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Life-Cycle Performance and Cost Analysis of Concrete Beams Designed with Cross Sections of Equal Durability

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Abstract: The durability resistance at the section corner is relatively weak in concrete beam bridges, therefore, the reinforcements at the corner would be corroded in advance. In order to delay the occurrence of the concrete disease in the corner, measures such as reinforcements adjusting would be taken. In this way, the durability resistance would be adjusted to be equal in the section, which is called equal durability design method. The service life would be extended by this means. A comparative analysis of life-cycle performance and cost of traditionally designed concrete beam and equal durability designed concrete beam would be carried out in this article. Probabilistic corrosion initiation time, cracking initiation time and sever-cracking time would be developed and life-cycle performance probability model would be established. The economics of equal durability design is conducted by life-cycle cost analysis. It was found that equal durability design got less limitation if the concrete cover depth is smaller. The equal durability design method provides a new idea to traditional design concept.

1 INTRODUCTION

A large number of field survey found that the corrosion-induced cracking appeared in the corner of the RC beam bridge. The durability resistance at the section corner is relatively weak. Apparently, the durability resistance on the cross section is not equal. It is not economical to overhaul or scrap if it is damaged in the corner parts but other parts are in good condition. This is why equal durability design method was raised. The purpose of equal durability design method is to make the corrosion expansion cracking times of each edge and angle approximately the same. In other words, the durability resistance of the whole section is generally the same by this means. The equal durability method helps to extend the service life of the concrete bridge and to reduce the overhaul times and costs.

The performance of a structure is not constant in the whole life cycle, it will gradually degenerate due to environment attacks such as the carbonization. The environment actions will lead to cracking and spalling that indicate the need for an assessment of existing safety, repair or replacement of damaged structural elements, or the need for more frequent inspections. All these cases will require the investment of additional resources.

Qu(1995) specified section based equal durability design method of RC beams to improve durability, reduce maintenance costs and extend service life. Initial costs of structures designed by using this method will be more expensive than that of designed by conditional method. However, it would be suggested that expected reductions in maintenance and repair costs and the extension of service life can justify its use on a life-cycle cost basis.

It is considered to be reasonable intuitionally to apply the equal durability design method to the concrete bridge design process. However, the performance of the components designed with cross sections of equal durability has not been studied. It is unclear that the performance of the equal durability designed member is better or not, compared with the traditional designed concrete components.

The present paper will focus on a life-cycle cost analysis of RC beams designed with sections of equal durability. A structural deterioration life-cycle probabilistic model including random variability of initial corrosion time, initial cracking time and sever cracking time is used to calculate probabilities of cracking for RC structures. It is assumed herein that the incidence of sever cracking will result in repair in order to extend the service life of structures. Time-dependent probabilities of sever-cracking are calculated over the lifetime of the structure (50 years). Life-cycle costs considering initial construction cost, inspection costs, maintenance costs and relative failure costs would be estimated in this paper.

2 SERVICE LIFE OF CONCRETE STRUCTURE

For the concrete bridge beams located in the general atmospheric environment, the general durability failure sign is that the crack width induced by corrosion of the reinforcements reaches a certain limit. Excessive cracking can lead to rapid structural degeneration, although it has not any significant effect on the structural capacity. So sever cracking (crack width equals to w_{lim}) is the significance of reparation.

There are three stages from the time putting into service to the time reaching durability ultimate state. Stage I: time to corrosion initiation, referred to herein as T_i; Stage II: time from corrosion initiation to first cracking, referred to herein as T_{cr1}; Stage III: time from first cracking to sever cracking, referred to herein as T_{cr}. Stage I is called corrosion initiation stage and it represents the time required for the carbonization front to reach the steel surface (carbonization remains is neglected). Stage II represents the period of corrosion initiation until crack initiation, when stresses resulting from the expansion of corrosion products exceed the tensile strength of concrete. The first crack is generally taken to be a hairline crack of approximately 0.05mm. Stage III represents the period of crack propagation. So a crack width of 1.0mm represents severe cracking in this paper. The sum of stage II and stage III is called corrosion propagation stage (corrosion propagation time is referred as T_{sp}).

