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Shear Resistance of Circular Concrete Members Reinforced with FRP Bars: Code Predictions and Numerical Analysis

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Abstract: Reinforced concrete (RC) members of circular cross-section are widely used in different types of structures such as piles and bridge pier columns. The noncorrosive fiber-reinforced polymer (FRP) bars are becoming increasingly needed in reinforcing concrete piles in aggressive environments. These piles are usually subjected to considerable shear stresses resulting from the lateral loads. The effective shear area of the circular concrete section reinforced with longitudinal FRP bars has not yet been investigated. In this paper, the FRP shear design methods were reviewed. These methods include the *American Concrete Institute design guide, ACI 440.1R-06*; *the Canadian Standards Association, CAN/CSA S806-12*; and the design recommendations of the *Japan Society of Civil Engineers (JSCE)*. The three methods for shear design prescribed in these guidelines were compared with experimental database obtained from previous studies on circular FRP-RC members. The results of comparison indicated that the CAN/CSA S806-12 predictions were close to the experimental results, while the other two design methods were over conservative. Also, this paper presents an introduction to the finite element modeling of concrete beams reinforced longitudinally with FRP bars.

1. Introduction and Literature Review

Traditional pile materials for bridge foundations and waterfront structures include steel, concrete, and timber. These pile materials have limited service life and high maintenance costs when used in harsh marine environments due to corrosion. The high cost associated with the corrosion of steel reinforcement command a solution that will attack the problem from its root cause. The use of fibre-reinforced polymers (FRPs) has increased during the last decade. Known to be corrosion resistant, FRP reinforcing bars provides a great alternative to steel reinforcement. FRP materials in general offer many advantages over the conventional steel, including one quarter to one fifth the density of steel, no corrosion even in harsh chemical environments, and greater tensile strength than steel (Benmokrane et al. 1995).

In fact, extensive research works have been conducted to investigate the behavior of concrete beams reinforced with FRP bars that have rectangular cross section. In addition, all the codes and guidelines provide flexural and shear design provisions and equations for the reinforced concrete beams based on they have rectangular cross section. This resulted from the common practical use of the rectangular reinforced concrete (RC) beams in civil engineering structures. Circular axi-symmetric flexural members are desirable in certain applications, such as concrete piles, utility and light poles, highway overhead sign structures, fender piles, bore piles, and bridge pier columns. As a result of lateral loads resulted from wind pressure, earth pressure, earthquake or vehicle impact, these piles are subjected to considerable shear loads. Thus, the pile should be inevitable designed to suppress a possible shear failure. The section geometry (rectangular, T or circular cross-section) strongly influences the member shear capacity, which determine the area that effectively resists the external shear load. For rectangular sections, the effective shear area is clearly defined as the area corresponding to the effective depth. This definition is not so distinctive in case of circular section (Merta and Kolbitsch 2006). The analysis of the circular cross-section is more complex than that the rectangular. The stresses, which are variable over the section depth, are also distributed along an area of variable width (Mohamed 2010). In addition, the bars are usually arranged throughout the depth, such that the cross-sectional area of reinforcement at any given depth is more difficult to calculate than in conventional RC sections. The American Concrete Institute (ACI 318M-11) recommends to estimate the effective shear area of the circular cross-section as the product of the diameter and the effective depth, whereas the effective depth is permitted to be taken as at least 0.8 times the diameter. In general, the CAN/CSA S806-12 recommends using the effective shear depth for RC members to be taken as the greater of 0.9 the effective depth or 0.72 the total thickness. Also, limited studies have investigated the shear models of circular RC members. Clarke and Birjandi (1993) has proposed to use the same shear design approach as given by the British Codes of Practice, BS 5400 for rectangular sections with a modification in the effective depth as the distance from the extreme compression fiber to the centroid of the tension reinforcement. The effective shear area is then defined as the area corresponding to the effective depth.

