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Characterization of 3D fluid-structure interaction effects in civil engineering applications

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Abstract: In recent years, the analysis of fluid-structure interaction problems has become of significant interest to civil engineering applications, in particular for hydraulic structures such as water gates, spillways and canal locks. The dynamic analysis of these types of structures can be carried out using finite element models in two or three dimensions. In many cases however, three dimensional (3D) coupled finite element models of structure-reservoir systems can be prohibitively complex, considerably increasing costs associated with actual modeling and analysis. Major difficulties arise mainly from the modeling of the fluid domain and associated boundary conditions. In this paper, we present a systematic characterization of 3D effects on the dynamic response of wall-water systems. For that purpose, we propose a new formulation based on a sub-structuring approach where the structure is discretized using conventional 3D-solid finite elements, while the fluid is modeled analytically by solving the wave equation governing hydrodynamic pressure. The proposed formulation is first validated against coupled finite element models including fluid-structure capabilities. The proposed method is then used to examine case studies of wall-water systems, investigate corresponding 3D hydrodynamic effects and assess the validity of reducing wall-water system's width for simplified analysis purposes.

1 Introduction

Appropriate account of fluid-structure dynamic interactions has been shown an important factor in the design and safety evaluation of earthquake-excited civil engineering structures vibrating in contact with water. The first work in this regard was conducted in 1933 by Westergaard (1933), which consisted of taking into account hydrodynamic loads by adding mass to the structures face in contact with water. These static masses were determined assuming a rigid structural block and incompressible fluid. The added-mass technique has been widely used and suggested by many codes of practice over the years due to its simplicity. Research work conducted since the 1970s by a number of researchers have shown that neglecting structural flexibility may lead to an inappropriate evaluation of hydrodynamic pressures applied to hydraulic structures during earthquake loading. Chopra and his collaborators (Chopra 1968, Fenves and Chopra 1984) proposed numerical procedures in which a sub-structuring approach (where the structure and fluid domain are analyzed separately) is used to obtain the hydrodynamic pressure distribution along the face of a dam in two dimensions. These methods take into account, among others, fluid-structure interaction, foundation-structure interaction and reservoir bottom absorption. Chopra and his collaborators also developed simplified methods for the seismic analysis of two-dimensional monoliths of gravity dams (Fenves and Chopra 1987). Bouaanani and collaborators generalized these methods to two-dimensional structures of arbitrary shapes (Miquel and Bouaanani 2011). In most cases however, available analytical and simplified methods for fluid-structure interaction problems have been limited to two-dimensional idealizations. Such approaches neglect the three-dimensional (3D) effects associated with the distribution of hydrodynamic pressure within the fluid domain.

The advent of the finite element method and of computers capable of handling large volumes of calculations has made possible the analysis of complex fluid-structure systems in both two and three dimensions, taking into account fluid-structure interaction effects. While these numerical methods are robust, they can also be prohibitively complex, especially in three dimensions. They also often require considerable resources in terms of specialized personnel, computer hardware and time devoted to computations. These complexities are exacerbated by the large scale of most civil engineering applications involving fluid-structure interaction effects, such as the seismic analyses of dam-reservoir systems, sea walls, water storages, breakwaters and navigation locks. For such applications, the development and analysis of three dimensional finite element models may not always be appropriate, especially in the early stages of project development. In light of this, there is still an obvious interest in developing simplified methods capable of accounting for 3D fluid-structure interaction effects. Along the same line, simplifying considerations such as plain strain assumption can be adopted to expedite the dynamic analysis of dry structures, i.e. without water. As such, only a reduced, usually unit-width section can be analyzed instead of the whole structure. In practice, this assumption is usually extended to include the fluid domain when analyzing the vibrations of hydraulic structures such as dams or spillways. The validity of this extension as well as the corresponding sensitivity to wall lateral boundary conditions have not been assessed in the literature.

2 Objectives and methodolgy

The main objective of this paper is to characterize the 3D effects of fluid-structure interaction on the dynamic response of typical civil engineering applications. An analytical method which simulates the 3D dynamic behaviour of wall-water systems is first proposed and validated against more advanced finite element solutions. The proposed technique is then used to conduct parametric studies illustrating 3D hydrodynamic effects in dynamically excited wall-water systems. The validity of reducing the width of such wall-water systems for simplified analysis purposes is also assessed.

