EFFECT OF LATERAL DEFORMATIONS ON VERTICAL STIFFNESS OF RUBBER BEARING ISOLATORS WITH RECTANGULAR CROSS SECTION

Moein Ahmadipour[1]; and M. Shahria Alam[2]

Abstract

Seismic isolation bearings are intended to lengthen superstructure's vibration period by their low lateral stiffness. Their vertical stiffness, however, should remain in a safe range in order to withstand gravity loads imposed by the superstructure. Part of the bearing's vertical load-carrying capacity is provided by closely-spaced internal steel shims and the rubber, as an incompressible material, bonded to the steel reinforcements. As the bearing undergoes lateral deformations, its vertical stiffness and, as a result, its load-carrying capacity, tends to decrease, which might have catastrophic consequences. In this research, the vertical stiffness of rectangular rubber bearings is studied. The effect of lateral deformations on the vertical stiffness is investigated through three methods: 1) The two-spring method, 2) Overlapping area method and 3) Linear interpolation method. The effect of manufacturing central holes is also investigated and found not to be considerable to be taken into account. The effect of lead core in lead-core rubber bearing isolators is also studied and found to be the dominant factor in determining the vertical stiffness. It is observed that, unlike the two-spring method, the two other methods suggest a linear trend for vertical stiffness reduction in bearings with a rectangular cross section. It is also shown that the overlapping area method suggests a steeper decrease and gives the most conservative prediction of the vertical stiffness.

Keywords: Rubber bearing, Base isolation, Vertical Stiffness

1 Introduction

Base isolation has been used since as early as 1970's to make new and existing structures more resistant against seismic events such as earthquakes [1]. Various isolation devices have been introduced since the invention of base isolation techniques and they are shown to be effective in helping structures to survive in moderate and severe ground motions [2,3]. Base isolation protects the structure from structural and non-structural damages by increasing its natural frequencies far from dominant earthquake frequencies. In addition to decoupling the structure from earthquake excitations, base isolation is also used for other phenomena such as pre-stressing relaxations, thermal movements, shrinkage and time-dependent actions like creep [4,5].

Among various types of isolators introduced (e.g., friction-based, elastomeric and etc.), rubber bearings are focused on in this study. Numerical simulation is used to gain a better understanding of rubber bearings' behavior under lateral cyclic loadings, using validated rubber material models. The initial vertical stiffness of lead-core rubber bearing isolators (LCRB) is studied. The effects of shear strains on the vertical

[1] Ph.D. Student, School of Engineering, Univ. of British Columbia, Kelowna, BC, Canada, V1V 1V7. E-mail: moein.ahmadipour@ubc.ca
[2] Associate Professor, School of Engineering, Univ. of British Columbia, Kelowna, BC, Canada V1V 1V7. E-mail: shahria.alam@ubc.ca

stiffness are then investigated and compared, using three different methodologies previously introduced in literature.

2 Material model validation

Rubber-like materials are capable of going through outsized strains with minor volume changes [6]. This property offers simultaneous high compressive strength and high lateral flexibility to elastomeric bearings, which is desired in base isolation. Rubber is the ruling material in these bearings and should be modeled carefully to capture the true behavior in numerical simulations. To prove the accuracy of the numerical simulation conducted using ANSYS finite element software [7], response of the elastomeric bearing is compared to that of experimental results conducted by [8], shown in Figure 1. A 3-parameter Mooney-Rivlin material model is employed to acquire the hyper-elastic behavior and a 2-parameter Prony material model for rubber's viscoelastic shear response. The slight discrepancy (less than 5%) observed between the simulated response and the experimental one, mostly in negative shear strains, can be assigned to difficulty of applying perfect symmetry conditions in real-life experiments and also to rate-dependent properties of rubber, which is an inevitable characteristic of such hyper-viscoelastic models [9,10].