Previous research work on the FRP RC flexural members without shear reinforcement has indicated that the concrete shear strength can be evaluated by taking into account the axial stiffness of the tensile reinforcement. Yost et al. (2001) investigated the shear capacity, V_{cf}, of concrete beams subjected to four-point bending and reinforced with glass FRP (GFRP) bars. Six different reinforcement ratios ranging from 2.10 to 4.32 times the balance reinforcement ratio, along with one steel design were tested. For each design, three identical specimens were fabricated, such that a total of 21 beams were tested. No shear or compression reinforcement was provided in the test specimens. The total length and depth of the test specimens were 2286 and 286 mm, respectively, while the width varied between 178 and 254 mm. The shear span-to-depth ratio for all test beams was approximately 4.0. The test results indicated that the shear strength was found to be independent of the amount of longitudinal GFRP reinforcement.

Razaqpur et al. (2004) tested seven beams in four-point bending to determine the concrete contribution to their shear resistance. The beams were reinforced only in the longitudinal direction with carbon FRP (CFRP) bars and had no shear reinforcement. The beams measured 2262 mm long, 225 mm depth and 200 mm wide. The test variables were the shear span-to-depth ratio, varying from 1.82 to 4.5, and the flexural reinforcement ratio, varying from 1.1 to 3.88 times the balanced strain ratio. The test results were analyzed and compared with the ACI440.1R-03, CAN/CSA S806-02, and the Japan Society of Civil Engineers (JSCE) FRP design recommendations. It was concluded that the ACI recommendations were extremely conservative whereas the Canadian and JSCE recommendations, albeit still conservative, were in closer agreement with the experimental data. Also, it was stated that the CAN/CSA S806-02 predictions were in a better agreement with experimental data than the JSCE predictions.

El-sayed et al. (2006) investigated the behavior and shear strength of concrete slender beams reinforced with FRP bars. A total of nine large-scale reinforced concrete beams without stirrups were constructed and tested up to failure. The beams measured 3250 mm long, 250 mm wide, and 400 mm deep and were tested in four-point bending. The test variables were the reinforcement ratio and modulus of elasticity of longitudinal reinforcing bars. The test specimens included three beams reinforced with GFRP bars, three with CFRP bars, and three control beams reinforced with conventional steel bars. The test results were compared with code prediction values, and design guidelines. It was found that the relatively low modulus of elasticity of FRP bars reduced the shear strength compared to the shear strength of the control steel RC beams. Also, it was concluded that the ACI 440.1R-03 design method provided very conservative predictions, particularly for beams reinforced with GFRP bars. In 2011, El-Sayed and Soudki used 112 shear test results of the available experimental database in the literature to evaluate the shear design provisions of ACI 440.1R-06; CAN/CSA-S806-02; the ISIS Canada design manual (ISIS-M03- 07); the BISE guidelines; and the design recommendations of the JSCE. The outcome of that study indicated that the average predictions of the concrete contribution to the shear resistance using the five methods varied by more than 70%. More accurate predictions were obtained using methods that account for the effect of axial stiffness of longitudinal bars as raised to a power of 1/3 in their equations. These methods included CSA-S806, BISE, and JSCE.

In this paper, three FRP shear design methods were presented and evaluated to predict the concrete contribution to the shear resistance of circular concrete beams reinforced with FRP bars. These methods include the ACI 440.1R-06; CAN/CSAS806-12; and JSCE. The three methods for shear design prescribed in these guidelines were compared with experimental database obtained from the literature on circular concrete beams reinforced longitudinally with FRP bars and without shear reinforcements. In addition, a nonlinear finite element model of FRP beams was developed using the commercial software ADINA finite element program to simulate the shear behavior of the circular concrete beams reinforced with GFRP.