3 Basic assumptions and modelling considerations

3.1 Systems studied

We consider typical structure-water systems of the type illustrated in Figure 1. The structure can have any geometry but a vertical face at contact with water. It can represent a dam, a gated spillway, or a navigation lock for example. A rectangular wall type structure is considered herein for simplicity. The reservoir has a rectangular geometry, with height H_{r}, width b_{r}, and length L_{r}. The wall has a height H_{s}, thickness L_{s}, and the same width b_{r} as the reservoir. It is fixed at its base, and can have free or restrained lateral ends. The fluid domain is bounded by rigid boundary conditions at lateral ends, upstream end and bottom. A free surface boundary condition is considered at the top of the reservoir. The Cartesian system illustrated in Figure 1 is adopted to describe the geometry of the system and develop the equations of the proposed analytical method as presented below.

3.2 Proposed analytical method

The proposed analytical method utilizes a sub-structuring technique in which the fluid domain and structure are first analyzed separately, using finite elements and an analytical formulation, respectively (Fenves and Chopra 1984, Bouaanani and Lu 2009), the structure is discretized using conventional 3D-solid finite elements, while the fluid is modeled analytically by solving the wave equation governing hydrodynamic pressure.

Assuming that water is incompressible, the hydrodynamic pressure p within the reservoir obeys the classical Helmholtz equation:

[1] $$\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} + \frac{\partial^2 \bar{p}}{\partial z^2} = 0$$

in which $\bar{p}$ is the complex-valued frequency response function for hydrodynamic pressure defined by $\bar{p}(x, y, z, \omega) = p(x, y, z, t)\mathrm{e}^{-\mathrm{i}\omega t}$, where ω is the exciting frequency, and (x, y, z) are the coordinates of a given point in the Cartesian system illustrated in Figure 1.

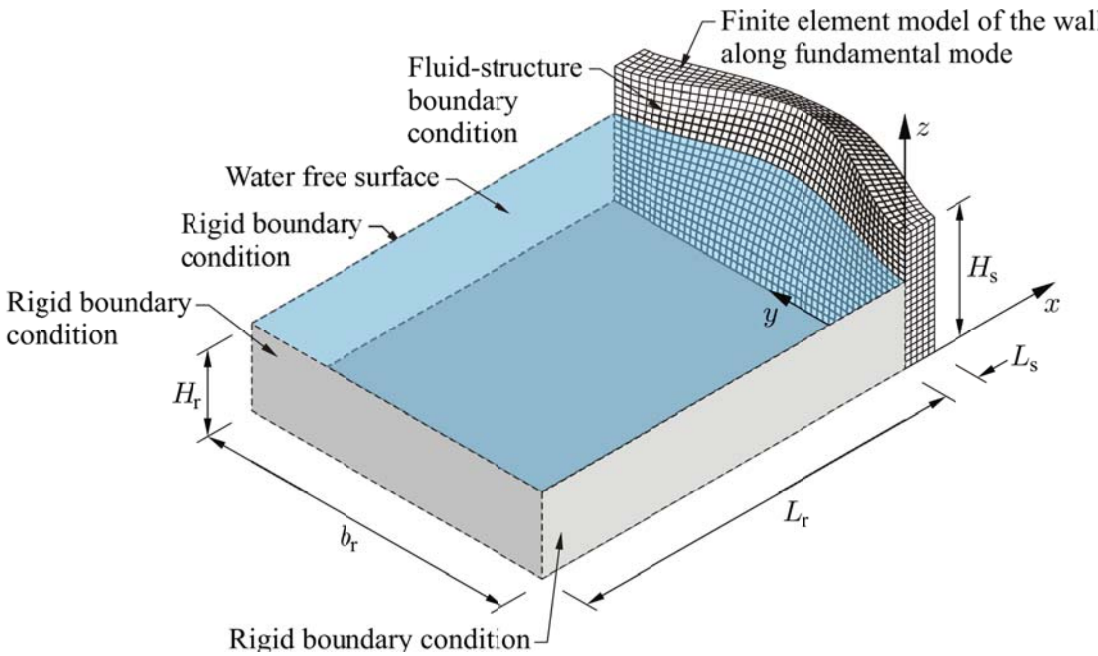

Figure 1: Typical wall-water system and boundary, geometry and boundary conditions.

