stress and Axial Strain

The peak axial stress on the stress-strain curve of actively confined concrete is the compressive strength of such concrete and the peak stress equation defines the failure of such concrete (Teng et al. 2007). Several researchers have used the Hognestad’s (1951) parabola shape; e.g., Miyauchi et al. (1997), Youssef et al. (1997), Miyauchi et al. (1999), Jolly and Lilistone (1998), Jolly and Lilistone (2000) and Miyauchi et al. (2006) to model the transition point in the initial part of the stress-strain curve of FRP-confined concrete. The other part of the stress-strain curve was achieved by joining the initial peak with the ultimate point through a straight line using Eq. (6).

$$[6] \quad f_{co} = f'_{c1}\left[2\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2\right] \text{for } \varepsilon_c \leq \varepsilon_{c1}$$

where $f'_{co}$is the axial stress, ε_c the axial strain and, ε_{co} the axial strain at peak stress of concrete.

Richart et al. (1928) introduced the generalised concept of stress-strain curve and peak stress f_{cc} (Eq. 7) and the corresponding peak strain ε_{co} (Eq.8) of confined concrete in which the failure strength of confined concrete by an active confinement. Mander et al. (1988) reported that the axial strain at maximum stress ε_{cc}, (Eq. 8) can be represented by function of the strength of confined concrete f_{cc}.

[7] $f'_{cc}/f'_{co}=1+k_1\left(f_l/f_{co}\right)$

[8] $\varepsilon^*_{cc}=\varepsilon_{co}\left[1+5\left(\frac{f'^*_{cc}}{f'_{co}}-1\right)\right]$

where f_{cc} is the maximum strength of confined concrete, $f'_{co}$the maximum strength of unconfined concrete, f_l is the lateral confining pressure and k_1 is the confinement effectiveness coefficient. Richart et al. (1928) found that k_1 decreases as the lateral confining pressure increases, which is equal to 4.1. The majority of the existing strength models adopted the general form of Eq. (7) for FRP-confined concrete.

To define the peak axial stress Mirmiran et al. (1997a), Spoelstra and Monti (1999), Fam and Rizkalla (2001) and, Chun and Park (2002) directly adopt the "five parameter" multiaxial failure surface provided by Willam and Warnke (1975) in the Eq. (9) proposed by Mander et al. (1988). While other four models, Harries and Kharel (2002) adopted the Eq. (10) proposed by Mirmiran and Shahawy (1997b). For the peak axial stress Marques et al. (2004) used Eq. (12) for normal strength concrete (NSC) $f'_{co} \leq 40$MPa; but for high strength concrete (HSC), Eq. (9) was modified by a factor introduced by Razvi and Saatcioglu (1999) in order to reduce the effectiveness in the improvement of axial strain for HSC. Aire et al. (2010) adopt Eq. (9) proposed by Mander et al. (1988) to define the peak axial stress. Binici's (2005) model adopted the Leon-Pramono (1989) criterion that reduces to Eq. (13) if the tensile strength of unconfined concrete is taken to be 0.1 times of its compressive strength.

Teng et al. (2007) and, Jiang and Teng (2007) proposed the linear function to define the peak axial stress Eq. (14) and the corresponding axial strain of Eq. (15). Albanesi et al. (2007) proposed the linear function to define the peak axial stress Eq. (16) and corresponding axial strain Eq. (17). Xiao et al. (2010) proposed the Eq. (18) and Eq. (19) for NSC and HSC by regression analysis to define the peak axial stress and the peak axial strain, respectively.

[9] $f'_{cc}=f'_{co}\left(2.254\sqrt{1+7.94\frac{f^*_l}{f'_{co}}}-2\frac{f^*_l}{f'_{co}}-1.254\right)$

[10] $f'_{cc}=f'_{co}+4.269f_l^{0.587}$

[11] $f'_{cc}=f'_{co}+4.269f_l^{0.587}$

[12] $f'_{cc}=(E_{\theta,}\varepsilon_\theta+E_c\varepsilon_c)$

[13] $f'_{cc}=f'_{co}\left[\sqrt{1+9.99\frac{f_l}{f_{co}}}+\frac{f_l}{f_{co}}\right]$

[14] $f'_{cc}=f'_{co}+3.5f_l$

[15] $\frac{\varepsilon_{cc}}{\varepsilon_{co}}=1+17.5\left(\frac{f^*_l}{f'_{co}}\right)^{1.2}$

