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Lateral-Torsional Buckling of Steel Web-Tapered Beam-Columns: Further Analytical Studies

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Abstract: Solutions for the inelastic lateral-torsional buckling of steel web tapered beam-columns are presented. The beam-columns are subjected to an axial force and to bending moments applied at one end of the member only or to both ends. Three different computational procedures are utilized; the first method is based on the finite difference method using a direct discretization of the differential equations of lateral-torsional buckling. The coefficients appearing in the finite difference equations are determined considering the reductions of the flexural and torsional stiffness due to yielding in the inelastic range. The effects of residual stresses are included. The resulting simultaneous equations are then set up to compute the buckling determinant which yields the critical load. The second method investigated is based on the finite element method using commercially available software ANSYS. The following parameters are studied: residual stresses, initial imperfections, elastic and inelastic analyses, and different support conditions. The results of the numerical solutions are presented and compared to the design equations in the last AISC Specification that explicitly addresses tapered members.

1 Introduction

Web-tapered members are structural members commonly used in typical one-story pre-engineered buildings. Appreciable savings in materials and in the cost of structural framing can be assumed by the use of elements having a tapering depth or flanges.

In the United States of America, the last specification that addressed tapered members is the 1999 American Institute of Steel Construction Specification (1999) for web tapered members which was based on a study performed in 1972. The contributors to the study were the Column Research Council, presently known as the Structural Stability Research Council, and the Welding Research Council, under the technical guidance of Lee *et al.* (1972) at the University of New York at Buffalo. The general design approach used in the 1999 Specification is to apply modification factors to convert the tapered members into appropriately proportioned prismatic members so that the prismatic AISC equations may be applied. From the practitioner's point of view, the 1999 AISC design equations for tapered members represent the use of existing basic formulas for prismatic members altered with the use of an additional factor. Furthermore, the additional factor will give the designer an inherent feeling for the increase in strength over a prismatic section.

At the same time, the "easy to use" 1999 AISC Specification (1999) is restricted to doubly symmetric I-shaped sections. The reason for this limitation was the inability to uncouple the torsional and flexural deformations due to varying locations of the shear center for singly-symmetric sections during Lee's study. The development was also limited to members with small tapering angles. This limitation was investigated in Lee's studies (1972) when he compared the normal stresses obtained using the prismatic analogy with an exact elastic solution using a tapered geometry. For practical considerations, the limiting tapering ratio has been restricted to 6. Moreover, the development is focused to members with flanges of an equal and constant area with webs that are not slender. However, what is of interest is that the current practice in the low-rise metal building industry is the use of flanges of unequal area and slender webs. Therefore, the 1999 AISC Specification (1999) does not appear to provide equations for web-tapered I-shaped beam geometries of proportions that are consistent with what has been the industry standard for metal buildings.

Jimenez (1998) and other researchers have performed studies on the topic of inelastic stability of tapered members and have shown that the 1999 AISC specification (1999) predict un-conservative results when determining the lateral-torsional buckling strength of tapered beams and beam-columns for certain slenderness values of typical tapered members since the equations provided in the 1999 specification for tapered members are identical to the equations developed by Lee in 1972. Figure 8 depicts an example of this un-conservative condition. The most recent version of the AISC Specification (2010) does not explicitly define the use of the AISC provisions for tapered members leaving the design professional without guidance on how to address this type of structural elements.

Recognizing this important need, the American Institute of Steel Construction launched a design guide – Steel Design Guide 25 (2011) devoted specifically for the design of tapered members. Additional verifications with other versions of the steel code as well as experimental results are needed to verify their full applicability.

The general behavior of a typical beam-column is illustrated in Figure 1, where the relationship between the applied end-moment M_o and the resulting end-slope θ is shown for a wide-flange member bent about its strong-axis, in which the length as well as the axial force P is assumed to remain constant as the moment M_o is increased from zero to its maximum value and past the maximum moment into the unloading zone.

The optimum performance of the beam-column is reached if failure is due to excessive bending in the plane of the applied moment, and this behavior is represented by the upper branch of the curve in Figure 1. The corresponding maximum moment is M_{o1max}. If no lateral bracing is provided, failure will be due to lateral-torsional buckling and the resulting moment is M_{o2max} represented by the lower branch of the curve in Figure 1. The additional incremental moment represented by M_{o2max} beyond M_{ocr} is small, and, therefore, the bifurcation point is considered to reasonably determine the buckling limit to the beam-column. The work described in this paper deals with the determination of the value of M_{ocr} for web-tapered beam-columns.