Based on the conceptual model above, the service life (T_s) can be defined as the total time to sever cracking, which is the sum of the corrosion initiation time (T_i), and the time of the crack propagation (T_{sp}), as follows:

[1] $$T_s = T_i + T_{sp} = T_i + T_{cr1} + T_{cr}$$

3 EQUAL DURABILITY DESIGN METHOD OF THE CONCRETE BEAM

3.1 Cross-sectional Equal Durability Design

The main inducing factor of the corrosion of reinforcements in general atmosphere environment is the completely carbonization of the protective cover.

There are obvious differences in carbonization depth on the different parts of the bridge deck section, particularly with regard to the corners and the edges. The carbonization rate of the corner is much faster than that of the edge. The carbonization front of the section at any time is as shown in Figure 1.

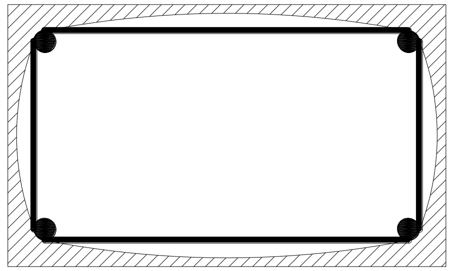


Figure 1: Carbonization front of Rectangular member section

According to the survey data and literatures, the cover caving caused by the corrosion of reinforcements appeared in the corner in tension firstly. There are three main reasons: a) the carbonization rate is much faster in the corner by two-way carbonization; b) the reinforcements in the angle can get oxygen and moisture from two directions, the corrosion speed is accelerated; c) the corrosion expansion capability is relatively weak in the angle area.

This kind of phenomenon prompted us to looking for methods to make the time reaching durability limit state probably the same between the corner and the edge. In other words, it promoted us to looking for methods to achieve the equal durability design.

3.2 Design Principles

Previous research showed that the carbonization depth at the corner is about $\sqrt{2}$ times the depth at the edge. So the occurrence of the corrosion of the corner reinforcements is earlier than the corrosion of the edge reinforcements.

So the principle of cross-sectional equal durability design is to make service life of the corner equal to that of the edge, which could be expressed herein as:

[2] $T_{s,c}=T_{s,e}$

where $T_{s,c}$ is the service life of the corner; $T_{s,e}$ is the service life of the edge.

3.3 Design Recommendation

It is reasonable to delay the corrosion of the corner reinforcements from the view of equal durability design. A possible way was proposed to achieve this point, which is to substitute FRP bars for corner reinforcements, which is shown as Figure 2.

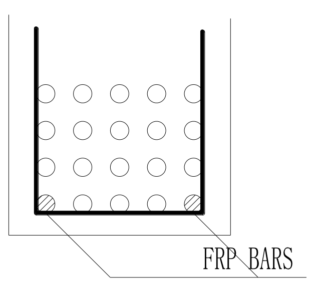


Figure 2 Design Recommendation

4 DURABILITY DEGRADATION MODEL OF CONCRETE BEAM

4.1 Corrosion Initiation Model

It is reasonable to assume that the material property, mix proportion, cement content and environment temperature are the same in different parts of the bridge beam, and so the effect of partial carbonation zone on the corrosion initiation time would be neglected. Consequently, corrosion occurs and performance degenerates when the carbonation front reaches the reinforcement bar surface.

The carbonation front is controlled by carbon dioxide diffusion that occurs through the concrete's pores, which depends on moisture, temperature, carbon dioxide concentration and concrete composition. The process of CO_2 diffusion can modeled based on Fick's first law. The initial model proposed by Tuutti(1982) is based on the diffusion law and considers that the carbonation rate is proportional to the square root of the time of exposure to the CO_2, which could be expressed as Equation 3:

[3] $x = K\sqrt{t}$

where, x is the carbonation depth(mm); K is carbonation coefficient(mm/year-1/2); t is the exposure time to CO_2(year).

