DAMAGE IDENTIFICATION USING FREQUENCY SENSITIVITY FUNCTIONS

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Abstract: The inevitable occurrence of damage in structures necessitates the availability of an efficient and reliable damage identification routine. The recent boom in construction and the surge in the scale of structures have proven that relying solely on the classical approach to damage detection lacks practicality. Vibration-based techniques, which emerged in the early 1970s, have been successfully utilized in many damage detection applications. One of the most widely known vibration-based methods relies on detecting damage through the measured shift in resonant frequencies. Although this technique has the advantage of simplicity of the measurement process, its sensitivity to damages in large scale structures has been a subject of investigation. Additionally, the influence of variations in ambient conditions has been found to be measurable. This paper presents a new technique for detecting damage in beam-type structures by utilizing the frequency sensitivity functions of the first few vibration modes. The proposed method was successfully tested on Finite Element models of single-spanned beams. The effect of changes in both uniform and gradient temperatures was also addressed in this research. The capability of the presented method in identifying damage in relatively long period structures was investigated by applying the proposed technique on a numerical model of a cable-stayed bridge. The method was also tested using the results of pervious experimental investigation in which damage was simulated at different locations of a cantilever beam. The presented technique showed promising capabilities in identifying the location and magnitude of damage.

Keywords: Damage detection; Natural frequency; Structural Health Monitoring; Cable-stayed Bridge; Temperature effect

1 INTRODUCTION

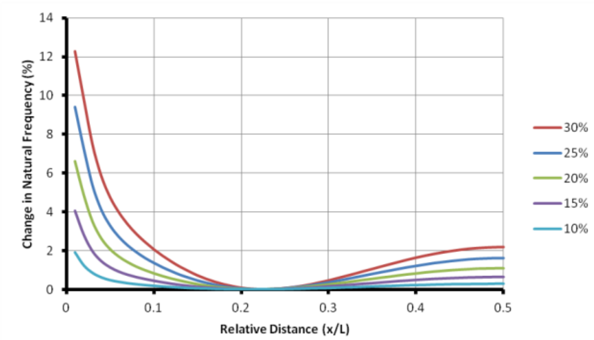
Identifying structural damage is generally a rather complex and sometimes prolonged process, especially in large-scale structures. Vibration-based damage identification techniques have been widely adopted in the past few decades due to their inherent global nature, which is reflected on both the economical and practical aspects of their applications (Abdel-Mooty, 2002). Despite the ongoing controversy regarding the effectiveness of such global techniques to detect damage, which is generally regarded as a local phenomenon, numerous modal-based applications have been reportedly successful in locating damage (Banks et al, 1996; Farrar et al, 1999). Out of all modal-based damage identification methods, techniques that relied on monitoring alterations in the resonant frequencies were found to be the most practical and cost efficient. This is attributed to the ease by which natural frequencies could be measured and the minimal instrumentation that is needed for such task (Salawu, 1997; Messina et al, 1998). Nonetheless, several investigations have revealed that when natural frequencies were employed to identify damage, success was merely limited to confirming the existence of damage without the ability to acquire useful information regarding its location or severity.

This paper introduces a new damage detection technique that is initially developed for beam-type structures. The proposed technique utilizes natural frequency shifts in terms of frequency sensitivity functions to identify damage. The introduced method is tested on a Finite Element (FE) model of a single-spanned Reinforced Concrete (RC) beam, in which damage is modeled as a crack with various depths at different locations of the beam. The technique is also examined using experimental data that are obtained from a previously conducted experimental investigation that was performed by other researchers. The proposed technique showed high capability in providing precise spatial information regarding the identified damage as well as its severity. Factors that have a substantial influence on the accuracy of the developed technique, such as the effect of fluctuations in the ambient conditions, are also addressed in this paper as a premise for a future extensive solution that would incorporate a proper filtration mechanism.