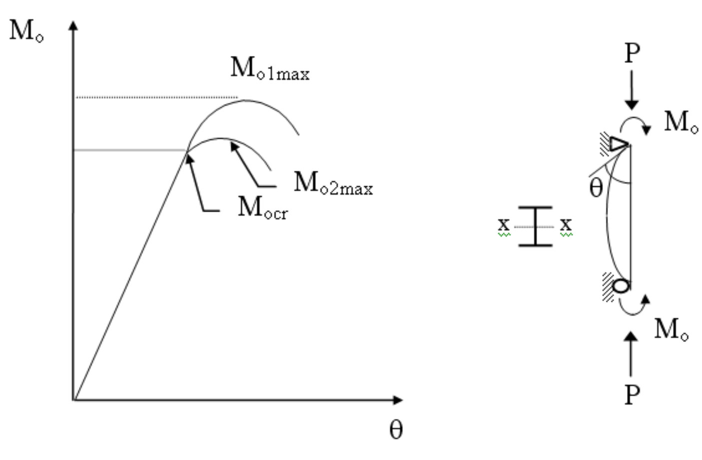


Figure 1: M-Θ curves for beam-columns

2 Differential Equations of Lateral-Torsional Buckling

The differential equations governing the lateral-torsional buckling of tapered members subjected to centroidal axial forces P and to end moments M_o and ρM_o are given in Jimenez (1998) and are repeated here for convenience:

$$\text{[1a]} \qquad B_x(z)\frac{d^2 v}{dz^2} + Pv - M_o\left[\rho + (1-\rho)\frac{z}{L}\right] = 0$$

$$\text{[1b]} \qquad B_y\frac{d^2 u}{dz^2} + Pu - \beta\left[M_o\left\{\rho + (1-\rho)\frac{z}{L}\right\} - P\ y_o(z)\right] = 0$$

$$\text{[1c]} \qquad C_w(z)\frac{d^3\beta}{dz^3} - \left[C_T(z) - \overline{K}(z)\right]\frac{d\beta}{dz} - \left[M_o\left\{\rho + (1-\rho)\frac{z}{L}\right\} - P\ y_o(z)\right]\frac{du}{dz} +$$

$$M_o\frac{u}{L}(1-\rho) = 0$$

The beam-column prescribed by the above differential equations is shown in Figure 2. It is subjected to end bending moments M_o at z = L and ρM_o at z = 0, where “z” is the coordinate axis along the undeformed, centroidal axis and “L” is the length of the member. The coefficient “ρ” is the ratio of the end moments. The deformations of the shear center are: “*u*” in the x-direction, “*v*” in the y-direction and the cross-section twists about the shear center an angle “β”. In Figure the smaller end will be denoted as end A and the larger end as end B.

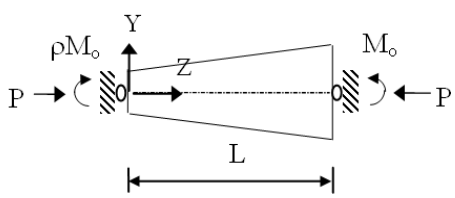


Figure 2: Loading condition

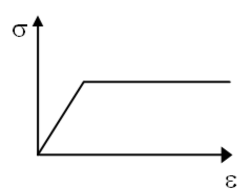


Figure 3: Stress-strain diagram

The stress-strain diagram of the material is shown in Figure 3. The coefficients $B_x(z)$, B_y, $C_T(z)$, $C_w(z)$, $y_o(z)$ and $\overline{K}(z)$ in the differential equations are defined as follows: $B_x(z)$ is the bending stiffness about the x-axis; B_y is the bending stiffness about the y-axis; $C_T(z)$ is the St. Venant’s torsional stiffness; $C_w(z)$ is the Warping stiffness; $y_o(z)$ is the distance between the centroid “C” and the shear center “S” in the plane of symmetry; $\overline{K}(z) = \int_A \sigma s^2 dA$ where: σ = is the stress on any cross-sectional element *d*A (positive in compression) and “s” is the distance of element *d*A from the shear center. These coefficients vary with respect to the coordinate “z” to account for the non-uniform variation of the cross-section properties along the length of the column. Also, when the beam-column is in the inelastic range the coefficients will vary with the different patterns of the yielding.