[16] $f'_{cu}\cong f'_{co}+3.609f_{lu}$

[17] $\varepsilon_{cu}\cong\varepsilon_{co}+18.045\varepsilon_{co}\frac{f_{lu}}{f'_{co}}$, for $\frac{f_{lu}}{f'_{co}}\leq 0.7$

[18] $\frac{f'_{cc}}{f'_{co}}=1+3.24\left(\frac{f_l}{f'_{co}}\right)^{0.80}$

[19] $\frac{\varepsilon_{cc}}{\varepsilon_{co}}=1+17.4\left(\frac{f^*_l}{f'_{co}}\right)^{1.06}$

where ε_{cc} is the axial strain at peak axial stress (f'_{cc}) of concrete under a specific constant confining pressure and ε_{co} is the axial strain at peak axial stress of unconfined concrete, f'_{co}. The existing AOMs and DOMs reviewed in this article are presented in Table 1 and 3, respectively, the equations that each model uses to estimate the peak stress and the corresponding strain. It can be observed that all the AOMs depend on Eqs. (8) and (14) in Richart et al.'s (1928) model modified by Mander et al. (1988) for the estimation of strain at peak stress (Table 1).

Table 1: Peak stress-strain equation of existing AOMs

Axial stress-strain curve equation	Peak stress Eq. no.	Peak strain Eq. no.	Lateral-to-axial strain relationship	Model
Popovics (1973)	[9]	[8]	Explicitly derived from FRP-confined concrete test results	Mirmiran and Shahawy (1997a)
Popovics (1973)	[9]	[8]	Implicitly adopted from an actively confined concrete mode	Spoelstra and Monti (1999) 'exact
Popovics (1973)	[9]	[8]	Implicitly derived from actively confined concrete test results	Fam and Rizkalla (2001)
Popovics (1973)	[9]	[8]	Implicitly adopted from an actively confined concrete mode	Chun and Park (2002)
Modified from Popovics (1973)	[10]	[8]	Explicitly derived from FRP-confined concrete test results	Harries and Kharel (2002)
Modified from Richard and Abbott (1975) and Popovics (1973)	[11]	[8]	Explicitly derived from FRP-confined concrete test results	Moran and Pantelides (2002)
Popovics (1973) with (ACI 363R-84)	[12]	[8]	Implicitly adopted from an actively confined concrete mode	Marques et al. (2004)
Modified from Popovics (1973)	[14]	[8]	Implicitly derived from actively confined concrete test result	Binici (2005)
Popovics (1973)	[14]	[8]	Explicitly derived from FRP-confined and actively confined concrete test result	Teng et al. (2007)
Popovics(1973)	[14]	[8]	Explicitly derived from FRP-confined and actively confined concrete test results	Jiang and Teng (2007)
Popovics(1973)	[10]	[8]	Implicitly adopted from an actively confined concrete model	Aire et al. (2010)
Popovics(1973)	[18]	[8]	Same as Teng et al. (2007)	Xiao et al. (2010)
Popovics(1973)	[9]	[8]	Same as Teng et al. (2007)	Hu and Seracino (2014)

Table 2: Peak stress-strain equation of existing DOMs

Peak stress	Eq.No.	Peak strain	Eq. No.	Reference
based on Richart et al. (1928) $f_{cc}=f_{co}+6f_{l}^{0.7}$	[20]	$\varepsilon_{cc}=\frac{f_{cc}-f_{co}}{E_2}$	[21]	Samman et al. (1998)
based on Richart et al. (1928) $\frac{f'_{cc}}{f'_{co}}=\left[1+3.5\left(\frac{f_{lu}}{f'_{co}}\right)^{0.85}\right]$	[22]	Based on Richart et al. (1928) $\varepsilon_{cc}=\varepsilon_{co}\left[1+\left(310.57\varepsilon_{lu}+1.9\right)\left(\frac{f'_{cc}}{f'_{co}}\right)^{0.85}\right]$	[23]	Tauntaji (1999)
based on Richart et al. (1928) $\frac{f'_{cc}}{f'_{co}}=1+2.2\left(\frac{f_{l}}{f'_{co}}\right)^{0.84}$	[24]	Calibrated based on Mander et al. (1988) $\varepsilon_{cc}=\varepsilon_{co}\left[1+\left(537\varepsilon_{lu}+2.6\right)\left(\frac{f'_{cc}}{f'_{co}}-1\right)\right]$	[25]	Saffi et al. (1999)