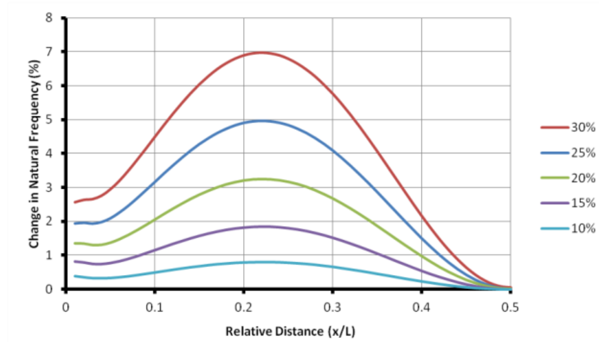
2 FREQUENCY SENSITIVITY FUNCTIONS

A substantial relationship between damage and frequency shifts does exit. This relationship is well known as the sensitivity function of the system for a certain mode of vibration. The mathematical derivation of the sensitivity functions is explained in detail by (Kim et al., 2003). An investigation is conducted in the present study on Finite Element (FE) models that were constructed to imitate the dynamic performance of a simply supported Reinforced Concrete (RC) beam. The span of the simulated beam is 20 m and has a depth of 2.0 m and a width of 0.5 m. The model was built using membrane-type area elements of size 10 x 10 cm having a modulus of elasticity of 28000 MPa. A 2-D modal analysis was performed in which only in-plane vibration modes were considered. A commercially available software package SAP2000 was used in this investigation.

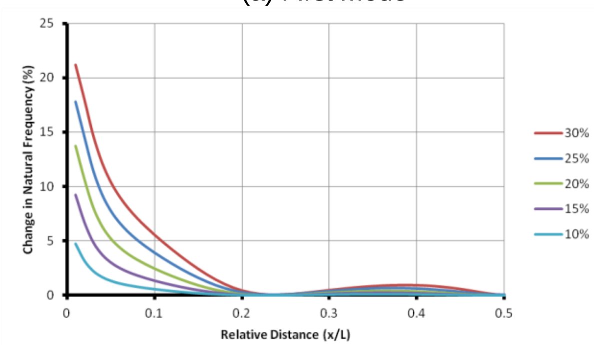
Figure 1 provides a clear representation of the sensitivity functions determined for the first four modes, which are extracted from the FE models described above with damage modeled on its bottom side. The data presented in these figures are for five different levels of damage, where the effective cross-sectional area of the beam is reduced by 10, 15, 20, 25 and 30%. The corresponding reduction in the moment of inertia is 27.1, 38.6, 48.8, 57.8 and 65.7% respectively. The sensitivity functions are plotted until the center of the span only because of the symmetry of the beam. The x-axis displays the distance along the beam in terms of fractions of the entire span, while the y-axis presents the change in Eigen frequencies due to damage as a percentage.



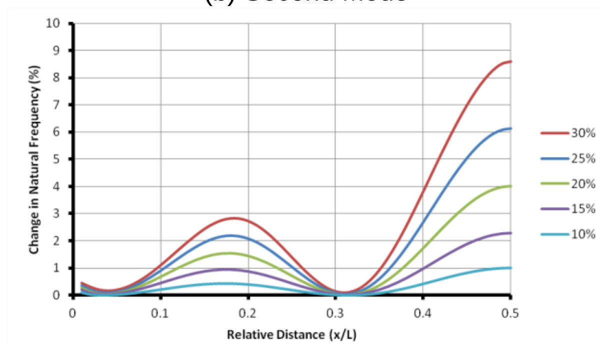
(a) First mode



(b) Second Mode



(c) Third Mode

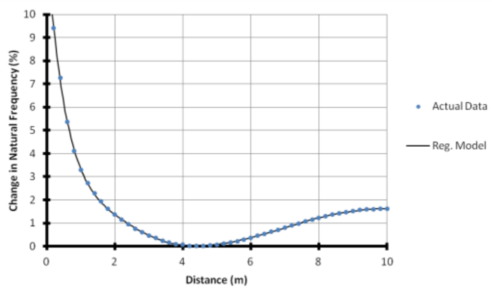


(d) Fourth Mode

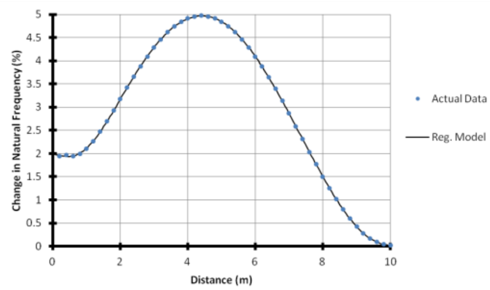
Figure 1: Sensitivity functions for the first four vibration modes

It can be noticed from Figure 1 that the frequencies of mode (1) and mode (3) are highly sensitive to damage near the supports. On the other hand, the second and fourth modes appear to be much more affected by damage that would occur in the regions of the ¼ span and mid span respectively. It can be also concluded that the first four modes sufficiently cover the entire span such that if damage is inflicted at any location along the span a noticeable shift in the resonant frequency of at least one mode would be detected. It is clear from these results that sensitivity functions could be employed to identify damage location and estimate its severity in an efficient and systematic manner when they are described in a mathematical form. However, relying on one mode only would not be sufficient to locate damage since each sensitivity function may have more than one solution for a given frequency shift. Therefore, the first four modes were elected to fulfill this purpose. In order to achieve a numerical representation for the sensitivity functions found for these modes, a regression analysis was performed.