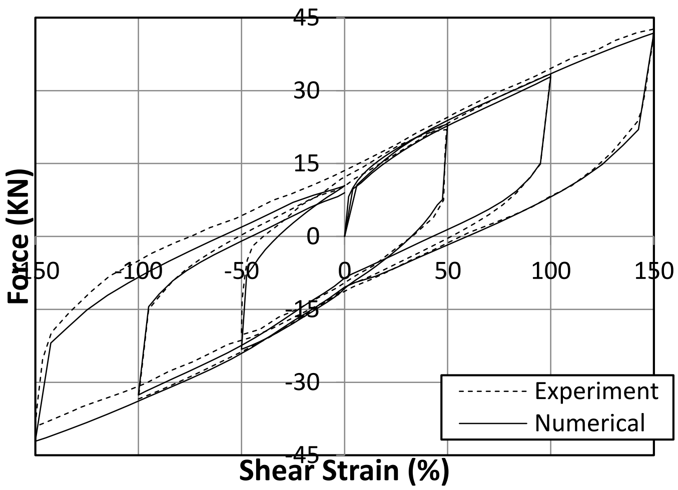

Figure 1. Rubber numerical material model validation with experimental results by [8]

3 Vertical stiffness of rubber bearings

Isolation bearings increase structure's natural vibration period by their low lateral stiffness. Their vertical stiffness, however, should remain in a safe range in order to withstand gravity loads from the superstructure. The closely-spaced internal steel shims provide part of that vertical stiffness and the rest is provided by rubber thanks to its incompressibility [11].

The vertical stiffness of bearings decrease as they undergo lateral deformations and their vertical strength decreases as well [12]. In large shear strains, the reduction in the vertical stiffness markedly increases. As a result, the vertical stiffness and load-carrying capacity should be calculated with respect to the initial vertical stiffness and horizontal deformations' consequences. This is investigated in this study using three methodologies, 1) The two-spring method, 2) Overlapping area method and 3) Linear interpolation method, as follows.

3.1 Two-spring method

Different techniques are introduced to calculate the vertical stiffness of rubber bearing isolators. The two-spring model is a mechanical model suggested and developed later on to consider the effects of horizontal shear deformations on the vertical stiffness [13,14]. This method employs a linear spring to accommodate bearing's vertical deformations and a rotational spring for the horizontal ones. Warn et al., afterwards, examined the sensitivity of this effect to assumptions engaged before in the formulations [11]. Based on definition, the vertical stiffness is obtained by dividing the vertical load by the deformation happening due to that load. Assuming a vertically applied load P, the total vertical deformation is calculated for the bearing, which is composed of two components caused by axial and shear loadings. Considering the bearing's shear rigidity and load P much lower than Euler's buckling load, and considering an incompressible material behavior for rubber, the vertical displacement due to horizontal deformations (δ_{vh}) is obtained by:

$$\delta_{vh} = \frac{3(GA_bh+PT_r)\Delta^2}{\pi^2 E_c I} \qquad (1)$$

In this formula, rubber's shear modulus of elasticity, rubber area bonded to supporting steel plates and total bearing height is denoted by G, A_b and h, respectively. T_r and E_c are the total rubber thickness and the compression modulus, respectively, and I is bearing's cross section's second moment of inertia. Δ also represents the lateral deformation. This vertical deformation is added up to the vertical deformation caused by the vertical compression load P, denoted by δ_{v0}, to get the total vertical deformation. Having the initial vertical stiffness defined as K_{v0}, the total vertical stiffness K_v is calculated by differentiating the inverse of the total vertical deformation with respect to the compression load, which would be re-written as:

$$K_v = K_{v0}\frac{1}{1+\frac{3A_b}{I}\left(\frac{\Delta^2}{\pi^2}\right)} \qquad (2)$$

Another factor affecting the vertical stiffness of rubber bearings is the central hole that is often made in their center for manufacturing purposes. This hole, if big enough, can reduce the vertical stiffness, which might not be desirable in most cases. To consider the hole's effect, the bonded area and the second moment of inertia used in equation (2) should be modified with respect to hole dimensions. Defining R and A_H as the hole's radius and cross-sectional area, respectively, the reduced bonded area can be written as $A_b' = A_b - A_H$. In this study, rectangular-cross section bearings are considered. By plugging the calculated modified bonded area and the modified second moment of inertia in equation (2), the modified vertical stiffness can be written in the form of equation (3) as follows, considering the effect of the central hole on the vertical stiffness, the results of which are shown and discussed in section 4 of this article.

$$K_v' = K_{v0}\frac{1}{1+\frac{3\left(WL-\pi(R)^2\right)}{WL^3/12-\pi(R)^4/4}\left(\frac{\Delta^2}{\pi^2}\right)} \qquad (3)$$

3.2 Overlapping area method

In this methodology, vertical stiffness of the bearing is assumed to be provided only by the vertical column of bearing projected under overlapping area of its deformed shape. This method, introduced by Buckle and Liu, suggests simply reducing the initial vertical stiffness of the bearing by the ratio of the overlapping area to the initial bonded area [15]. That is,

$$K_v = K_{v0}\left(\frac{A_{overlap}}{A_b}\right) \qquad (4)$$

in which $A_{overlap}$ is the overlapping area explained above. Figure 2 shows the concept of overlapping area methodology for a bearing with rectangular cross section. Based on this methodology's prediction, the bearing would exhibit zero vertical load-carrying capacity at lateral displacements of $\Delta = L$ and further.