2. Experimental Database

To evaluate the code design equations, the experimental database of 6 circular FRP-RC beams were used. A selected 5 tested circular RC beam specimens that have been investigated by SHI Xiao et al. (2012) were included in the experimental database. These beams were prepared without stirrups and were reinforced longitudinally with FRP bars. The five circular beams had the same concrete strength (42 MPa), reinforcement type and dimensions. These beams were prepared with two variables namely: longitudinal reinforcement ratio (ρ_f) and shear span to depth ratio (*a/d*). GFRP bars (12 mm diameter) were used to reinforce the beam specimens. The manufacturer specified tensile strength and modulus of elasticity of these bars were 541.80 MPa, and 33.2 GPa, respectively. The beam specimens were simply supported and had three different total lengths, namely 1200, 900 and 750mm, corresponding to the clear test span of 810, 675 and 530 mm, respectively, with constant diameter equal to 300 mm. The shear span to depth ratios were 0.66, 0.86 and 1.0. Also, the longitudinal reinforcement ratios were 1.92, 2.24 and 2.54%. It was found that the mode of failure was the type of shear failure with several major inclined cracks extended from the loading point to the support. Table 1 shows the details of the specimens included in the database.

Mohamed 2010 conducted an experimental investigation to study the flexural and shear behavior of 10 beam specimens reinforced internally with steel or FRP bars and with or without stirrups, and also using FRP tube. One control specimen was prepared without shear reinforcement and reinforced longitudinal with GFRP bars that failed in shear as shown in Figure 1. The beam had a total length of 1920 mm and was tested under four-point bending over a simple supported with shear span equal to 640 mm. The depth of the cross-section of the beam was 203 mm. Six sand coated GFRP bars (16 mm diameter) were used to reinforce the beam specimen. GFRP bars properties had tensile strength and modulus of elasticity 683 MPa and 48.2 GPa, respectively. The experimental observation showed that crack formation was initiated in the flexural span between the two concentrated loads where the flexural stress is highest and shear stress is zero. The cracks were vertical perpendicular to the direction of the maximum principle tensile stress induced by pure bending. As the load increased, additional flexural cracks opened within the shear span, and hence, diagonal tension failure at the shear span was the final failure mode (Mohamed 2010).

Table 1. Experimental database of test specimens without shear reinforcement

Reference	Specimens No.	f'_c *(MPa)*	b_w *(mm)*	*D* (mm)	*a/d*	E_{fl} *(GPa)*	ρ_f %
SHI Xiao et al. (2012)	B-1	42	300	300	0.66	33.2	2.24
	B-2	42	300	300	0.86	33.2	2.24
	B-3	42	300	300	1.0	33.2	2.24
	B-4	42	300	300	1.0	33.2	1.92
	B-5	42	300	300	1.0	33.2	2.56
Mohamed (2010)	B-6	45	203	203	3.50	80	3.67

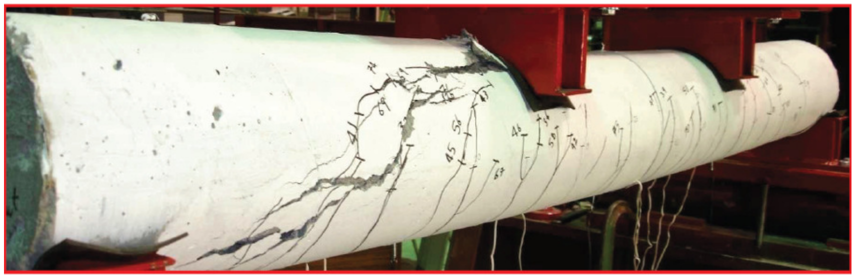

Figure 1. Shear failure mode of FRP-circular concrete beam by Mohamed (2010)

3. Review of the Current Design Provisions

The current shear design provisions for concrete structures reinforced with FRP bars are based on the design formulas of members reinforced with conventional steel bars considering some modifications to account for the substantial differences between FRP and steel reinforcement. These provisions are based on the shear capacity of a reinforced concrete beam that is taken to be the sum of the shear capacity of the concrete component V_c and the shear reinforcement component V_s.