Calibrated based on Toutanji (1999) $\frac{f'_{cc}}{f'_{co}}=\left[1+2.3\left(\frac{f_l}{f'_{co}}\right)^{0.85}\right]$	[26]	Calibrated based on Mander et al. (1988) $\varepsilon_{cc}=\varepsilon_{co}\left[1+\left(537\varepsilon_{lu}+2.6\right)\left(\frac{f'_{cc}}{f'_{co}}-1\right)\right]$	[27]	Toutanji's revised Matthys et al. (2006)
based on Richart et al. (1928) $f'_{cc}=f'_{co}\left(\frac{1.5+(D/H)}{2}\right)+5.29f_l$ $f_D=f'_{co}\left(\frac{1.5+(D/H)}{2}\right)$	[28]	2nd region- $\varepsilon^{*}_{lu}=0.6\varepsilon_{flu}$ $\varepsilon_{cc}=\varepsilon_{co}17.65\left(\frac{f_l}{f_D}\right)^{0.7}$ fl obtained from $\varepsilon^{*}_{lu}=0.6\varepsilon_{flu}$	[29]	Chastre and Silva (2010)
$f'_{cc}=f'_{co}+3.1f'_{co}v_c\left(\frac{2E_{frp}t_{frp}}{DE_c}\right)+f_{lu}$	[30]	$\varepsilon_{cu}=\varepsilon_{co}+0.01\left(\frac{f_{lu}}{f'_{co}}\right)$	[31]	Karbhari and Gao (1997)
$f'_{cc}=f'_{co}+k_{e1}4.1f_{lu}$, if $f'_{co}\leq 50$ MPa $k_{e1}=0.85$	[32]	$\varepsilon_{cu}=\varepsilon_{co}\left\{1+10.6\left(\frac{f_{lu}}{f'_{co}}\right)^{0.373}\right\}$	[33]	Miyauchi et al. (1997)
based on Mander et al. (1988) $\frac{f'_{cc}}{f'_{co}}=0.2+3\left(\frac{f_{lu}}{f'_{co}}\right)^{0.5}$	[34]	$\varepsilon_{cu}=\varepsilon_{co}\left[2+1.25\left(\frac{E_c}{f'_{co}}\right)\left(\sqrt{\frac{f_{lu}}{f'_{co}}}\right)\varepsilon_{lu}\right]$	[35]	Spoelstra and Monti (1999)
$\frac{f'_{cc}}{f'_{co}}=\left[1.1+\left(4.1-0.75\frac{f'^{2}_{co}.d}{2E_{frp}.t_{frp}}\right).\left(\frac{f_{lu,a}}{f'_{co}}\right)\right]$	[36]	$\varepsilon_{cu}=\varepsilon_{co}\frac{\varepsilon_{h,rup}+\varepsilon_o}{\mu_{tu}}$	[37]	Xiao and Wu (2000)
$\frac{f'_{cc}}{f'_{co}}=1+3.3\frac{f_{lu,a}}{f'_{co}}$ if $\frac{f_l}{f'_{co}}\,^{3}0.07$	[38]	$\frac{\varepsilon_{cu}}{\varepsilon_{co}}=1.75+12\left(\frac{f_{lu,a}}{f'_{co}}\right)\left(\frac{\varepsilon_{h,rup}}{\varepsilon_{co}}\right)^{0.45}$	[39]	Lam and Teng (2003)
$\frac{f'_{cc}}{f'_{co}}=1+\frac{f_{lu,a}}{f'_{co}}\tan^{2}\left(1+45^{o}+\frac{\varphi}{2}\right)$	[40]	$\frac{\varepsilon_{cc}}{\varepsilon_{co}}=\left[1+2.24\tan^{2}\left(45^{o}+\frac{\varphi}{2}\right)\frac{f_{lu,a}}{f'_{co}}\right]$	[41]	Li et al. (2003)
$\frac{f'_{cc}}{f'_{co}}=1+2.25\left(\frac{f_{lu,a}}{f'_{co}}\right)^{1.25}$	[42]	$\varepsilon_{cu}=0.003368+0.259\left(\frac{f_{lu}}{f'_{co}}\right)\sqrt{\frac{f_{frp}}{E_{frp}}}$	[43]	Youssef et al. (2007)
$\frac{f'_{cc}}{f'_{co}}=1+3.5(\rho_K-0.01)\rho_{\varepsilon}$, if $\rho_K\geq 0.01$	[44]	$\frac{\varepsilon_{cu}}{\varepsilon_{co}}=1.75+6.5\left(\rho_K\right)^{0.8}\left(\rho_{\varepsilon}\right)^{1.45}$	[45]	Teng et al. (2009)