When the carbonation depth is equal to the cover depth, the reinforcements started to corrode without regard to the partial carbonation zone. So the corrosion initial time could be calculated from Equation 4:

[4] $t=(C/K)^2$

where C is the cover depth(mm).

Zhang and Jiang(1998) has developed a practical mathematical model to predict the carbonation depth. Carbonation coefficient K could be calculated as follows:

[5] $K = 839\left(1 - RH\right)^{1.1}\sqrt{\frac{w/c - 0.34}{cc}C_0}$

where RH is relative humidity; w/c is the water cement ratio; cc is the cement content(kg/m^3); C_0 is the CO_2 concentration(%); in Equation 4, carbonation time t is counted in days.

For the corner of the section, the carbonation coefficient is larger than that of the edge. The carbonation coefficient at the corner (K_c) is $\sqrt{2}$ times that at the edge (K_e), which can be expressed as follows (Qu 1995):

[6] $K_c = \sqrt{2}K_e$

4.2 Corrosion Propagation Model

4.2.1 Crack Initiation

In this paper, crack initiation time is referred to be the period of corrosion initiation to first crack, which is generally be taken as a hairline crack of approximately 0.05mm.

El Maaddawy and Soudki(2007) has proposed a mathematical model that predicts the time from corrosion initiation to corrosion cracking. The time can be get from Equation 7:

[7] $$T_{cr1}=\left[\frac{19.5\left(D+2\delta_0\right)\left(1+v+\varphi\right)}{i_{corr}E_{ef}}\right]\left[\frac{2Cf_{ct}}{D}+\frac{2\delta_0 E_{ef}}{\left(1+v+\varphi\right)\left(D+2\delta_0\right)}\right]$$

where T_{cr1} is the time from corrosion initiation to crack initiation which is given in years; δ_0 is the thickness of the porous zone (mm); E_{ef} is the effective elastic modulus of concrete that is equal to $E_c/\left(1+\phi_{cr}\right)$, E_c is the elastic modulus of concrete, ϕ_{cr} is the concrete creep coefficient (2.35); v is the Poisson's ratio of concrete(0.18); *C* is the concrete cover depth (mm); *D* is the diameter of the steel reinforcing bar (mm); $\varphi=D'^2/2C\left(C+D'\right)$; $D'=D_0+2\delta_0$.

4.2.2 Crack Propagation

Crack propagation time is referred to be the period of crack initiation to excessive cracking (crack width reaches w_{lim}).

Mullard and Stewart(2009) developed an empirical crack propagation model based on test data, which can be used to predict the timing of excessive corrosion-induced cover cracking for RC structures in chloride of carbonated environments. The model was expressed by Equation 8:

[8] $$T_{cr}=k_R\frac{w_{lim}-0.05}{k_c r_{crack}}\left(\frac{i_{corr(\exp)}}{i_{corr(real)}}\right)$$

[9] $$k_R\approx 0.95\left[\exp\left(-\frac{0.3i_{corr(\exp)}}{i_{corr(real)}}\right)-\frac{i_{corr(\exp)}}{2500i_{corr(real)}}+0.3\right]\qquad k_R\geq 0.25$$

[10] $$r_{crack}=0.0008exp\left(-1.7\varphi_{cp}\right)$$

[11] $$\varphi_{cp}=C/\left(Df_{ct}\right)\quad 0.1\leq\varphi_{cp}\leq 1.0$$

where k_R is the rate of loading correction factor which reflects the influence of the loading rate on crack propagation. k_R can be determined empirically by Equation 9. k_c is the confinement factor which represents an increase in crack propagation due to the lack of concrete confinement around the corner bars. If the reinforcing bar is in an internal location then k_c=1; if the reinforcing bar is in the corner then k_c=1.3. r_{crack} is the rate of crack propagation in mm/hour, which is defined as the slope of the crack width versus time graph between the time of first cracking and the time of excessive cracking. φ_{cp} is the cover cracking parameter which incorporates cover depth *C*, bar diameter *D* and concrete tensile strength f_{ct}.