3.1 ACI 440.1R-06 Design Guidelines

The shear resistance of concrete element V_c reinforced with FRP bars can be determined by the ACI 440.1R-06 as follows:

$$V_c = \frac{2}{5}\sqrt{f_c'}\, b_w c \tag{1}$$

where f_c' is concrete compressive strength; b_w is the beam web width; and c is cracked transformed section neutral axis depth, $c = k.d$. The factor k is calculated as follows:

$$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f \tag{2}$$

where ρ_f is the FRP reinforcement ratio and $n_f = E_f / E_c$, E_f and E_c are the modulus of elasticity of longitudinal FRP bars and concrete, respectively.

3.2 The Canadian Building Code CAN/CSA S806-12

According to the *CAN/CSA S806-12* code requirements, the concrete shear contribution can be determined as follows:

$$V_{cf} = 0.05\, \lambda\, \phi_c\, K_m K_r \left(f_c'\right)^{\frac{1}{3}}\, b_w d_v \tag{3}$$

where V_{cf} is the concrete shear contributions, b_w is the beam web width, d_v is the effective shear depth, taken as the greater of 0.9 d or 0.72 *h*, λ is a factor to account for concrete density, and ϕ_c is the concrete resistance factor. For K_m and K_r are calculated as follows:

$$K_m = \sqrt{\frac{V_f.d}{M_f}} \quad \leq \ 1.0 \tag{4}$$

$$K_r = 1 + \left(E_f \ \rho_{fw}\right)^{\frac{1}{3}} \tag{5}$$

where,

$$0.11 \ \phi_c \sqrt{f_c'} b_w d_v \leq V_{cf} \leq 0.22 \ \phi_c \sqrt{f_c'} b_w d_v \leq V_{cf} \tag{6}$$

For sections located within a distance of 2.5d from the face of a support where the support reaction causes compression in the beam parallel to the direction of the shear force at the section, the value of V_{cf} shall be calculated as the value determined according to Equation (3) multiplied by factor K_a as given in the following equation:

$$K_a = \frac{2.5}{M_m / V_f.d} \ \geq \ 1.0 \tag{7}$$

where, K_a shall not exceed than 2.5.

For members with effective depth greater than 300 mm and with no transverse shear reinforcement or less transverse shear reinforcement than minimum required by code, the value of V_{cf} shall be calculated as the value determined according to Equation (3) multiplied by factor K_s as given in the following equation:

$$K_s = \frac{750}{450 + d} \quad \leq \ 1.0 \tag{8}$$

3.3 JSCE Design Recommendations

The concrete shear capacity can be determined according to the following equation:

$$\mathrm{V_{cf}} = \ \beta_d \beta_p \beta_n f_{vcd} b_w d \, / \, \gamma_b \tag{9}$$

such that:

$$f_{vcd} = 0.2\left(f_c'\right)^{1/3} \quad \leq 0.72 \ N / mm^2 \tag{10}$$

$$\beta_d = (\frac{1000}{d})^{1/4} \quad \leq 1.5 \tag{11}$$

$$\beta_p = (100 \rho_f E_f \, / \, E_s)^{1/3} \quad \leq 1.5 \tag{12}$$

$$\beta_n = 1 + \frac{M_o}{M_d} \quad \leq 2 \qquad for \quad N'_d \geq 0 \tag{13}$$

$$\beta_n = 1 + \frac{2M_o}{M_d} \quad \geq 0 \qquad for \quad N'_d < 0 \tag{14}$$

where γ_b is the safety factor =1.3, M_o is the decompression moment, M_d is the design bending moment, and N'_d is the design axial compression force.

4. Comparison the experimental database with V_c design equations

The experimental databases were compared with the predicted shear strength values as given in Table 2. Equations 1, 3 and 9 were used to predict the concrete shear strengths of the tested circular FRP-RC beams according to the ACI 440.1R-06 design guideline, CSA-S806-12 code and JSCE code, respectively. Considering in all calculation of V_c, the material reduction and safety factors were set equal to 1.0. Also, the design axial compressive force N'_d as well as the decompression moment M_o were taken equal to zero. The ratio of experimental shear strength to predicted results V_{exp}/V_{pred} was calculated for each specimen in the database. Table 2 presents the V_{exp}/V_{pred} ratios for each specimen, and also includes the overall statistical values of average, standard deviation and coefficient of variation.