Eq. [1] is combined with a free surface boundary condition at the top of the reservoir, rigid boundary conditions at its bottom, and lateral and upstream sides, and a compatibility boundary condition at the fluid-structure interface (Bouaanani and Lu 2009). Applying the principle of separation of variables and using these boundary conditions, we show that the frequency response functions for hydrodynamic pressure under the effect of a harmonic ground acceleration $a_{\mathrm{g}}^{(x)}\mathrm{e}^{\mathrm{i}\omega t}$ along the x direction can be obtained as:

[2] $$\bar{p}_0(x, y, z, \omega) = -\sum_{m=0}^{N_{\mathrm{r}}}\sum_{n=0}^{N_{\mathrm{r}}} \frac{4 \times (-1)^n \, \alpha \, \rho_{\mathrm{r}} \, a_{\mathrm{g}}^{(x)} \, \delta_{m0}}{(2n+1)\, \pi \, \kappa_{mn}} \, \Gamma_{mn}(x) \cos(\mu_m \, y) \cos(\lambda_n \, z)$$

[3] $$\bar{p}_j(x, y, z, \omega) = -\sum_{m=0}^{N_{\mathrm{r}}}\sum_{n=0}^{N_{\mathrm{r}}} \frac{2 \, \alpha \, \rho_{\mathrm{r}} \, I_{j,mn}}{b_{\mathrm{r}} H_{\mathrm{r}} \, \kappa_{mn}} \, \Gamma_{mn}(x) \cos(\mu_m \, y) \cos(\lambda_n \, z)$$

where $\alpha = 1$ when $m = 0$ and $\alpha = 2$ when $m > 0$, δ_{m0} is the Kronecker symbol, ρ_{r} is the mass density of water, N_{r} is the number of reservoir acoustical modes included in the analysis, and parameters λ_n, μ_m, Γ_{mn} and $I_{j,mn}$ are given by

[4] $$\lambda_n = \frac{(2n+1)\pi}{2H_\mathrm{r}}$$

[5] $$\mu_m = \frac{m\,\pi}{b_\mathrm{r}}$$

[6] $$\Gamma_{mn}(x) = \frac{\mathrm{e}^{\kappa_{mn}\,(x+L_\mathrm{r})} + \mathrm{e}^{-\kappa_{mn}\,(x+L_\mathrm{r})}}{\mathrm{e}^{\kappa_{mn}\,L_\mathrm{r}} - \mathrm{e}^{-\kappa_{mn}\,L_\mathrm{r}}}$$

[7] $$\kappa_{mn} = \sqrt{\mu_m^2 + \lambda_n^2}$$

and

[8] $$I_{j,mn} = \int_0^{H_\mathrm{r}} \int_0^{b_\mathrm{r}} \psi_j^{(x)}(0,y,z)\,\cos(\mu_m\,y)\,\cos(\lambda_n\,z)\,\mathrm{d}y\,\mathrm{d}z$$

in which $\psi_j^{(x)}(0,y,z)$ is the j th structural mode shape taken at the fluid-structure interface.

We show that the complex-valued frequency response function for hydrodynamic pressure can be expressed as (Fenves and Chopra 1984, Bouaanani and Lu 2009):

[9] $$\bar{p}(x,y,z,\omega) = \bar{p}_0(x,y,z,\omega) - \omega^2 \sum_{j=1}^{N_\mathrm{s}} \bar{Z}_j(\omega)\,\bar{p}_j(x,y,z,\omega)$$

where $\bar{p}_0$ designates the frequency response function for hydrodynamic pressure due to rigid body motion of the structure, $\bar{p}_j$ the frequency response function for hydrodynamic pressure due to horizontal acceleration $\psi_j^{(x)}(0,y,z)$ of the wall's face, $\bar{Z}_j$ is the vector of generalized coordinates and N_s is the number of structural modes considered in the analysis. The frequency response functions for displacements and accelerations along the x direction can be expressed as (Bouaanani and Lu 2009):

[10] $$\bar{u}(x,y,z,\omega) = \sum_{j=1}^{N_\mathrm{s}} \psi_j^{(x)}(x,y,z)\,\bar{Z}_j(\omega)\,; \qquad \bar{\ddot{u}}(x,y,z,\omega) = -\omega^2 \sum_{j=1}^{N_\mathrm{s}} \psi_j^{(x)}(x,y,z)\,\bar{Z}_j(\omega)$$

The vector $\bar{\mathbf{Z}}$ of generalized coordinates $\bar{Z}_j$, $j = 1 \ldots N_\mathrm{s}$, can be obtained by solving the following system of equations (Bouaanani and Lu 2009):

[11] $$\bar{\mathbf{S}}\,\bar{\mathbf{Z}} = \bar{\mathbf{Q}}$$

where the elements of matrix $\bar{\mathbf{S}}$ and vector $\bar{\mathbf{Q}}$ are given for $n = 1 \ldots N_\mathrm{s}$ and $j = 1 \ldots N_\mathrm{s}$ by:

[12] $$\bar{S}_{nj}(\omega) = \Big[-\omega^2 + \big(1 + \mathrm{i}\eta_\mathrm{s}\big)\,\omega_n^2\Big]\,\delta_{nj} + \omega^2 \int_0^{H_\mathrm{r}} \int_0^{b_\mathrm{r}} \bar{p}_j(0,y,z,\omega)\,\psi_n^{(x)}(0,y,z)\,\mathrm{d}y\,\mathrm{d}z$$

[13] $$\bar{Q}_n(\omega) = -\boldsymbol{\psi}_n^\mathrm{T}\,\mathbf{M}_\mathrm{s}\,\mathbf{1}^{(x)} + \int_0^{H_\mathrm{r}} \int_0^{b_\mathrm{r}} \bar{p}_0(0,y,z,\omega)\,\psi_n^{(x)}(0,y,z)\,\mathrm{d}y\,\mathrm{d}z$$

in which η_s, $\mathbf{M}_s$, ω_n and ψ_n denote, respectively, the hysteretic damping coefficient, mass matrix, natural frequency and corresponding mode shape of the dry wall, i.e. without water. The modal properties of the dry wall can be obtained using standard finite elements or analytical formulations. The former approach is adopted in this work. We note that a convergence study has to be conducted to determine a sufficient number of structural (N_s) and reservoir (N_r) modes to use in each analysis case. The proposed procedure is programmed and is applied next to wall-water systems.

4 Results and discussion

4.1 Validation of the proposed analytical method

The proposed analytical method is first validated against a coupled finite element model with fluid-structure interaction capabilities. For illustration purposes, we consider a wall-water system with the following dimensions (Figure 1): $b_r = 5$ m, $H_r = 2$ m, $L_r = 2$ m, $H_s = 2$ m, and $L_s = 0.5$ m. A hysteretic damping $\eta_s = 0.1$ is assumed. For validation purposes, a finite element model of the wall-water system is constructed using the ADINA software (ADINA 2010). The structural wall is modelled using 3D displacement-based 8-node solid elements, while the reservoir is modelled using potential-based 3D 8-node fluid elements. Fluid-structure interaction at the wall-reservoir interface is taken into account through special interface elements programmed in the software (ADINA 2010). A large bulk modulus is considered to neglect water compressibility.

Figure 2 compares the frequency-response curves of hydrodynamic pressures computed using the proposed method and finite elements at a point on the wall-reservoir interface with coordinates $x = 0$ m, $y = 2.5$ m and $z = 0.7$ m. In this figure, hydrodynamic pressures are normalized by hydrostatic pressure $\rho_r g H_r$ and frequencies are normalized by the fundamental frequency of the structure alone ω_1. Figure 2 clearly shows that the agreement between the proposed analytical method and the finite elements method is excellent.

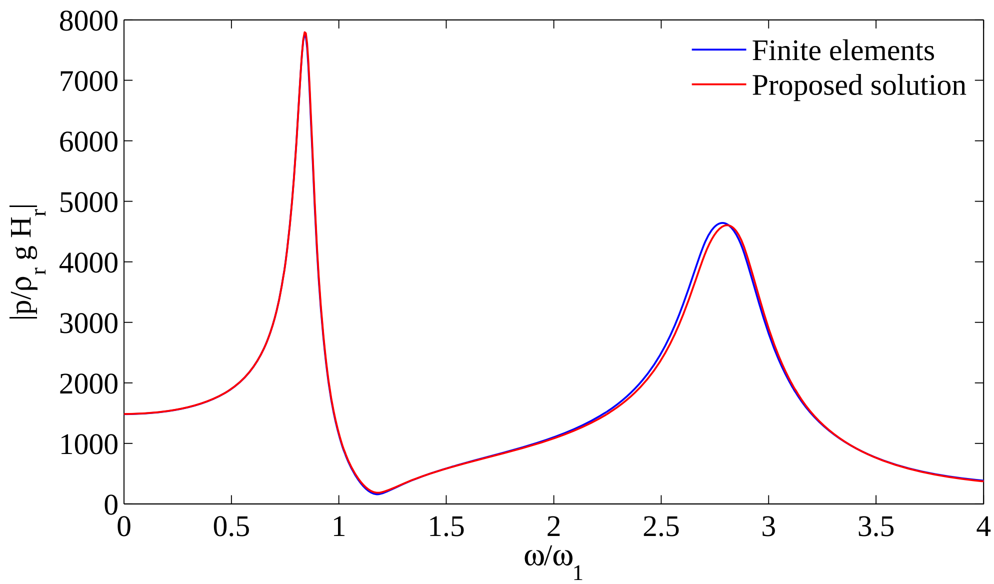

Figure 2: Hydrodynamic pressures obtained using the proposed method and finite elements.