4.3 Probability Model of Life-Cycle Performance

The serviceability limit state in this article is that the crack width induced by corrosion reaches a certain limit w. Stewart and Val(2003) has developed the corrosion damage time function:

[12] $$G\left(w,t\right)=T_s-t=\left(T_i+T_{sp}-t\right)$$

where T_i is the time to corrosion initiation; T_{sp} is the time from corrosion initiation to sever cracking.

The probability of the corrosion failure in the time interval (0,*t*) which can be called the corrosion failure probability, is equal to the probability of corrosion-induced cover crack width reaches *w*. The probability can be expressed as:

[13] $$p(w,t) = p\left[G(w,t)<0\right]$$

5 LIFE-CYCLE COST ANALYSIS

5.1 Life-Cycle Cost

All attributes and consequences concerning a structure such as design, construction, inspections, maintenance, failure, can be expressed in monetary terms. According to the research results in the literature(Peng and Stewart, 2008), the effect of carbonation-induced corrosion for RC beams is negligible for flexural and shear limit states. That is to say, the probabilities of failure for ultimate limit states will be very low compared to probabilities of serviceability failure. Hence, in the following analysis serviceability failure caused by severe cracking is considered as the most influential mode of failure for the estimation of life-cycle costs for RC structures. The life-cycle cost of a structure to time T, *LCC(T)*, may be expressed as:

[14] $$LCC(T) = C_C + C_{IN}(T) + C_M(T) + E_{SF}(T)$$

where C_C is the initial construction cost which is a one-time cost can be calculated as the summation of costs due to design and construction; $C_{IN}(T)$ the cost of inspections; $C_M(T)$ the expected cost of maintenance; $E_{SF}(T)$ is the expected cost of severe cracking during service life *T*, including damages, cost of life, injury, user delay, etc.

Costs may occur at any time during the lifetime, so in order to obtain consistent results it is necessary for all costs to be discounted to a present value. Discount rates are influenced by a number of economic, social and political factors, it is generally ranging from 4% to 10% in practice.

5.2 Evaluation of Expected Costs of Sever Cracking

The expected cost of sever cracking can be estimated as(Val and Stewart, 2003):

[15] $$E_{SF}(T) = \sum_{i=1}^{T/\Delta t} \Delta P_{f,i} \frac{C_{SF}}{(1+r)^{i\Delta t}}$$

where Δt is the time between inspections; C_{SF} is the cost associated with the occurrence of sever cracking (i.e. repair, user losses, etc.); r is the discount rate; $\Delta P_{f,i}$ is the probability of a sever cracking incident between the *(i-1)*-th and *i*-th inspections, which can be calculated by the following recursive formula

[16] $$\Delta P_{f,i} = \left\{P_f(i\Delta t) - P_f\left[(i-1)t\right]\right\} + \sum_{j=1}^{i-1} \Delta P_{f,j}\left\{P_f\left[(i-j)\Delta t\right] - P_f\left[(i-j)-1\Delta t\right]\right\}$$

where $P_f(t)$ is the cumulative distribution function for the time of first sever cracking.

According to the above repair strategy assumption, the maximum possible number of sever cracking incidents is equal to the number of inspections during service life of the structure, i.e. $n_{\max} = T / \Delta t$.

For the conditional designed reinforced concrete beams, the probability got from the Equation 16 is specific to the reinforcements in the section corner for the anticipation. For the equal durability designed section, the bars in the corner (FRP bars) could not corrode, so the probability got from Equation 16 is specific to the reinforcements in the edge.