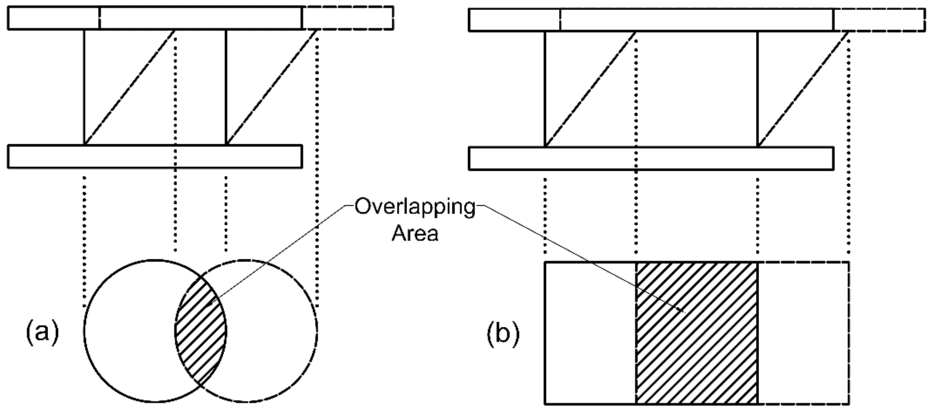

Figure 2. Overlapping area concept for (a) circular and (b) rectangular rubber bearings

3.3 Linear interpolation method

A linear piecewise function is suggested by Warn et al. to predict the vertical stiffness as the bearing undergoes horizontal deflections [11]. Unlike the overlapping area method, this method suggests a residual vertical stiffness for the bearing and a linear interpolation is employed to obtain the vertical stiffness between the bearing's initial and final stiffness values, as a function of the horizontal displacement. The final stiffness, k_{vf}, is considered to be the one found from the two-spring method at a horizontal deflection of $\Delta = L$. The following formula, aroused from this concept, are used in order to compare the results and show the preciseness of this simple regression.

$$\begin{cases} K_v = K_{v0}\left[1 + \frac{\Delta}{L}\left(\frac{K_{vf}}{K_{v0}} - 1\right)\right] & for\ \Delta \leq L \\ K_v = K_{vf} & for\ \Delta > L \end{cases} \quad (5)$$

The discussed methodologies for calculating the vertical stiffness are applied, the results of which are found in the **Results and discussion** section of this article.

3.4 Vertical stiffness of lead-core rubber bearings

Regular rubber bearings are known to be effective in terms of seismic isolation purposes. However, it has been shown that their performance can be improved by adding a lead plug to the isolator, introducing another type of isolators, lead-core rubber bearings (LCRBs). The lead plug enhances bearing's energy dissipation capacity and, at the same time, augments bearing's vertical stiffness, as a comparatively strong metal. Different lead grades are used in LCRBs. Lead's material properties affect the vertical stiffness of the bearing to a high extent. In this research, first, a sensitivity analysis is conducted in order to see the effect of rubber layers' thickness in regular rubber bearings (without lead core), and then investigate the effect of lead core properties on the vertical stiffness. The results are shown and discussed later in this article.

4 Results and discussion

As discussed above in section 3.1, the central hole often made on rubber bearings' cross section for manufacturing purposes can affect the vertical stiffness of the bearing, the formulations of which are derived earlier and the results of which will be shown in this section. Four different hole sizes are examined, which are represented by R/L ratios of 0.00, 0.10, 0.15 and 0.25, given that L is the bearing's smaller plan dimension. This assumption leads to a smaller second moment of inertia and, as a consequence, a more conservative approach, since the bearing is intended to operate in both horizontal axes. A R/L ratio of 0.00 represents that no hole exists in the bearing and a R/L ratio of 0.25 characterizes a hole whose diameter covers half bearing's width which is big more than enough to represent an extreme real-life case.