Table 2. Experimental to predicated shear strength ratios.

Reference	Beam	f'_c (MPa)	b_w (mm)	D (mm)	ρ_f %	a/d	V_{exp} (KN)	V_{exp} / V_{pred}		
								ACI 440.1R	CSA-S806-12	JSCE-1997
SHI Xiao et al. (2012)	B-1	42	300	300	2.24	0.66	150	4.0	1.21	2.79
	B-2				2.24	0.83	190	5.06	1.54	3.53
	B-3				2.24	1.0	149	3.96	1.20	2.76
	B-4				1.92	1.0	149	4.25	1.26	2.94
	B-5				2.54	1.0	168	4.23	1.29	3.01
Mohamed (2010)	B-6	45	203	203	3.67	3.5	77	2.46	1.71	1.93
Average								4.0	1.3	2.8
Standard deviation								0.85	0.21	0.52
Coefficient of variation (%)								21	15	18

From Table 2, it can be noticed that all the three design methods provided conservative predictions with the mean value of V_{exp}/V_{pred} greater than 1.0. Also, it can be seen that the design method CSA-S806-12 which account for the effect of axial stiffness of the longitudinal bars raised to a power of 1/3 in their equations provides more accurate predictions than other methods, where the mean value of V_{exp}/V_{pred} is 1.37. This result is consistent with the test results conducted by El-Sayed et al. 2006 on reinforced concrete beams without stirrups and reinforced in the longitudinal direction with GFRP, CFRP and steel bars. Also, this result is in a good agreement with the evaluation study of the shear strength of 112 FRP-RC beam that has been conducted by El-Sayed and Soudki 2011. Table 2 showed that the JSCE code provided more conservative predictions than that obtained using CSA-S806-12, with average ratio of V_{exp} /V_{pred} equal to 2.8. On the other hand, the ACI 440 design method provides the most conservative predictions with average ratio of V_{exp}/V_{pred} equal to 4. This high level of conservatism is expected as this method considers that the concrete shear strength of a section is provided only by the uncracked concrete above the neutral axis (Tureyen and Frosch 2003). However, the level of conservatism obtained in this study for the circular beams (V_{exp}/V_{pred} equal to 4) is higher than that obtained for the beams with rectangular section (V_{exp}/V_{pred} equal to 1.87, El-Sayed and Soudki 2011). This can be attributed to the fact that the neutral axis depth of the circular beam is lower than that for the equivalent beam with rectangular section. Hence, more experimental research works are needed to re-evaluate and modify the ACI 440-1R-06 design shear equation to account for the geometry of the circular cross-section.

5. Finite Element Analysis

A nonlinear finite element model of FRP beams is carried out using the finite-element software package ADINA. The finite element method used to analyze and simulate the behaviours of GFRP RC beams. In the analysis, the software formulations for the concrete and FRP bars are employed. These are described in detail in the ADINA theory and modeling guide (ADINA R&D Inc. 2006), and are summarized below. In additional, this model will be used to investigate the capability of the ADINA software to simulate the shear reinforcement effect on the behaviour of FRP reinforced circular concrete beams. In order to verify this model, the finite element analysis was verified through comparison with the available experimental data obtained from the static test conducted on circular concrete beam reinforced with GFRP bars (SHI Xiao-quan el at. 2012).

5.1. Geometrical Modeling

The specimens were circular section, 1200 mm in length and the diameter was 300 mm. The typical specimen dimensions are shown in Figure 2. The three-dimensional (3D) finite-element mesh used for the circular GFRP-RC beams is shown in Figure 3. Eight-node 3D solid elements were used to represent the concrete with three degrees of freedom at each node. The FRP bar was modeled using two-node truss elements with three translational degrees of freedom at each node.