6 ESTIMATION OF EXPECTED LIFE-CYCLE COST

6.1 Design Strategis

Two design strategies are considered in this paper: the first one is traditionally designed, and the second one is equal durability designed (Figure 2). For the traditionally designed concrete beam, the durability of the corner is relatively weaker than that of the other parts. So the corners are the reference points. For the equal durability designed concrete beam which shown as the Figure 2, the reinforcements in the corner would no longer corrode. So the reference point is the edge of beam.

In this paper, it is assumed that the diameters of the reinforcements are the same-20mm. The strength grade of the concrete is C25 and the concrete compressive strength is 16.7MPa.

6.2 Relative Data

The construction cost C_C including the costs of material and labour is a one-time cost in early stages of construction. It is assumed to be C_{C1} for the conditional designed reinforcement concrete beam (design strategy I) and C_{C2} for the equal durability designed reinforcement concrete beam(design strategy II). Other costs are all normalized to construction cost.

It is assumed that the inspection cost is 0.5% of the construction cost based on a routine inspection every 1 year. The maintenance cost is assumed to be 1% of the construction cost.

The cost associated with the occurrence of sever cracking contains the repair costs, user losses, etc. In some circumstances, user losses are much greater than direct repair or replacement. It is difficult to make generalizations about these costs. It is assumed that the cost due to sever cracking is equal to 20% of the construction cost.

The discount rate is assumed to be 4% in this paper, and the service life is considered to be 50 years.

Statistical parameters of random variables related to the models of corrosion initiation, propagation and sever cracking are summarized in Table 1, which are collected from Li etc.(1997).

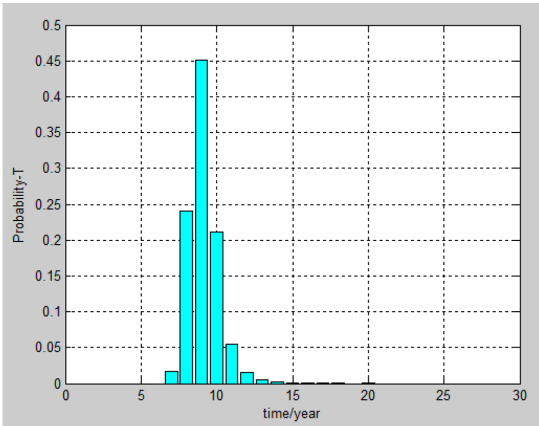
Table 1: Statistical parameters of random variables(ratio of actual value to design value)

PARAMETER	MEAN	VAR	DISTRIBUTION
Diameter of Reinforcements	1	0.0247	normal
C25 Concrete Comprehensive Strength	1.5868	0.3059	normal
Concrete Cover Depth	1.0178	0.0504	normal

6.3 Service life of the Two Strategies

The three times, including corrosion initiation time, cracking initiation time and sever cracking time, could be inferred from Monte Carlo simulation.

The simulation results of service life of the two strategies (cover depth are all 30mm) could be showed as Figure 5 and Figure 6. Results of three times of two design strategies are shown in Table 2.



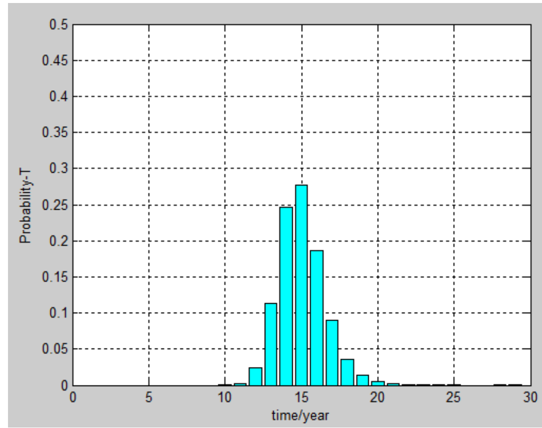


Figure 5: Service Life probability density of strategy I   Figure 6: Service Life probability density of strategy II