In order to develop the DOMs, the researchers derived the peak stress-strain equation based on the experimental data using Richart et al. (1928) Eq. (7) (Eqs. 20, 22, 24, 26, 28) where Toutanji's (1999) model revised by Matthys et al. (2006) for FRP-confined concrete columns based on two parts confinement model. The peak strain at corresponding stress of Saafi et al. (1999) and Toutanji's (1999) models are based on Mander et al.'s (1988) Eq. (8) and derived using experimental data (Eq. (23) and (25)). Matthys et al. (2006) revised Toutanji's (1999) model Eq. (23) but for the 2nd section the strain values are multiplied by 0.6 (Eq. (25)). Chastre and Silva (2010) developed the Eq. (28) based on experimental tests while Karbhari and Gao (1997), Miyauchi et al. (1997), Samaan et al. (1998), Spoelstra and Monti (1999), Xiao and Wu (2000) Lam and Teng (2003), Li et al. (2003), Youssef et al. (2007), Teng et al. (2009) proposed axial strain at peak stress by their own test results (Table 2).

4. STRESS-STRAIN RELATIONSHIP

4.1. Analysis-Oriented Model (AOMs)

The confinement offered by FRP casing to concrete core is passive rather than active due to the confining pressure from the FRP casing is produced by increasing the lateral expansion of the concrete core (Jiang and Teng 2007). The AOMs reviewed in this literature for FRP-confined concrete are theoretical models which means the confining pressure is applied externally and remains constant as the axial stress increases. Ahmad and Shah (1982), and Madas and Elnashai (1992) employed the concept of a passive-confinement stress-strain model from an active-confinement (steel coffined concrete) base model through an incremental approach. The first attempt made by Mirmiran and Shahawy (1997a) to develop passive-confinement stress-strain model for FRP-confined concrete. Their model follows the technique proposed by Madas and Elnashai (1992). Mirmiran and Shahawy (1997b) proposed the new model, but they did not specified the active-confinement base model; and hence Lam and Teng (2004) also omitted the active confinement from the assessment of the stress-strain model. Teng et al. (2007) assumed Mander et al. (1988) model as the active-confinement model as base model introduced by Mirmiran and Shahawy (1997b). A number of active confinement model has been proposed by following the work of Mirmiran and Shahawy (1997a, 1997b) such as, Fam and Rizkalla (2001), Chun and Park (2002), Harries and Kharel (2002), Marques et al. (2004), Binici (2005) and Teng et al. (2007).

Most of the existing AOMs developed, for example, Saadatmanesh et al. (1994), Mirmiran et al. (1996), Spoelstra and Monti (1999), Teng et al. (2007), Xiao and Teng (2010), Jiang and Teng (2007), Fam and Rizkalla (2001), Chun and Park (2002), Marques et al. (2004), Aire et al. (2010) and, Hu and Seracino (2013) except Harries and Kharel (2002), Binici (2005) adopted the stress-strain model proposed by Popovics (1973) Eq. (8)), and later adopted by Mander et al. (1988) to estimate the shape of the actively confined concrete base curves. In the model of Popovics' (1973) the stress-strain curve of concrete is defined using an energy balance approach as Eq. (46). Majority of the model reviewed in this article that adopted the original or modified version of Popovics' (1973) model to calculate this constant r using Carreira and Chu (1985) in Eq. (46). The stress-strain equation of actively-confined concrete defined by Eq. (46) is modified version of Popovics' (1973) original version using a factor r which was employed in Harries and Kharel's (2002) model to control the slope of the decreasing part. The model of Binici (2005) adopts the three separate equations to define the full stress-strain curve. Most of the reviewed models in the existing literature use the ACI 318 (1995) equation to estimate the initial elastic modulus of concrete. Few of the models adopt the different equations for the determination of E_c (Spoelstra and Monti 1999, Marques et al. 2004, Binici 2005, and Aire et al. 2010).