4.2 Characterization of 3D hydrodynamic effects

In this section, we use the proposed analytical method described above to characterize 3D hydrodynamic effects in wall-water systems such as the one described in Figure 1. We first consider a wall-water system, designated hereafter as the reference system, with a large ratio $b_r/H_r = 10$, and the following dimensions: $b_\mathrm{r} = 300\ \mathrm{m}$, $H_\mathrm{r} = 30\ \mathrm{m}$, $L_\mathrm{r} = 150\ \mathrm{m}$, $H_\mathrm{s} = 30\ \mathrm{m}$, and $L_\mathrm{s} = 3\ \mathrm{m}$. A hysteretic damping $\eta_s = 0.1$ is considered and the wall is assumed fixed at its lateral ends.

The validity of reducing the width of a wall-water system as well as the corresponding sensitivity to wall lateral boundary conditions are addressed next for the reference wall-water system described above. For this purpose, in addition to the reference system, we also consider sub-systems with reduced widths b_r of $8H_\mathrm{r}$, $6H_\mathrm{r}$, $4H_\mathrm{r}$, $2H_\mathrm{r}$ and $1H_\mathrm{r}$. In order to study the effect of the lateral boundary conditions applied to the lateral sides of the wall, the dynamic analyses of the reduced systems are carried out for two cases: (i) a restrained case where the lateral ends of the wall are maintained fixed, and (ii) an unrestrained case where the lateral ends of the wall are free to vibrate. We note that the lateral ends of the reference system remain fixed.

Figure 3 illustrates the frequency response functions for hydrodynamic pressure computed midway along the width of the wall, i.e. at $0.5b_\mathrm{r}$, at a point on the wall-reservoir interface located at $z = 2.0\ \mathrm{m}$. The results are shown for laterally restrained walls, and frequencies ranging from 0 to 11 Hz, which are of most interest in civil engineering applications. We observe that reducing the overall width of the system while maintaining fixed boundary conditions on the lateral ends of the walls affects resonant frequencies as well as the amplitudes of hydrodynamic pressures. In particular, the overestimation of the fundamental frequency of the wall-water system increases with decreasing reduced width b_r.

It can be seen from Table 1 that the fundamental frequency of the reduced system with $b_\mathrm{r} = 1H_\mathrm{r}$ is about 600% higher than that of the reference wall-water system. These overpredictions of resonant frequencies could lead to overly conservative designs of hydraulic structures when the response spectrum method is used for example.

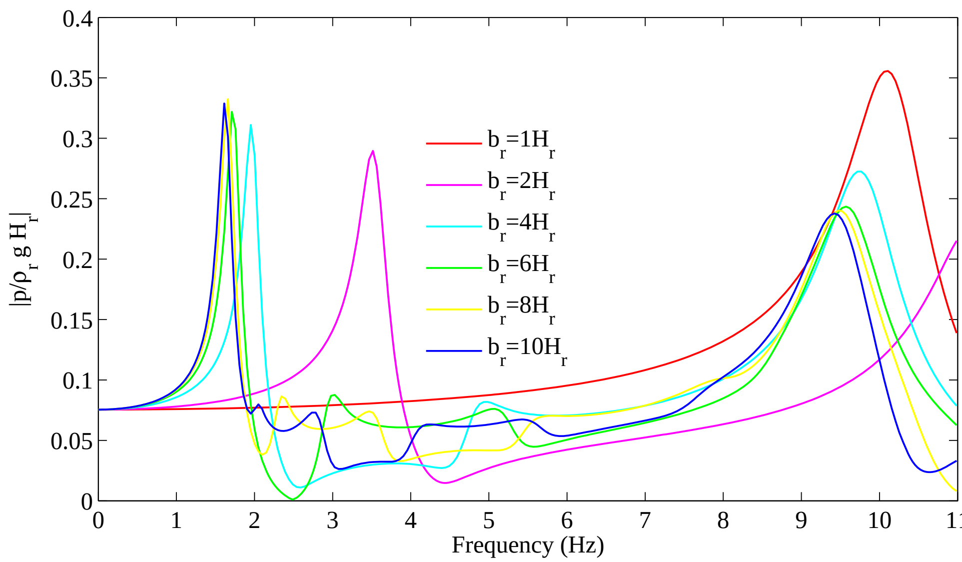

Figure 3: Hydrodynamic pressure for varying reservoir widths and laterally restrained reduced walls.

The same observations apply to Figure 4 which shows the frequency response functions for horizontal acceleration determined midway along the width of the wall, at a point on the wall-reservoir interface located at the top of the wall, i.e. $z = 30\ \mathrm{m}$.