3 Design Strength of Tapered Members using Advanced Analysis

Solutions for the elastic and inelastic lateral-torsional buckling of steel web tapered beam-columns were computed using advanced analyses. The beam-column elements are subjected to an axial force and to bending moments applied at both ends of the member. A computational procedure based on the finite difference method using a direct discretization of the differential equations of lateral-torsional buckling was utilized using a Fortran program developed by the author performing a lateral-torsional analysis using equations (1b) and (1c) and the in-plane strength check utilizing equation (1a) through a numerical integration routine. The coefficients appearing in the finite difference equations are determined considering the reductions of the flexural and torsional stiffness due to yielding in the inelastic range. The effects of residual stresses are included. The resulting simultaneous equations are then set up to compute the buckling determinant which yields the critical load.

The lateral-torsional buckling of tapered beam-columns is determined by using equations (1b) and (1c) where the cross-section coefficients are variable with respect to "z". The finite difference equations corresponding to the equations (1b) and (1c) at each station by first-order central differences becomes:

[2a] $$u_{i-1}[\mathrm{By}(i)]+u_i\left[\mathrm{P}h^2-2\mathrm{By}(i)\right]+u_{i+1}[\mathrm{By}(i)]+\beta_i\left[\lambda(i)h^2\right]=0$$

[2b] $$u_{i-1}\left[-\lambda(i)h^2\right]+u_i\left[\frac{2\mathrm{Mo}(1-\rho)h^2}{n}\right]+u_{i+1}\left[\lambda(i)h^2\right]+\beta_{i-2}\left[-\mathrm{C_w}(i)\right]+$$

[2c] $$\beta_{i-1}\left[2\mathrm{C_w}(i)+\mathrm{C_T}(i)h^2\left(1-\frac{\overline{\mathrm{K}}(i)}{\mathrm{C_T}(i)}\right)\right]+\beta_{i+1}\left[-\left\{2\mathrm{C_w}(i)+\mathrm{C_T}(i)h^2\left(1-\frac{\overline{\mathrm{K}}(i)}{\mathrm{C_T}(i)}\right)\right\}\right]+$$

$$\beta_{i+2}\left[\mathrm{C_w}(i)\right]=0$$

$$\text{where}: \lambda(i)=\mathrm{Py_o}(i)-\mathrm{M}(i), \qquad \mathrm{M}(i)=\mathrm{Mo}\left[\rho+(1-\rho)\frac{i}{n}\right]$$

The ends of the beam-column are allowed to rotate, the end sections are free to warp, and the ends of the member are not permitted to twist or to translate. These boundary conditions can be written as follows: $u_o = 0$, $u_n = 0$, $\beta_{-1} = -\beta_1$, $\beta_{n+1} = -\beta_{n-1}$, $\beta_o = 0$, $\beta_n = 0$.

This leads to a set of simultaneous algebraic equations in the lateral displacement u and the rotation β at a number of discrete points spaced at $h = L/n$, in which n is an odd number to which the beam-column is divided. This set of simultaneous equations may be written in matrix form; In this equation the matrix $[\mathrm{A}]\begin{Bmatrix} u \\ \beta \end{Bmatrix}=0$, is a set of the coefficients A_{ij} representing combinations of the cross-section properties (B_y, $C_T(z)$, $C_w(z)$, $y_o(z)$, and $\overline{\mathrm{K}}(z)$, the load parameters (P and M_o) and the length of the member (L). In order to compute the stiffness of a cross-section it is necessary to know how much of the section is plastic and how much of the section is elastic, and where the corresponding regions are located on the cross-section. The non-dimensionalized M/M_y, ϕ/ϕ_y, P/P_y, relationships about the strong-axis for an I-shape section have been determined by Jimenez (1998). Figure 4 shows these relationships for the following cases of yielding:

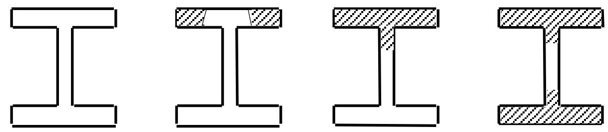
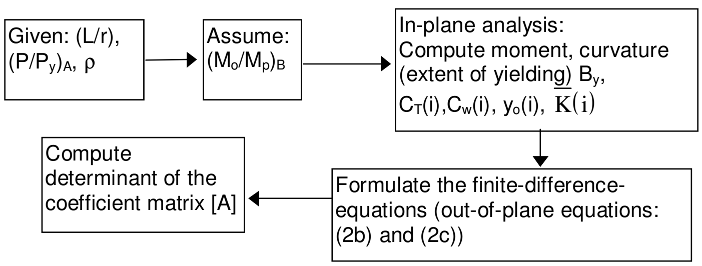


Figure 4: Yielded patterns for wide-flange cross-section.