4.2. Design Oriented Models (DOMs)

Toutanji (1999) proposed a model (Eq. 47) for FRP-confined concrete by modifying Ahmad and Shah's (1982) model which was for concrete confined by steel spirals. Toutanji's (1999) proposed the model based on experimental results of circular cross-section concrete confined using FRP sheets, while Saafi et al. (1999) group's proposed the models based on experimental results of FRP confining tubes. Therefore, the peak stress and corresponding strain equations are different (Table 2). Matthys et al. (2006) revised Toutanji (1999) model, considering for the 2nd part of the failure strain in the hoop direction corresponding to 0.6 of the ultimate strain of the CFRP material. Ahmad and Shah (1982) proposed the models using the 1st region and Richart et al.'s (1928) Eq. (7) in which the failure strength of concrete confined by a hydrostatic fluid pressure (active confinement) following Eq. (7) for the 2nd region modified by Toutanji (1999) (Table 3). Richard and Abbott (1975) proposed a confinement model to describe the elastic plastic (bilinear) constitutive law by four-parameter formulation of FRP-confined concrete columns Eq. (48). The other researchers, Samaan et al. (1998), Moran and Pantelides (2005) and Chastre and Silva (2010) proposed the models based on the most famous that were derived based on the Eq. (48). They calibrated the model with experimental results (Eq. (33)), where the both stress-axial strain and stress-lateral-strain relations are of bilinear type with a shape factor. The reference plastic stress is calculated from Eq. (25). Zohrevand and Mirmiran (2013) proposed the new confinement by calibrating the Samaan et al. (1998) and Lam and Teng (2003) model (Eq.49) for ultra-high performance concrete confined by FRP with experimental test results.

Table 3: Stress-strain relation of each DOMs

Stress-strain relation	Eq.No	Model
Popovics (1973) and Mander et al. (1988) $f'_{co}=\frac{(\varepsilon_c/\varepsilon_{cc})r}{r-1+(\varepsilon_c/\varepsilon_{cc})^r}$,where $r=\frac{E_c}{E_c-(f'_{cc}/\varepsilon_{cc})}$	[46]	Mirmiran and Shahawy (1997a,1997b), Spoelstra and Monti (1999), Fam and Rizkalla (2001), Teng et al. (2007)

Toutanji (1999) based on Ahmad and Shah (1982) and Richart et al. (1928) $f_c(\varepsilon_c)=\dfrac{E_1\varepsilon_c}{1+\left[\dfrac{E_1}{f_a}-\dfrac{2}{\varepsilon_{1c}}+\dfrac{E_1E_2\varepsilon_{1c}}{f_a^2}\right]\varepsilon_c+\left[\dfrac{1}{\varepsilon_{1c}^2}-\dfrac{E_1E_2}{f_a^2}\right]\varepsilon_c^2}$ $f_c(\varepsilon_c)=\dfrac{E_{11}\varepsilon}{1+\left[\dfrac{E_{11}}{f_a}-\dfrac{2}{\varepsilon_{11}}+\dfrac{E_{11}E_{21}\varepsilon_{11}}{f_a^2}\right]\varepsilon_1+\left[\dfrac{1}{\varepsilon_{11}^2}-\dfrac{E_{11}E_{21}}{f_a^2}\right]\varepsilon_1^2}$	[47]	Toutanji (1999), Saafi et al. (1999), Toutanji revised (2006)
Richard and Abbott (1975) $f_c(\varepsilon_c)=\dfrac{(E_1-E_2)\varepsilon_c}{\left[1+\left(\dfrac{(E_1-E_2)\varepsilon_c}{f_0}\right)^n\right]^{\frac{1}{n}}}+E_2\varepsilon_c\ ,$ $f_c(\varepsilon_c)=\dfrac{(E_{11}-E_{21})\varepsilon_1}{\left[1+\left(\dfrac{(E_{11}-E_{21})\varepsilon_1}{f_{0l}}\right)^{nl}\right]^{\frac{1}{nl}}}+E_{21}\varepsilon_1$	[48]	Samaan et al. (1998), Chastre and Silva (2010), Lam and Teng (2003), Teng et al. (2009), Wu et al. (2009), Wu and Wang (2010), Yu and Teng (2011)
$f_c=0.67\dfrac{f'_{co}}{\gamma_m}\left\{2\dfrac{\varepsilon_c}{0.002}-\left(\dfrac{\varepsilon_c}{0.002}\right)^2\right\}+E_p\varepsilon_c$	[49]	Jolly and Lilliston (1998), Lillistone and Jolly (2000)
$f_c=\begin{cases}E_c\varepsilon_c+2\nu_cf_{l,}\ f_c<1.1f^*_{co}\\ f_c=1.1f^*_{co}+k_cf_{l,},\ \geq 1.1f^*_{co}\end{cases}$	[50]	Xiao and Wu (2000)
$f'_{c1}=\left(1.2+3.85\dfrac{f_{lu}}{f'_{co}}\right)f'_{co}$	[51]	Wang and Wu (2011)
Modified from Samman et al. (1998) and, Lam and Teng (2003) $f_c=E_{11}\varepsilon_c-\dfrac{(E_{11}-E_{21})^2}{4f_{ol}}\varepsilon_c^2$ for $0\leq\varepsilon_c\leq\varepsilon_{c1}$ for parabola $f_c=f_{ol}+E_{21}\varepsilon_c$ for $\varepsilon_t\leq\varepsilon_c\leq\varepsilon_{cu}$ for straight line	[52]	Zohrevand, and Mirmiran (2013)