It is also important to investigate the effect of reduced reservoir width on the distribution of hydrodynamic pressure along reservoir height. The distributions of hydrodynamic pressures corresponding to the fundamental frequencies of each reduced wall-water system are illustrated in Figure 5. These distributions, which are evaluated at the center lines of the restrained walls, reveal that the profiles of hydrodynamic pressures are generally not very sensistive to the reduction of reservoir width, although the smallest considered width of $b_r = 1H_r$ corresponds to the more distinct profile and maximum hydrodynamic pressure.

	Fundamental Frequency (Hz)	
Reduced width	Laterally restrained wall	Laterally unrestrained wall
$1H_r$	10.26	1.56
$2H_r$	3.56	1.56
$3H_r$	2.34	1.56
$4H_r$	2.00	1.56
$5H_r$	1.81	1.56
$6H_r$	1.76	1.56
$7H_r$	1.71	1.56
$8H_r$	1.66	1.56
$9H_r$	1.66	1.56
$10H_r$	1.66	-

Table1: Fundamental frequencies for varying reservoir widths and restrained and unrestrained reduced walls

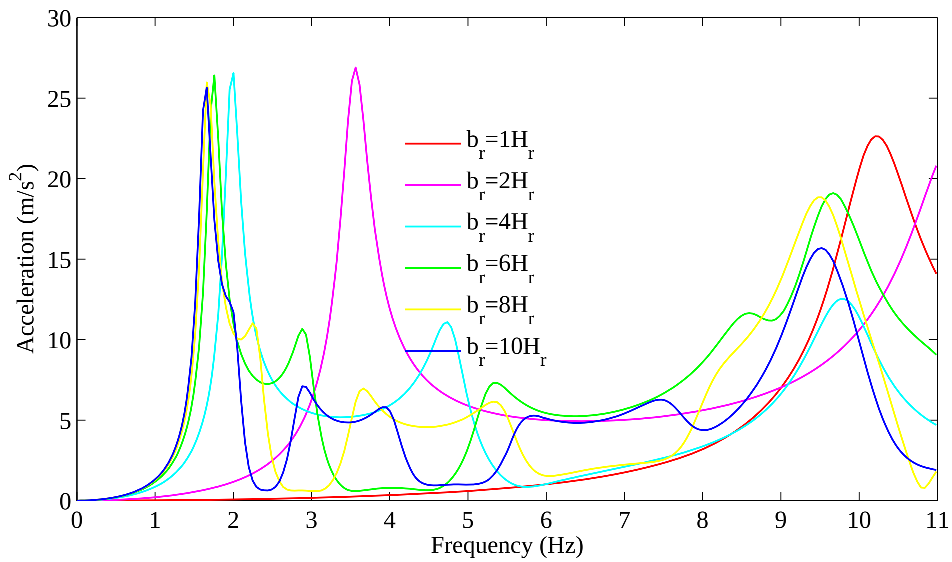

Figure 4: Horizontal acceleration for varying reservoir widths and laterally restrained reduced walls.

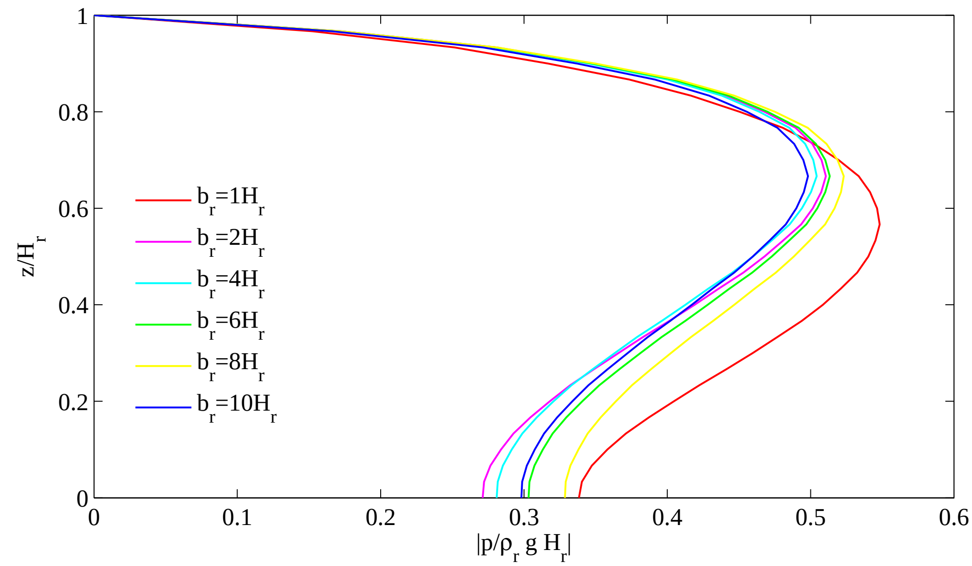

Figure 5: Pressure distrbutions for varying reservoir widths and laterally restrained reduced walls.