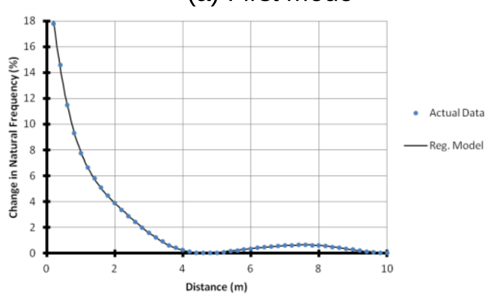
For illustration purposes, the regression models generated for the sensitivity functions of the first four modes of a 20 m long beam are presented in Figure 2. In addition, the actual data obtained from the analysis are also plotted along with the sensitivity functions. It can be noticed that a complete correlation between the actual data and the obtained regression models is achieved for each mode with a value of (r^2) ranging from 0.998 to 1.0. Figure 2 displays the regression models calculated for sensitivity functions in which damage is simulated as a 50 cm deep crack on the bottom side of the beam.



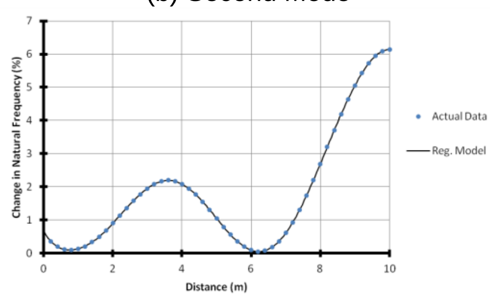
(a) First mode



(b) Second Mode



(c) Third Mode



(d) Fourth Mode

Figure 2: Regression models for the found sensitivity functions

By having a mathematical formulation that describes the sensitivity functions in terms of the location of damage (x) for every damage level assumed, and given the value of frequency shifts for each of the first four modes which are acquired experimentally during testing, all possible values of (x) can be calculated from the sensitivity function of each of the four modes. The value of (x) that is repeated in all four modes would be then taken as the location of the damage that caused the frequency shifts. This process is repeated for all defined levels of damage until the common value of (x) is found. Only one level of damage is expected to result in a real solution which represents the location of the damage. The level of damage at which this solution was found indicates its severity.

Solving these equations for damage location would be a rather complicated process if it was carried out manually. However, this technique is proposed for a computerized application that could be possibly integrated in an online automated damage detection system that would be able to identify damage in real time.