In outline form, the steps that are used in computing the critical moment M_{ocr} for steel web-tapered beam-columns are as follows:



This process is repeated for different load levels until a zero value for the determinant is found.

To create the finite element model using ANSYS (2010), a commercially available finite element program, several steps had to be performed including element selection, laying out the mesh and determining boundary conditions. The finite element mesh is comprised of BEAM188 elements. BEAM188 elements are suitable for analyzing slender to moderately stubby/thick structures. This element is based on Timoshenko beam theory. Shear deformation effects are included. The BEAM188 is a quadratic beam element in 3-D. This element is well-suited for linear, large rotation, and/or large strain nonlinear applications. Furthermore, the provided stress stiffness terms enable the elements to analyze flexural, lateral, and torsional stability. The cross-section associated with the element may be linearly tapered. Elasticity and plasticity models are supported.

4 Comparison between Advanced Analysis and the AISC Specification

Solutions for the elastic and inelastic lateral-torsional buckling of steel web tapered beam-columns were computed using both the finite difference method and a commercially available finite element program. The beam-column elements are subjected to an axial force and to equal bending moments applied at both ends of the member. Figure 5 compares the Finite Difference (FD) solution with the ANSYS solution for a typical tapered beam-column subjected to the forces shown. In this case γ represents the taper ratio, L/r_x represents the slenderness parameter about the x axis, r_x is the radius of gyration about the x axis, and M_p is the plastic moment.

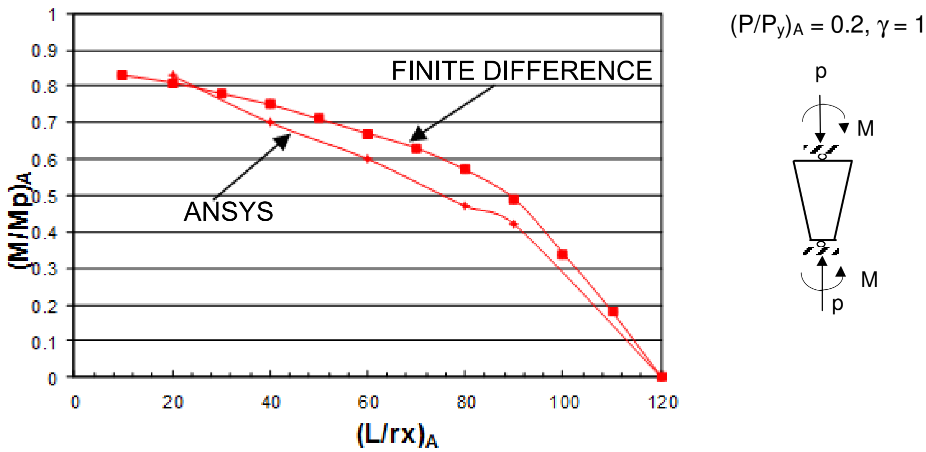


Figure 5: Comparison between FD and ANSYS models (LTB of Tapered Beam-Column).

It appears that the ANSYS solution produces more conservative results for the slenderness ratios between 40 to 90. This behavior is due to the gradual yielding in the ANSYS model versus the four defined yielded patterns from Figure 4 utilized in the FD method. Figure 6 corresponds to the overall buckled shape of the tapered beam-column as depicted by ANSYS. Yielding of both flanges at the smaller end is evident. A close-up look of the smaller end is depicted in Figure 7.