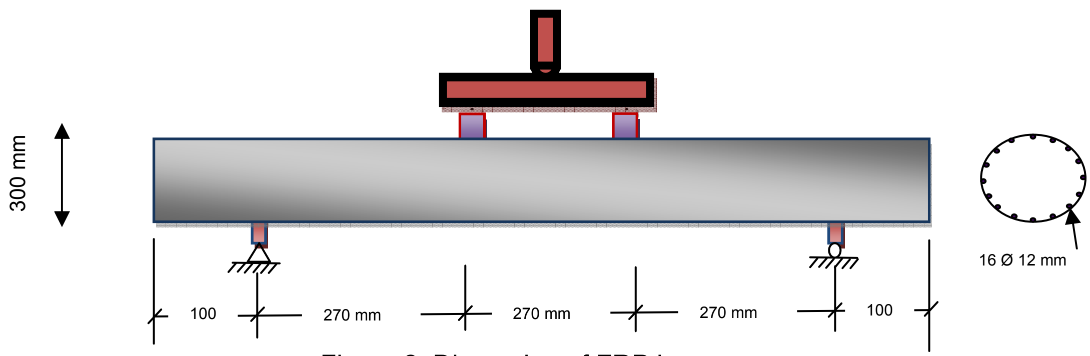

Figure 2. Dimension of FRP beams

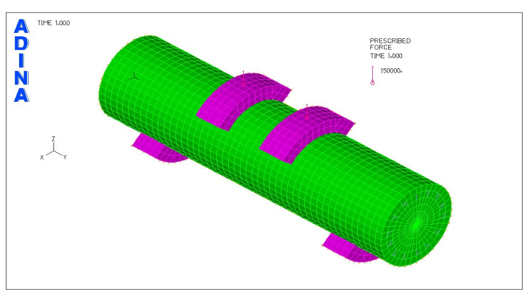

Figure 3. Finite element mesh

5.2. MATERIAL MODELLING

5.2.1 Concrete constitutive model

A plasticity-based concrete constitutive model is used in this analysis. The model utilizes the classical concepts of the theory of plasticity. A complete representation of the model is defined considering the following aspects: strain rate decomposition into elastic and inelastic rates, elasticity, yield, flow and hardening. The concrete model in compression is elastic until the initial yield surface is reached. The initial yield defines the elastic limit at which the linear elastic constitutive relationships are valid. The concrete constitutive model addresses the tensile behaviour of the concrete by considering several aspects. These aspects are cracking, shear modulus degradation, fracture energy and tension stiffening. Concrete behaviour in tension is linear until the cracking stress is reached. The tension stiffening model is defined as linearly descending after the post-peak point at which the concrete is cracked (K.W. Neale et al. 2005).

5.2.1 GFRP constitutive model

The typical stress–strain relationship for reinforcing FRP bar was a linear elastic tensile model. A rupture point on the stress–strain relationship defines the maximum stress and strain of the FRP bars as shown in Figure 4.

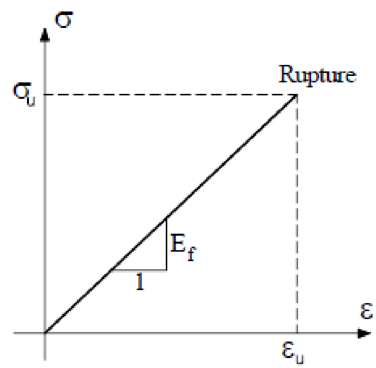

Figure 4. Typical stress−strain relationship for FRP bars

5.3 Loading

The FRP-RC beam was subjected to a vertical load (P) at two points. The value of (P) was varied from zero to the ultimate load capacity corresponding to the beam failure. The beam was incrementally loaded using 100-150 time steps.

5.4 Material Properties

The material properties for the FRP bars are presented in Table 3.