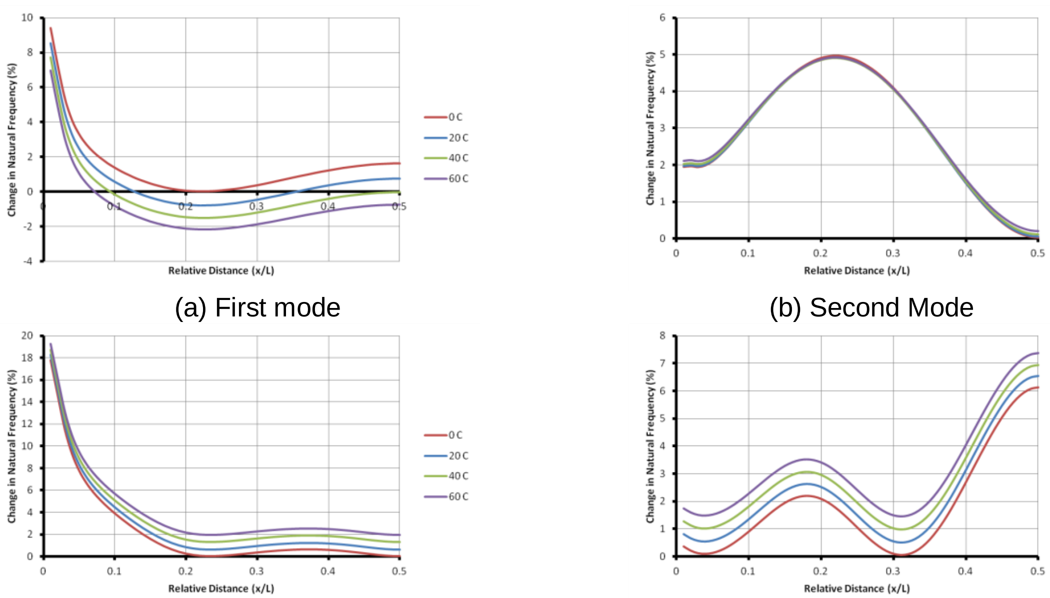
3 EFFECT OF TEMPERATURE VARIATION ON FREQUENCY SENSITIVITY FUNCTIONS

It has been generally accepted that the sensitivity of resonant frequencies to changes in the ambient conditions is one of the major drawbacks that hinder the spreading of its applications in the field of damage detection (Carden et al, 2004; Zhou et al, 2011). Changes in temperature, humidity, wind speed or even loading conditions can produce frequency shifts similar to that induced by damage (Basseville et al, 2011). Temperature fluctuations can reach 20 C^{o} on a daily basis from one part of a bridge to the other while it can vary by up to 50 C^{o} throughout the year in many places (Xu and Wu, 2007). Such fluctuations result in changes in the stiffness of the affected system. This is a result of the changed size of the structural elements, which produces a tangible amount of internal forces if restrained. Additionally, the properties of elasticity of the materials were found to experience minor alteration with temperature as well, leading to a different cause for stiffness change. It is expected that the effect of temperature variations on the resonant frequencies would be different from one mode to another. Additionally, the influence of daily variations in temperature might be different from changes that take place around the year. Both seasonal and daily temperature changes are discussed in detail below.

3.1 Uniform temperature variations

Temperature varies throughout the year in a relatively slow and steady tone, which allows for such variation to be considered uniform within the structure (Xu and Wu, 2007). The magnitude of temperature variations through the year would adversely affect the coherence of any experimentally acquired modal data, and if not properly handled, the entire process of Experimental Modal Analysis (EMA) and any subsequent damage identification operations would be deemed trivial. However, it has been consistently reported in several studies that the effect of such temperature changes on small-scale structures such as short-span bridges is negligible.

In the present study, an investigation is conducted to understand how sensitivity functions are influenced by uniform temperature fluctuations. A FE model was created for this purpose using area elements having the same properties and cross section as explained in the previous examination. The span of this model was selected to be 20 m and damage was modeled as a 50 cm deep crack at different locations along the span on the bottom side. Sensitivity functions were extracted and plotted corresponding to a change in temperature with values of zero, 20, 40 and 60 C^{o}. Additionally, the modulus of elasticity of RC was reduced for each model in correspondence to the assigned temperature change according to (Xu et al., 2007). The resulting sensitivity functions of the first four modes are presented in Figure 3.



(a) First mode (b) Second Mode

(c) Third Mode (d) Fourth Mode

Figure 3: Shift in sensitivity functions with uniform temperature variation

By observing the sensitivity functions displayed in Figure 3, it can be seen that the change in the resonant frequency of the fundamental mode decreases by the increase of the reference temperature. This is the result of the increase of the modal stiffness of the first mode with temperature. Consequently, the resulting increase in the first mode's modal stiffness reduces the effect of any introduced damage on the frequency of this fundamental mode.

The sensitivity functions of the second mode displayed in Figure 3 shows that temperature change appears to have no tangible effect on the resonant frequency. Finally, the sensitivity functions of the third and fourth modes respectively exhibit a uniform shift with temperature change. This shift is a result of the reduced elastic properties of the material. It can be also noticed that temperature variations increases the frequency changes resulting from damage for these two modes indicating the reduction that occurred to the modal stiffness.

3.2 Gradient temperature variations

In addition to the seasonal temperature fluctuations, beam-type structures such as bridges also experience temperature variations on a daily basis. This type of temperature variations is best represented in any analysis as a gradient temperature change, which simulates the different levels of exposure to heat between the top and bottom extreme fibers. For a better understanding of how resonant frequencies are influenced by daily temperature changes, a FE model was constructed for a 20 m spanned RC beam. The cross-section of the modeled beam and the properties of the model are consistent with the previous models discussed above. The model was subjected to gradient temperatures of 5, 10 and 15 C^o. The temperature was assumed to vary linearly along the depth of the section from its maximum value at the top of the section reaching to zero at the bottom. In a separate case, the gradient temperature was modeled in an inverted direction, with the maximum at the bottom to resemble the heat radiations during night time as shown in Figure 4 (Mosavi et al., 2012).