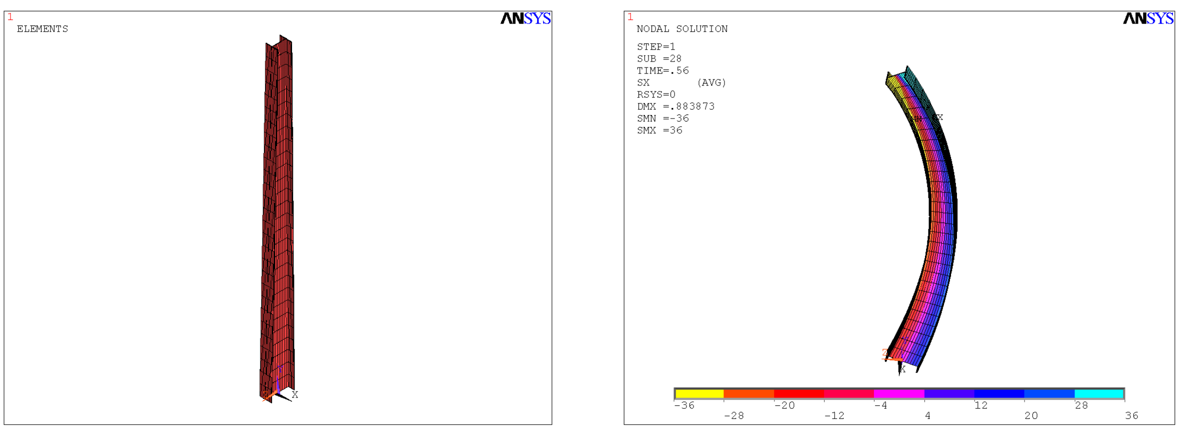


Figure 6: Overall view of tapered member before and after buckling.

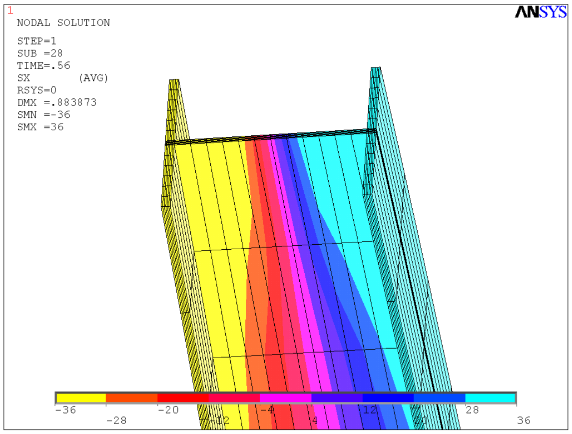


Figure 7: Close up view of yielded flanges/web at the smaller end of a tapered beam-column.

Comparisons were made between the 1999 AISC-LRFD code and this study. Figure 8 illustrates a typical case of a tapered beam-column subjected to compressive axial load and end moments for lateral-torsional buckling (LTB). The beam-column problem is treated in the 1999 Specification in the form of an interaction equation. It can be seen that for values of M/M_p greater than about 0.4 the predictions of the interaction equation are unsafe. The un-conservative results predicted by the use of the beam equation in the AISC Specification are typical for different tapering ratios with unsafe discrepancies up to 25 % between the advanced analysis and the Specification.

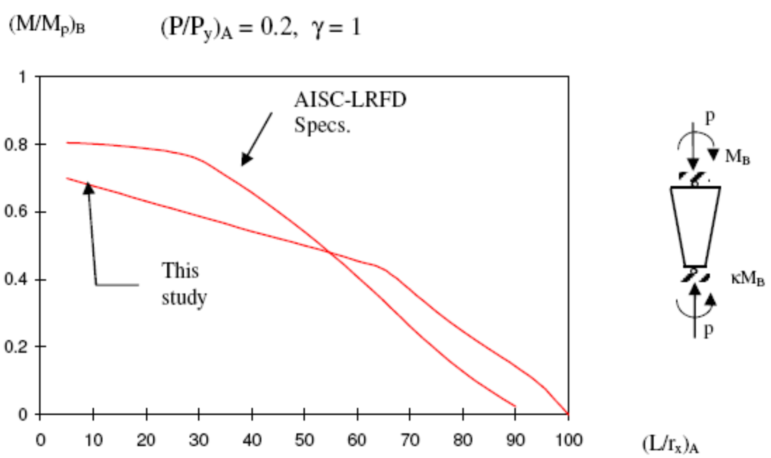


Figure 8: LTB strength of beam-column.

5 Conclusions

This paper presents studies for the out-of-plane behavior of tapered beam-columns using ANSYS (2010) and the Finite Difference method. It was shown that the ANSYS solution produces slightly more conservative results due to the progressive yielding of the flanges and web. It was found that for medium to short beams and beam-columns, the 1999 Specification (1999) predicts strengths on the non-conservative side, with maximum discrepancies of about 25% between advanced analysis approaches and those given the specifications. Furthermore, additional studies are needed to evaluate the use of the 2010 AISC provisions (2010) combined with the Design Guide 25 (2011) with previous specifications for tapered members.

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