Figure 3 plots the results of vertical stiffness reductions for considered hole sizes in shear strains up to 150%. Interestingly, as the shear strain increases, the reduction in the vertical stiffness is observed to be less in bearings with larger central holes. It is also observed that the effect of hole radius is magnified as the shear strain increases. Figure 4 shows the change in the vertical stiffness with respect to shear strain for different central hole dimensions. Although the vertical stiffness decreases noticeably with shear strain, the effect of the central hole is found to be a maximum of only 4% in the worst case, which is happening at the maximum shear strain. As a result, it can be concluded that the effect of the central hole is small enough to be neglected in vertical stiffness calculations.

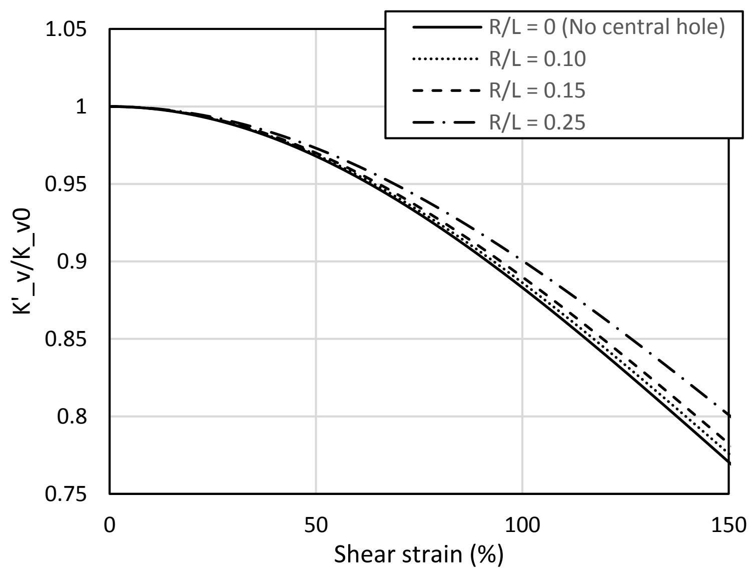

Figure 3. Stiffness reduction of rectangular bearing using the two-spring method with respect to different central hole dimensions

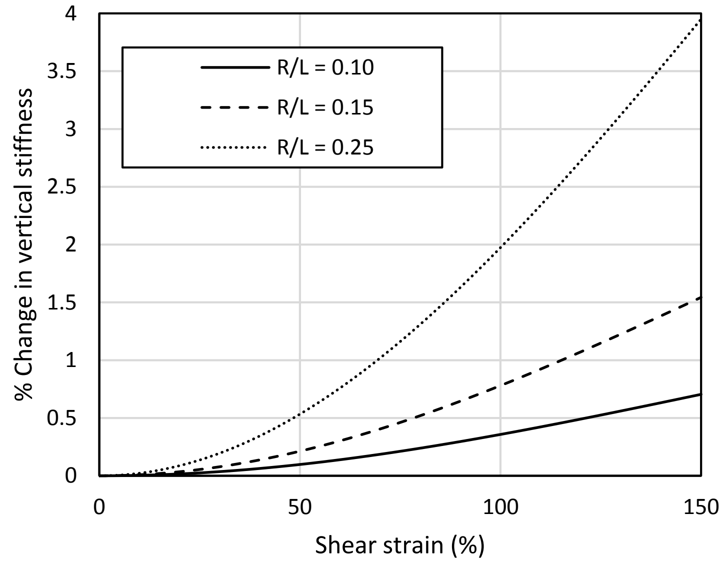

Figure 4. % Change in vertical stiffness with respect to shear strain for different central hole dimensions

Three methodologies were discussed earlier in this article for vertical stiffness calculation of rubber bearings with respect to horizontal lateral deformations. As predicted, the vertical stiffness and, as a result, the vertical load-carrying capacity of the bearing reduces as it undergoes lateral deformations. The stiffness reduction trends due to lateral deflection for the three procedures are illustrated in Figure 5. As observed, the last two methods, the overlapping area and the linear interpolation, both suggest a linear trend for vertical stiffness reduction for a bearing with rectangular cross section. However, the overlapping area method suggests a steeper decrease and gives the most conservative prediction.