Table 3: Properties of reinforcement bars (SHI Xiao-quan el at. 2012)

	GFRP Bar (12 mm Diameter)
Tensile modulus(GPa)	33.2
Tensile strength (MPa)	541.80

5.5 Results and Discussion

A comparison between the finite element analysis and experimental results SHI Xiao et al. (2012) was calibrated in terms of the load-deflection relationship and the ultimate load carrying capacity. Table 4 shows a comparison between the finite element analysis and experimental results in terms of the failure load and the maximum deflections. Figure 5 presents the load-deflection relationship for the experimental and finite element analysis. It is evident from this figure that there is a good correlation between the results obtained from the finite element analysis and the experimental test results.

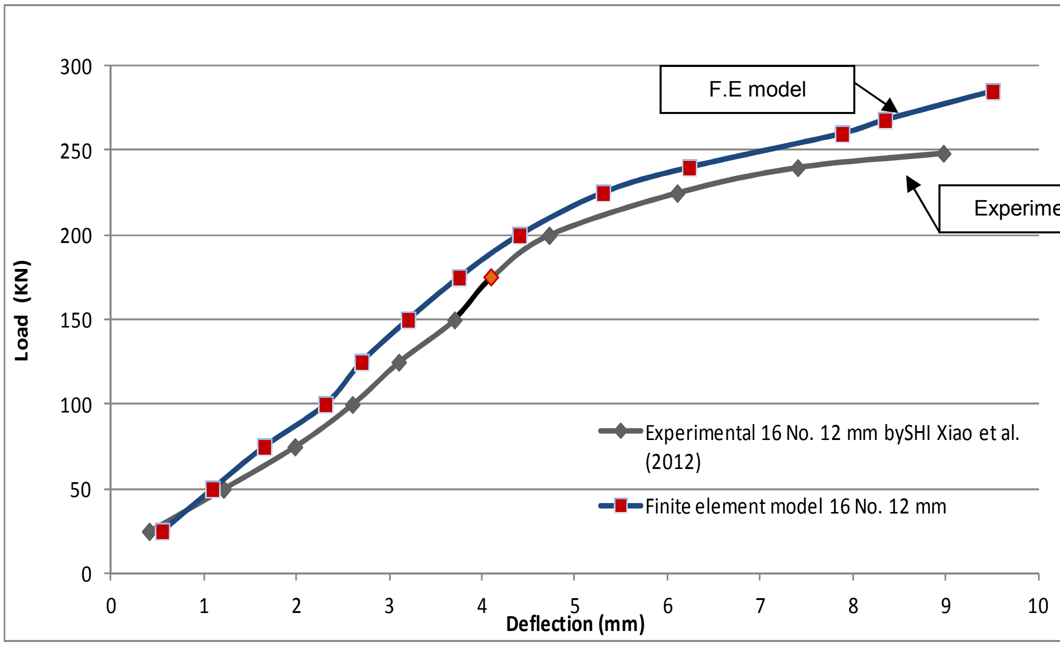

Figure 5. Comparison between experimental and FE load-deflection relationship

Table 4 Experimental and FE result comparisons

Model	ρ_f %	Failure load (kN)	Max deflection (mm)
Experimental	2.56	**248.5**	8.97
Finite element model	2.56	**285.0**	9.5

6. Conclusions

The experimental database of 6 shear test results of circular FRP-RC beams was used to evaluate the shear design equations. The three design methods (ACI 440-1R-06, CSA-S806-12, and JSCE code) provided conservative predictions with the mean value of V_{exp}/V_{pred} greater than 1.0. The result of comparison between the available experimental database and code predications showed that the design method of CSA-S806-12 for beams without shear reinforcement provided conservative and accurate predictions (mean value of V_{exp}/V_{pred} equal to 1.3). Also, the JSCE design method provided a mean value of V_{exp}/V_{pred} equal to 2.82. On other hand, the ACI 440.1R-06 design method provided highly conservative results with the mean value of V_{exp}/V_{pred} equal to 4. The high significant difference in the mean value of the predictions using the different design methods is attributed to the differences in the original formulation of the equations derived for rectangular section. Hence, more research works are needed to evaluate the design equations on the FRP-RC beam with circular cross-section. The finite-element modeling has been presented to simulate the nonlinear load-deflection behavior and failure load of circular RC beams reinforced with GFRP bars and without transvers stirrups. The comparisons between the finite element model and the referenced experimental test results showed a good agreement in terms of the load-deflection relationship. However, more research works are needed to simulate and include the effect of dowel action of FRP bars on the mode of failure.