Table 2: Three Times of the Two Design Strategy (Cover Depth 30mm)

	Strategy 1		Strategy 2	
time/year	MEAN	VAR	MEAN	VAR
Ti	4.99	0.245	10.00	0.996
Tcr1	0.57	1.35e-8	0.73	1.44e-8
Tcr	2.98	0.592	3.91	1.129
T	8.55	1.009	14.64	2.527

It can be seen from Figure 5, Figure 6 and Table2 that the service life of the equal durability designed beam is much longer than that of the traditional designed beam. The mean service life of design strategy II is about 1.712 times that of the design strategy I. That is to say, the durability performance of the strategy II is better than that of the strategy I. The first repair would be delayed by equal durability design.

6.4 Present Value of Life-Cycle Costs

The present value of life-cycle costs of two design strategies (cover depth are all 30mm) is shown as Table 3. It can be seen from this table that the failure cost and the life-cycle cost are related with the construction cost. The proportion of strategy II (0.2050) is smaller than that of strategy I (0.3867). So it is economical for the equal durability design if the construction cost is not 1.119 (1.7089/1.5272) times greater than that of traditional designed beam.

Table 3: Present Value of Life-Cycle Cost (Cover Depth 30mm)

	Strategy I	Strategy II
Construction Cost	C_{C1}	C_{C2}
Inspection Cost	$0.1074C_{C1}$	$0.1074C_{C2}$
Maintenance Cost	$0.2148C_{C1}$	$0.2148C_{C2}$
Failure Cost	$0.3867C_{C1}$	$0.2050C_{C2}$
Life-Cycle Cost	$1.7089C_{C1}$	$1.5272C_{C2}$

6.5 Effects of Cover Depth on Life-Cycle cost

The present value of life-cycle costs for two design strategies are shown as Table 4 and Table 5. It can be seen that when the cover depths increases, the life-cycle cost decreases for both strategies.

Table 4: Present Value of Cost for Design Strategy I

	25mm	30mm	35mm
Construction Cost	C_{C1}	C_{C1}	C_{C1}
Inspection Cost	$0.1074C_{C1}$	$0.1074C_{C1}$	$0.1074C_{C1}$
Maintenance Cost	$0.2148C_{C1}$	$0.2148C_{C1}$	$0.2148C_{C1}$
Failure Cost	$0.5216C_{C1}$	$0.3867C_{C1}$	$0.2923C_{C1}$
Life-Cycle Cost	$1.8438C_{C1}$	$1.7089C_{C1}$	$1.6145C_{C1}$

Table 5: Present Value of Cost for Design Strategy II

	25mm	30mm	35mm
Construction Cost	C_{C2}	C_{C2}	C_{C2}
Inspection Cost	$0.1074C_{C2}$	$0.1074C_{C2}$	$0.1074C_{C2}$
Maintenance Cost	$0.2148C_{C2}$	$0.2148C_{C2}$	$0.2148C_{C2}$
Failure Cost	$0.2955C_{C2}$	$0.2050C_{C2}$	$0.1381C_{C2}$
Life-Cycle Cost	$1.6177C_{C2}$	$1.5272C_{C2}$	$1.4603C_{C2}$

The comparison of life-cycle costs between traditional design and equal durability design is shown as Table 6. The critical proportion is referred to C_{C2}/ C_{C1}. If the construction cost of strategy II is not larger than this proportion, it is economical to use equal durability design method. It can be seen that as the cover depth increases, the critical proportion decreases. That is to say, the equal durability design method would be more dominant when the cover depth is smaller. When the cover depth increases, the construction cost is more limited for the equal durability design.