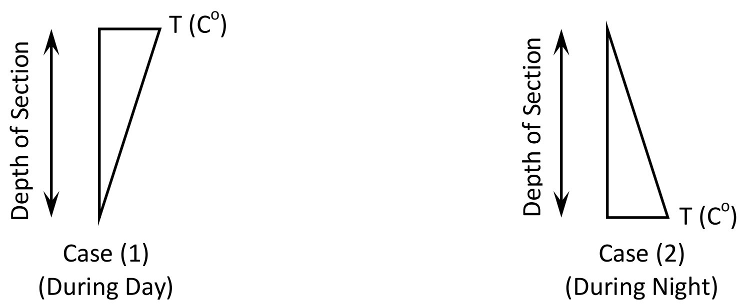
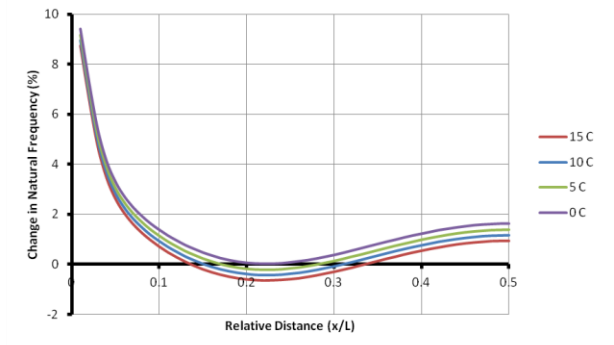


Figure 4: Gradient temperature profiles applied in the analysis

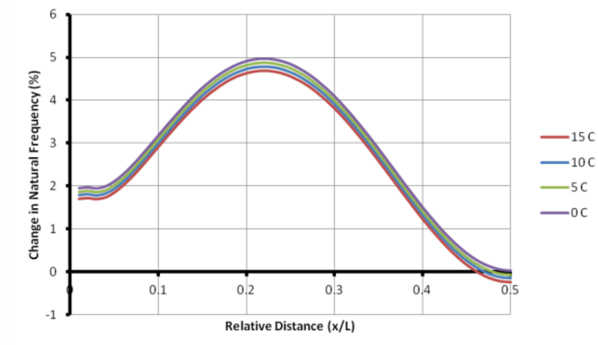
In the present investigation, the effect of temperature on the elasticity properties of the materials is found to be small and is therefore ignored. Accordingly, this investigation only considers the effect of change in the beam size with temperature variations.

By performing an analysis for both cases as described above, it was found that the effect of gradient temperature changes on the sensitivity functions in both cases were identical. Therefore, the analysis results were presented only once in Figure 5. The results displayed in Figure 5 presents a clear indication that the effect of gradient temperature variations has a similar pattern on resonant frequencies as that of the uniform temperature fluctuations.

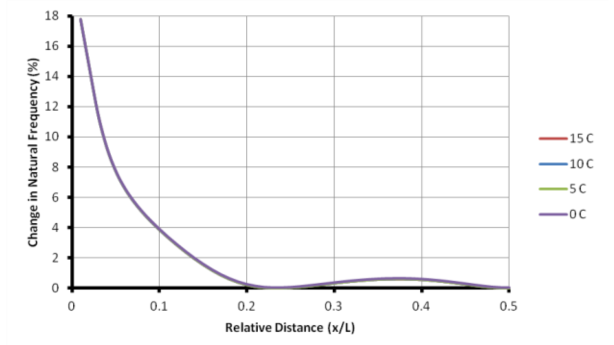
It can be observed in Figure 5 that the presence of a gradient temperature change reduces the damage induced-change in the resonant frequency of the first mode as a result of the increased modal stiffness of that mode. It can be also noticed that the sensitivity function of the second mode is mildly affected by the temperature variations. Additionally, it is seen that the frequencies of the third and fourth modes are hardly affected by the presence of a gradient temperature change.



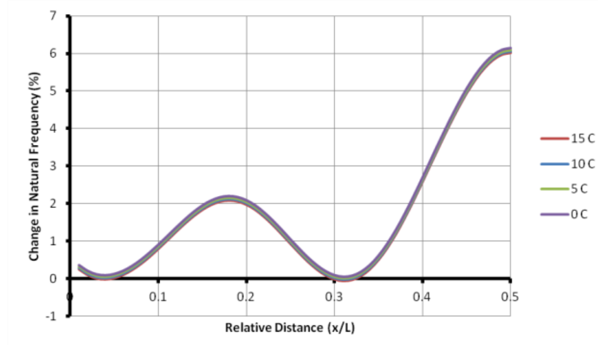
(a) First mode



(b) Second Mode



(c) Third Mode



(d) Fourth Mode

Figure 5: Shift in sensitivity functions with gradient temperature variation

The reason that the sensitivity functions of the third and fourth modes are not influenced by the temperature variations is that these two modes have proven to be affected only by the reduction in the elasticity properties when a beam of the same span was tested with uniform temperature change. Since both gradient and uniform temperature changes have shown to have similar effects on all sensitivity functions, it is expected that the sensitivity functions of modes (3) and (4) would have exhibited a shift had the elasticity properties been modified according to the assigned gradient temperatures.