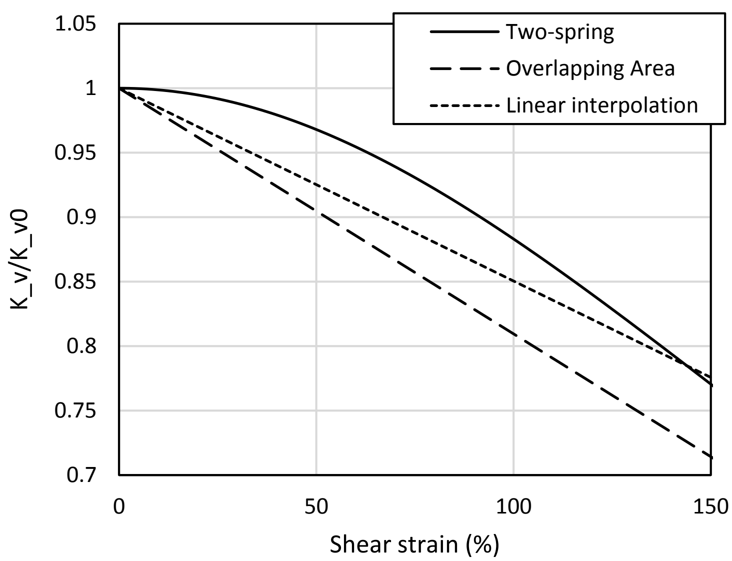

Figure 5. Comparison of stiffness reduction methods for rectangular rubber bearing

As part of a sensitivity analysis, the effect of rubber layers' thickness on bearing's initial vertical stiffness is also assessed. Three rubber bearings with three different rubber layer thicknesses are examined numerically to find out the effect of bearing's physical characteristics on the vertical stiffness. It is observed that as the rubber layer thickness decreases, which means a larger number of rubber layers have to be implemented, vertical stiffness of the bearing increases. As thinner rubber layers are used, the potentiality of rubber's bulging decreases – that is, less deformation takes place in rubber layers. As a result, a higher stiffness will be granted to the bearing. At the same time, implementing more rubber layers means increasing number of steel shims used as the reinforcements in between. This also contributes to a higher vertical stiffness for the bearing. Figure 6 shows variations of rubber bearing's initial vertical stiffness with the thickness of rubber layers.

For the case of lead-core rubber bearings (LCRBs), however, the lead core contributes to the vertical stiffness considerably much more than the physical parameters discussed. Its order of contribution in this study was found to be about a hundred times larger than that of the laminated pad, on average. In such a situation, the dominant component for determining the vertical stiffness would be the lead plug. Radius of the lead plug and its modulus of elasticity govern how stiff the bearing would be vertically. Figure 7 draws a simultaneous comparison between the effect of rubber layers' thickness and lead core's radius on the vertical stiffness. The dominant effect of the lead core is clearly observed.

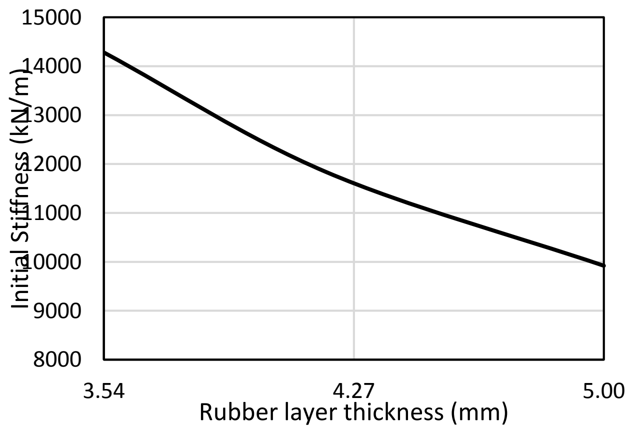

Figure 6. Variations of the initial vertical stiffness with thickness of rubber layers