The effects of the lateral boundaries of the reduced walls are investigated next. Figures 6 and 7 show, respectively, the frequency response functions for hydrodynamic pressure and horizontal acceleration computed at the same points as previously but for reduced walls with unrestrained lateral ends.

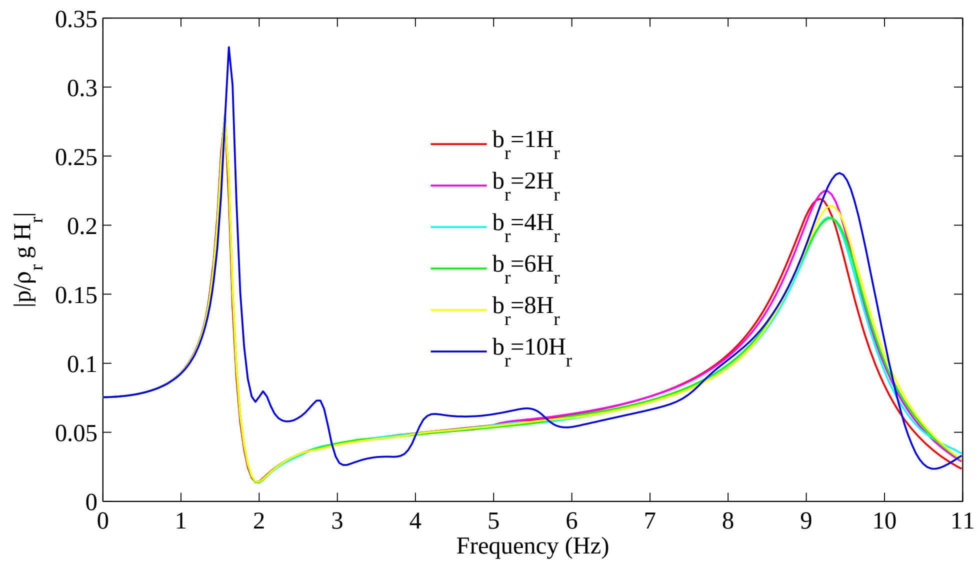

Figure 6: Hydrodynamic pressure for varying reservoir widths and laterally unrestrained reduced walls.

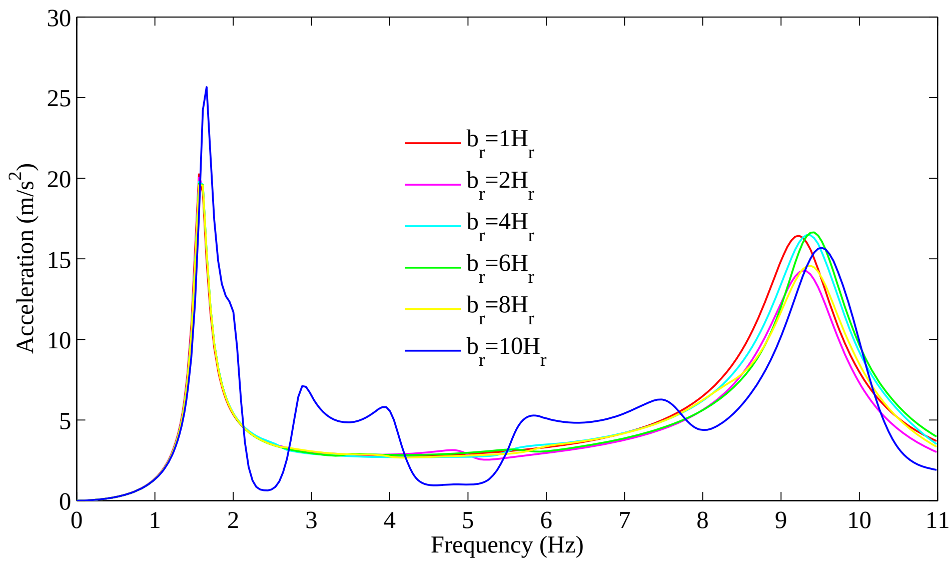

Figure 7: Horizontal acceleration for varying reservoir widths and laterally unrestrained reduced walls.