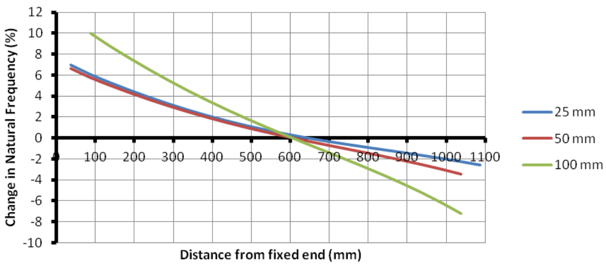
However, the properties of elasticity of the materials were not changed in this investigation as discussed above because of their small effect. An increase in temperature of 15 C^o, which is the utmost temperature change in this specific study, would result in a reduction of a little above 1% in the modulus of elasticity of the RC at the extreme fiber exposed to heat radiation. Accordingly, the accuracy of the results obtained from this investigation is not expected to be compromised as a result of such approximation.

4 EXPERIMENTAL INVESTIGATION

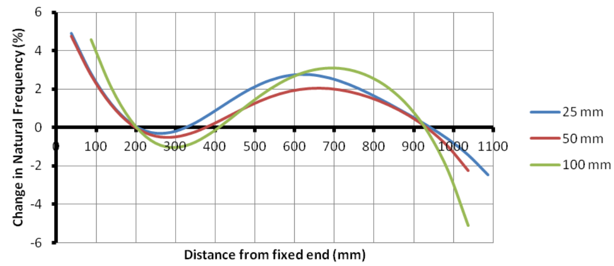
An experimental investigation that was previously conducted by Abdel Mooty and Hashad (2004) is used to verify the damage identification technique proposed in this study. The experimental work conducted involved measuring the natural frequencies of six identical steel cantilever beams with damage inflicted at different locations in addition to one undamaged beam that served as the reference specimen. All seven specimens tested had a clear span of 1100 mm and a cross-section of 57 x 6.1 mm. The excitation was carried out using an impact hammer and the resulting response was measured by the means of six accelerometers that were arranged along the span of the tested specimens.

A FE model is constructed for the tested cantilever beams using commercially available software package SAP2000, in order to formulate the sensitivity functions for the first three modes. The beam is modeled using area elements of size 5 x 4.74 mm, and all translation and rotation Degrees of Freedom (DOF) were restrained for the joints at the support end. The material properties assigned were those obtained experimentally as reported in (Abdel Mooty and Hashad, 2004). However, when modal analysis was carried out for the undamaged case, some discrepancies were realized between the natural frequencies obtained experimentally and those calculated from the numerical model. This is possibly due to the added mass of the transducers, which is not considered in the FE model, (among other minor deviations). Consequently, the FE model was updated by adjusting the mass of the beams until a match was achieved. To resemble the actual dynamic behavior, the modes considered in the FE analysis were the first three out of plane modes.

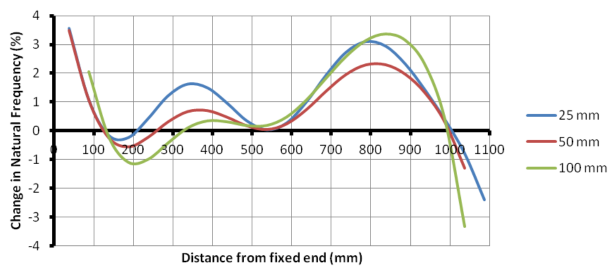
The frequency sensitivity functions for the first three modes were derived for each one of the defined damage levels using the updated FE model. Figure 6 shows the sensitivity functions of the first three modes for all three damage levels. It is noticed that changes in the level of damage causes a relative distortion to the pattern of the function as opposed to the findings discussed earlier in this study where changes in the damage severity was found to scale the function without altering its shape. The explanation for this is that damage was simulated by removing a portion of the beam, which results in both stiffness and mass reduction. In contrast, in the previously discussed investigation, damage was modeled as a crack, causing a reduction in the stiffness alone.