5. ESSENTIAL ASPECTS INFLUENCING MODEL'S PERFORMANCE

Ozbakkaloglu et al. (2013) conducted a statistical assessment on FRP-confined concrete cylinders tested under monotonic axial compression. They reported that the average values of the average absolute error and the mean square error of DOMs were found to be lower compared to the AOMs for FRP-confined concrete. Based on these values the DOMs performed well compared to the AOMs for the estimation of peak axial-strength and peak axial-strain enhancement ratios. The DOMs performed well because of the fact that the most of these DOMs were developed from the large test results or databases of FRP-confined concrete. These large test results (or databases) allowed them to directly deduce the main parametric effects on the behavior of FRP-confined concrete. On the contrary, the AOMs were proposed based on formulas from active (or steel) confined concrete to define the dilation relationship of FRP-confined concrete. According to the Jiang and Teng (2007), and Ozbakkaloglu et al. (2013) the implicitly assumed equation does not exactly define the relationship of FRP-confined concrete. On the other hand, Mirmiran and Shahawy (1997) and, Jiang and Teng (2007) found that the AOMs that adopted explicitly derived dilation behaviors for FRP-confined concrete performed well compared to the DOMs of FRP confined concrete.

6. CONCLUSIONS

A comprehensive review of the existing models from the open literature has been conducted in this article. The reviewed models have been classified in two categories AOMs and DOMs. It has been observed that the majority of the existing models for assessing the compressive strength of FRP-confined concrete circular columns are based on the earlier confinement model that was derived experimentally by Richart et al. (1928) for concrete under active hydrostatic pressure. It was found that actively (or steel) confined concrete models are not able to capture the behavior of FRP-confined concrete because of the fact that the FRP confining device exerts a continuously increasing pressure as opposed to steel at the yield state. The stress-strain curves of FRP-confined concrete circular section, which consists of strain-hardening or strain–softening, depends on the confinement efficiency of FRP materials. Based on the reviewed models, the DOMs perform better compared to the AOMs for the prediction of ultimate axial-stress and corresponding strain. In general, the DOMs performance increases with an increase of the number of databases used in the model development. The explicitly derived dilation behaviors of AOMs perform better compared to the implicitly adopted dilation behaviors. The models that use the hoop rapture strain ($\mu_{h,rup}$) are more accurate compared to the ones that directly use the ultimate tensile strain of fibers (ε_f).

Acknowledgments
The first author would like to acknowledge the financial support provided by the Mitacs and POLYRAP Engineered Solutions, Kelowna, Canada under the accelerate Ph.D. fellowship program and the Ministry of Human Resources Development (MHRD), Govt. of India, New Delhi. The first author is also grateful to the S.V. National Institute of Technology (SV NIT), Surat, Govt. of India to grant the study leave for the Ph.D. program at the University of British Columbia (UBC), Canada.

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