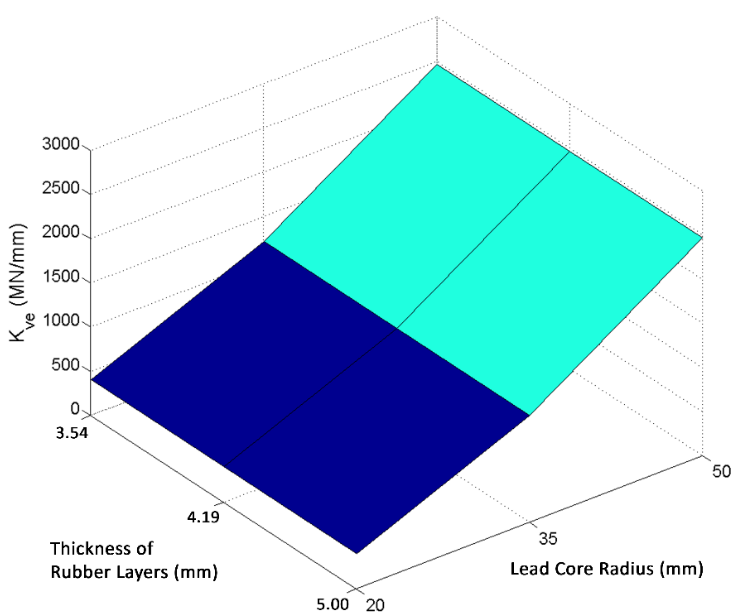

Figure 7. Vertical stiffness of LCRBs vs. thickness of rubber layers and lead core radius

5 Summary and conclusion

Vertical stiffness of rectangular rubber bearings is studied in this research. The effect of lateral deformations on the vertical stiffness is investigated through three methods: 1) The two-spring method, 2) Overlapping area method and 3) Linear interpolation method. The effect of manufacturing central hole size is also examined. Based on the research conducted, the following conclusions are drawn:

- As the bearing undergoes larger lateral deflections, the effect of the central hole on the vertical stiffness also increases.
- The reduction in vertical stiffness is observed to be smaller in bearings with larger central hole diameters. However, the effect of central hole was found to be small enough to be neglected in vertical stiffness calculations.
- Unlike the two-spring method, the overlapping area method and the linear interpolation method suggest a linear trend for vertical stiffness reduction with shear strain.
- The linear interpolation method suggests a residual vertical stiffness for the bearing. However, using the overlapping area method, the bearing loses all its vertical load-carrying capacity at lateral displacements of $\Delta = L$ and further. In the two-spring method, the vertical stiffness is inversely related to Δ and continuously decreases as Δ increases.
- The overlapping area method offers the simplest and, at the same time, the most conservative approach to predict the vertical stiffness reduction with respect to lateral deformations.
- Thinner rubber layers and, as a consequence, a higher number of rubber layers contributes to a higher vertical stiffness for bearings.
- For the case of lead-core rubber bearings, the lead core plays a dominant role in providing the vertical stiffness. Its order of contribution is about 100 times larger than that of the laminated pad.

6 References

[1] Hwang JS, Chiou JM. An equivalent linear model of lead-rubber seismic isolation bearings. Eng Struct 1996;18:528–36.

[2] Jangid RS, Datta TK. Seismic behaviour of base-isolated buildings: A state-of-the-art review. Proc ICE - Struct Build 1995;110:186–203.

[3] Kelly JM. Aseismic base isolation: review and bibliography. Soil Dyn Earthq Eng 1986;5:202–16.

[4] Cardone D, Gesualdi G. Experimental evaluation of the mechanical behavior of elastomeric materials for seismic applications at different air temperatures. Int J Mech Sci 2012;64:127–43.

[5] Mitoulis SA, Tegos IA, Stylianidis KC. Cost-effectiveness related to the earthquake resisting system of multi-span bridges. Eng Struct 2010;32:2658–71.

[6] ANSYS Documentation 2006.

[7] ANSYS Mechanical APDL 2012.

[8] Abe M, Yoshida J, Fujino Y. Multiaxial Behaviors of Laminated Rubber Bearings and Their Modeling. I: Experimental Study. J Struct Eng 2004;130:1119–32.

[9] Bhuiyan AR, Ahmed E. Analytical expression for evaluating stress-deformation response of rubber layers under combined action of compression and shear. Constr Build Mater 2007;21:1860–8.

[10] Khajehsaeid H, Arghavani J, Naghdabadi R, Sohrabpour S. A visco-hyperelastic constitutive model for rubber-like materials: A rate-dependent relaxation time scheme. Int J Eng Sci 2014;79:44–58.

[11] Warn G, Whittaker A, Constantinou M. Vertical Stiffness of Elastomeric and Lead–Rubber Seismic Isolation Bearings. J Struct Eng 2007;133:1227–36.

[12] Kelly J. Tension Buckling in Multilayer Elastomeric Bearings. J Eng Mech 2003;129:1363–8.

[13] Kelly JM. Earthquake-Resistant Design with Rubber. Second Edi. London: Springer-Verlag London Limited; 1997.

[14] Koh CG, Kelly JM. Effects of axial load on elastomeric isolation bearings. 1987.

[15] Buckle IG, Liu H. Experimental determination of critical loads of elastomeric isolators at high shear strain. NCEER Bull 1994;8:1–5.