(a) First Mode



(b) Second mode



(c) Third mode

Figure 6: Sensitivity functions of the first three modes for all three defined levels of damage

A regression analysis was performed for all sensitivity functions extracted in order to obtain a mathematical representation that could be used in the damage identification routine. The regression analysis was performed using commercially available software tool Evolver by employing sinusoidal functions for a better correlation. The final function describing each sensitivity function is composed of the superposition of 10 individual sinusoidal functions.

The derived regression models are implemented into the damage identification process. The proposed damage identification technique is tested by using the obtained sensitivity functions to calculate the location of damage for each value of the experimentally acquired set of natural frequencies. This process is repeated for each of the six damage scenarios investigated experimentally. The calculated damage locations are then compared with the actual locations in which damage was inflicted to measure the effectiveness of the damage detection method.

The proposed damage identification technique was successfully capable of identifying the location and magnitude of damage in four out of the six studied damage scenarios. The accuracy by which the detected damage was located is sufficiently high. The damage cases that were not detected are those where overlap occurred between the sensitivity functions of all three damage levels for the considered modes at these particular locations. This means that for the same set of frequency shifts more than one damage level was detected. The pattern distortion that caused this overlap is a result of the nature of the damage, which involves both changes in mass and stiffness, as opposed to the type of damage that results in an alteration in the stiffness alone as discussed above. It is therefore concluded that the proposed damage identification technique is generally effective when applied to investigate damage that causes stiffness changes only, while it may lose its ability to identify the level of damage at some locations when the inflicted damage results in both mass and stiffness changes.

5 APPLYING THE PROPOSED METHOD ON A FE MODEL OF A CABLE-STAYED BRIDGE

For further investigating the capacity of the proposed method to identify damage in different types of structures, an investigation is conducted on a FE model constructed for a cable-stayed bridge that is located in Egypt. The geometric and material properties of the bridge are reported in the literature by Abozeid (2005). A complete description of the 3-D finite elements model is reported in the literature by Thabit (2014). This cable-stayed bridge was particularly chosen due to its inherent nature as a long period structure, which would provide deep insight on the ability of the proposed technique to identify damage even for minute changes in the resonant frequencies. A schematic elevation of the bridge is shown in Figure 7.

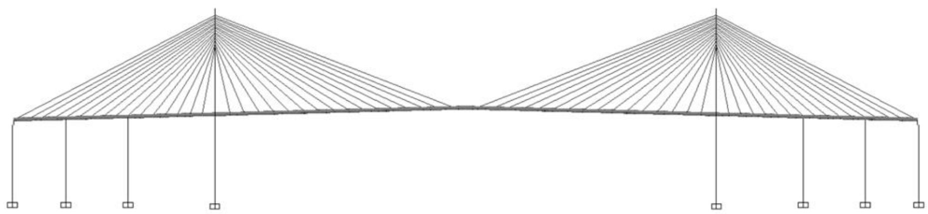
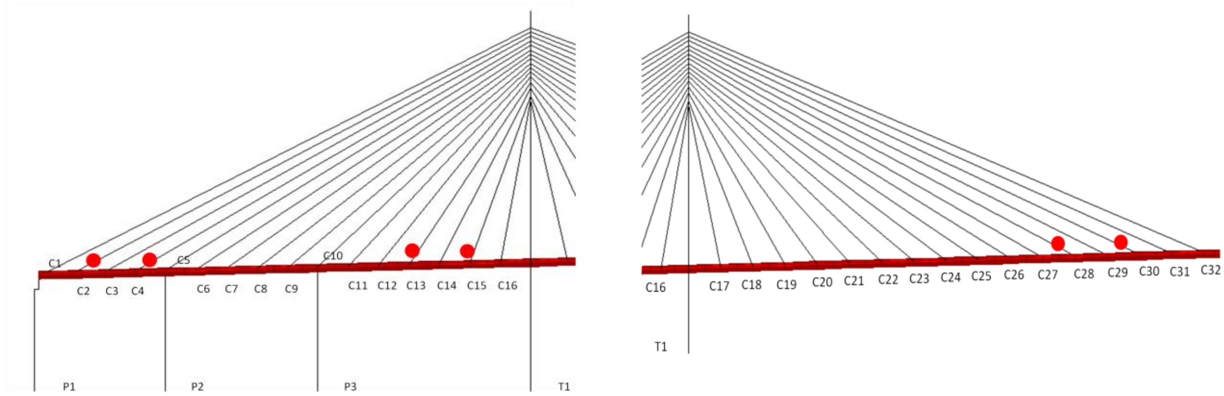