7. References

ACI Committee 440, (2006), "Guide for the Design and Construction of Concrete Reinforced with FRP Bars," ACI 440.1R-06, American Concrete Institute, Farmington Hills, Mich., 44p.

ADINA. (2006). Theory Manual, version 8.4. ADINA R&D, Inc Wastertown, USA.

Ahmed K. El-sayed and Khaled Soudki (2011).” Evaluation of Shear Design Equations of Concrete Beams with FRP Reinforcement” Journal of Composites for construction,Vol.15, No.1, pp. 9-20.

Benmokrane, B., Chaallal, O., and Masmoudi, R., (1995), "Flexural Response of Concrete Beams Reinforced with FRP Reinforcing Bars,(2010)" ACI Structural Journal, vol. 91, no. 2, pp. 46–55.

Canadian Standard Association (CSA), "Canadian Highway Bridge Design Code," CAN/CSA S6-06, Addendum, Rexdale, Ontario, Canada.

El-Sayed, A. K., El-Salakawy, E. F., and Benmokrane, B., (2006), “Shear Strength of FRP-Reinforced Concrete Beams without Transverse Reinforcement," ACI Structural Journal, Vol. 103, No.2, pp. 235 - 243.

Mohamed H. (2010), “Axial and Flexural Behaviour of Reinforced Concrete-Filled FRP Tubes: Experimental and Theoretical studies,” Ph.D thesis, sherbrooke, Québec, Canada.

Merta, I., and Kolbitsch, A. (2006).” Shear Area of Reinforced Concrete Circular Cross-Section Members” 31st Conference on Our World In Concrete & Structures: 16 - 17 August.

Merta, I., (2008).” Shear strength model of reinforced concrete circular cross-section members”. journal of Structural Engineering, Mechanics and Computation, 3, pp.1297-1302.

Japanese Society of Civil Engineers (JSCE), (1997), "Recommendation for Design and Construction of Concrete Structures Using Continuous Fiber Reinforcing Materials," Concrete Engineering Series 23, Edited by A. Machida, Tokyo, Japan, 325p.

Neale, K.W., Ebead, U.A., Abdel Baky, H.M., Elsayed, W.E., and A. Godat, A., (2005).” Modelling OF Debonding Phenomena In FRP-Strengthened Concrete Beams And Slabs” Proceedings of the International Symposium on Bond Behaviour of FRP in Structures (BBFS 2005), 45-54.

Razaqpur, A. G., Isgor, B. 0., Greenaway, S., and Selley, A. (2004), "Concrete Contribution to the Shear Resistance of Fibre Reinforced Polymer Reinforced Concrete Members," Journal of Composites for Construction, ASCE, Vol. 8, No.5, pp 452-460.

SHI Xiao-quan, ZHANG Zhi-qiang , LI Zhi-ye (2012) “Experimental Study of the Shear Capacity of Glass Fiber Reinforced Polymer Reinforced Concrete Beam with Circular Cross Section”, journal of Shanghai Jiaotong University, V.17, No.4, pp. 408-414, China.

Tureyen, A. K., and Frosch, R. J. (2003). “Concrete shear strength: Another perspective.” ACI Struct. J., 100(5), 609–615.

Uffe G. Jensen, Linh C. Hoang, Henrik B. Joergensen, and Lars S. Fabrin (2010). “Shear strength of heavily reinforced concrete members with circular cross section” , Journal of Structural Engineering, ASCE, No.32, pp. 617- 626.

Yost, J. R., Gross, S. P., and Dinehart, D. W., (2001), "Shear Strength of Normal Strength Concrete Beams Reinforced with Deformed GFRP Bars," J. of Composites for Construction, ASCE, Vol. 5, No.4, pp. 263-275.