Figures 6 and 7 show that the effects of reducing the lateral width of the wall-reservoir system are much less significant when the lateral sides of the reduced walls are unrestrained. While a reduction of the width b_r of the system does affect the amplitude of the fundamental mode response with respect to the reference system, the fundamental frequency does not vary, as shown in Table 1. This is due to the fact that the fundamental response of the cantilever dry wall is not affected by the reduction of its width. Figures 6 and 7 also reveal that there is almost no variation in the amplitude of the first mode resonant response between the models of reduced widths. Figure 8 illustrates the distribution of hydrodynamic pressures corresponding to the fundamental frequencies of each reduced wall-water system considering laterally unrestrained reduced walls. The distributions are determined at the center lines of the walls as previoulsy. It can be seen that the hydrodynamic pressure profiles corresponding to the reduced systems are almost identical, and that they are all lower than the profile corresponding to the reference system.

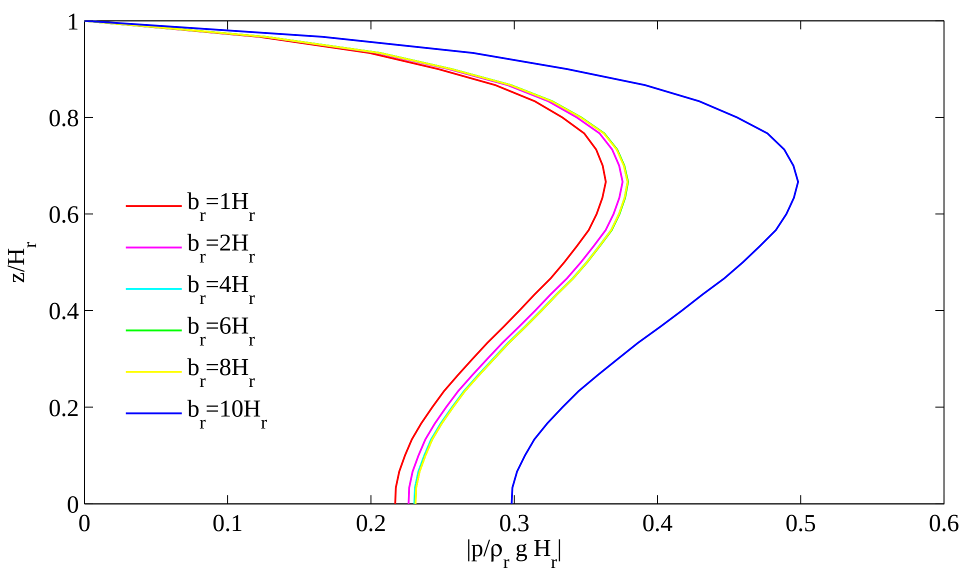

Figure 8: Pressure distrbutions for varying reservoir widths and laterally unrestrained reduced walls.

5 Summary and conclusions

The analysis of fluid-structure interaction problems is important for civil engineering applications such as water gates, spillways and canal locks. This paper presented a new formulation based on a sub-structuring approach where the structure is discretized using conventional 3D-solid finite elements, while the fluid is modeled analytically by solving the wave equation governing hydrodynamic pressure. The basic assumptions and mathematical background of the proposed method was provided and the technique was first validated against coupled finite element models including fluid-structure capabilities. An excellent agreement was found by comparing the results obtained using both methods. The developed technique was then applied to examine case studies of wall-water systems, investigate corresponding 3D hydrodynamic effects and assess the validity of reducing wall-water system's width for simplified analysis purposes. Reduced wall-water systems with various dimensions and boundary conditions were considered. We observed that reducing the overall width of the wall-water system while maintaining fixed boundary conditions on the lateral ends of the reduced walls affects resonant frequencies as well as the amplitudes of hydrodynamic pressures. In particular, the overestimation of the fundamental frequency of the wall-water system was found to increase with decreasing reduced width. The profiles of hydrodynamic pressures along reservoir height were found to be generally less affected by the reduction of reservoir width at fundamental frequencies of the reduced wall-water systems. The effects of reducing the lateral width of the wall-reservoir system are much less significant when the lateral sides of the reduced walls are unrestrained. While a reduction of the wall-water system's width was shown to affect the amplitude of the fundamental mode response with respect to the reference system, practically identical fundamental frequencies of the reduced systems were found. Overall, this work clearly illustrates the importance of 3D hydrodynamic effects on the dynamic response of the type of wall-water systems studied. We also showed that the reduction of wall-water system's width for purpose of simplified analysis may provide inaccurate results especially when predicting resonant frequencies. Ongoing work investigates the effects of water compressibility on the results.

Acknowledgements

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