Figure 7: Schematic elevation of the analyzed cable-stayed bridge

Several idealizations have been implemented to the finite element model to simplify the modeling and analysis procedures. These idealizations affected the precision of the modal analysis results in comparison with the experimental results reported earlier by Abouzeid (2005). To overcome such setback, the constructed FE model was updated by modifying some of the material properties of the bridge components to have closer agreement between the FE and experimental results of the natural frequencies of the first five modes of the bridge. The material properties that were modified included the modulus of elasticity of the steel members and stay cables, modulus of elasticity of the pylon concrete, and the stiffness of the bearings. Table 1 shows the comparison between the analytical and experimental results before and after updating the model. The chosen damage type to be investigated was cable damage, and was modeled in the FE model as loss of the axial stiffness of the cables. Despite the lack of resemblance to reality, damage was assumed to occur symmetrically at each pair of cables on both sides of the deck in a simultaneous manner to reduce the complexity of the analysis at this stage. Only the vertical bending modes were employed since the results of the sensitivity analysis clearly displayed that those modes were highly influenced by the axial stiffness of the cables .Detecting the damage in six cables was investigated in the present study. The location of the selected six cables is shown in Figure 8.

Table 1: Comparison between the analytical and experimental natural frequencies

Mode	Natural Frequency (hz)			Difference (%)	Description
	FE ANALYSIS		EMA		
	Before Update	After Update			
1	0.228	0.249	0.256	2.7	Longitudinal floating mode
2	0.303	0.273	0.269	1.5	1st transverse bending mode
3	0.354	0.285	0.281	1.4	2nd transverse bending mode
4	0.405	0.301	0.293	2.7	3rd transverse bending mode
5	0.505	0.406	0.403	0.74	1st Vertical bending mode



(a) Cable numbering on the west side of the pylon (b) Cable numbering on the east side of the pylon

Figure 8: selected cables for damage detection by the finite elements model

The sensitivity functions were extracted and a numerical representation was derived by performing a nonlinear regression analysis as discussed above. By performing the damage identification process for a selected number of cables, it was found that damage was accurately detectable in cables supporting the mid-span region of the deck (cables 2, 4, 28, and 30) while it was not detected at the cables near the pylon (cables 13 and 15). Damage was possibly easier to detect in stay cables away from any vertical pylon supports since the amplitude of the deck oscillations in the vertical bending direction would not be significant in the proximity of such supporting structures.

Although frequencies of torsional bending modes were also found to show some sensitivity to cable damage, the impact of damage in cables on the resonant frequencies is much less, and the pattern of the sensitivity functions of the torsional modes matches those of the vertical bending modes to a high extent. Consequently, cables that are not possibly detected using the sensitivity functions of the vertical bending modes would probably be undetected if the torsional modes were employed instead. Therefore, torsional modes should not be viewed as a replacement for the vertical bending modes with respect to detecting damage in cables. Alternatively, a non-frequency-based approach could be sought for this particular application.

6 CONCLUSION

A new damage identification technique is introduced is introduced in this paper that employs the sensitivity functions of the first few number of modes. The proposed technique has the potential capacity to provide spatial information regarding the detected damage as well as its severity.

The sensitivity function of each mode was found to have a unique shape. It is also noticed that all damage levels for every mode have the same unique shape of this specific sensitivity function but with a different scale. This is essentially true when the simulated damage resulted in loss of stiffness only. However, when damage caused a reduction in both stiffness and mass the extracted sensitivity functions were found to take different shapes for the same damage type.

Sensitivity functions can be built for the structure under consideration through numerical simulation of damage. Mathematical expressions for the sensitivity function can be derived using nonlinear regression analysis which can be used as reference for detecting damage.

This technique was successfully tested on Finite Element models of single-spanned beams where damage presented by crack of different depths was simulated in the model. The presented method was also tested on experimental data that is obtained from a previously conducted experimental investigation performed by other researchers. The advantage of using this method is that it relies basically on measuring natural frequency shifts, which is considered a simple, low cost and rather accurate process from a practical perspective.

The effect of temperature changes on the derived sensitivity functions is studied. This effect can be incorporated into the proposed damage detection technique to alleviate the adverse effect of temperature changes on the damage detection accuracy. The simplicity of this technique makes it easily programmable and viable for an online monitoring system.

When performed on a Finite Element model of a cable-stayed bridge, the proposed damage identification technique was capable to detect damage simulated in stay-cables near the center of the span while it was not able to identify damage at the stay-cables near the support.

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