Table 6 Present Value of Life-Cycle Cost for Both Strategies

	25mm	30mm	35mm
Strategy1	$1.8438C_{C1}$	$1.7089C_{C1}$	$1.6145C_{C1}$
Strategy2	$1.6177C_{C2}$	$1.5272C_{C2}$	$1.4603C_{C2}$
Critical Proportion	1.140	1.119	1.106

7 CONCLUSION

The present paper focused on the life-cycle serviceability reliability of concrete beam. A comparative life-cycle analysis between traditionally designed concrete beam and equal durability designed concrete beam is presented in conjunction with a probabilistic life-cycle cost analysis. Probabilistic corrosion initiation time, initial cracking time and sever cracking time are considered in the sever-cracking failure model. Using the developed framework, the life-cycle cost of both design strategies are evaluated. The service life would be extended using the equal durability design method. The failure cost of the equal durability designed beam is comparatively smaller than that of the traditionally designed beam. The life-cycle cost analysis showed that it is economical if the construction increasement of the equal durability design is controlled in a range. The equal durability design got more advantages when the concrete cover depth decreases.

REFERENCES

Alipour, A., Shafei, B., and Shinozuka, M. 2010. Life Cycle Cost Analysis of Reinforced Concrete Highway Bridges in Chloride Contaminated Environments. *Structures Congress*, Orlando, Florida, United States.

Andrade, C., Alonso, C., and Molina, F.J. 1993. Cover Cracking as a Function of Rebar Corrosion: Part 1-Experimental Test. *Materials and Structures*, 26:453-441.

El Maaddawy, T., and Soudki, K. 2007. A Model for Prediction of Time from Corrosion Initiation to Corrosion Cracking. *Cement & Concrete Composite*, 29:168-175.

Li, Y.H., Bao, W.G., Guo, X.W., and Cheng, X.Y. 1997. Highway Bridge Reliability and Probability Limit State Design. (In Chinese) *China Communication Press*, Beijing, China.

Mullard, J.A., and Stewart, M.G. 2009. Corrosion-Induced Cover Cracking of RC Structures: New Experimental Data and Predictive Models. *Research report No. 275.05.2009. ISBN No. 9870 9806 0345 3.* The University of Newcastle, Australia

Otieno, M., Beushausen, H., and Alexander, M. 2012. Prediction of Corrosion Rate in Reinforced Concrete Structures – a Critical Review and Preliminary Results. *Materials and Corrosion*, 63(9): 777-790.

Peng, J.X., and Stewart, M.G. 2008. Carbonation-induced corrosion damage and structural safety for concrete structures under enhanced greenhouse conditions. *Research report No. 270.11.2008. ISBN No. 9870 9805 03555*, The University of Newcastle, Australia.

Qu, W.J. 1995. Durability Evaluating and Life Predicting on Existing Concrete Bridges. (In Chinese) *Southwest Jiaotong University*, Chengdu, China.

Reale, T., and O'Connor, A. 2012. A Review and Comparative Analysis of Corrosion-Induced time to First Crack Models. *Construction and Building Materials*, 36:475-483.

Stewart, M.G., and Val, D.V. 2003. Multiple Limit States and Expected Failure Costs for Deteriorating Reinforced Concrete Bridges. *Journal of Bridge Engineering*, 8(6): 405-415

Tuutti, K. 1982. Corrosion of steel in concrete. *Swedish Cement and Concrete Research Institute*, Stockholm, Sweden.

Val, D.V., and Stewart, M.G. 2003. Life-Cycle Cost Analysis of Reinforced Concrete Structures in Marine Environments. *Structural Safety*, 25:343-362

Yalsyn, H., and Ergun, M. 1996. The prediction of corrosion rates of reinforcing steels in concrete. *Cement and Concrete Research*, 26(10): 1593-1599.

Zhang, Y., and Jiang, X.L. 1998. Practical Mathematical Model of Concrete Carbonation Depth based on the Mechanism of Carbonation. (In Chinese) *Industrial Construction*, 28(1): 16-19, 47.