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Patch Delamination Effect on Stress Intensity Factor of Cracked Steel Plates Repaired with Bonded Fiber Reinforced Polymers

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Abstract: Effect of bonded repair patch delamination on stress intensity factor (SIF) has been studied using finite element analysis (FEA) of cracked steel plates repaired with carbon fiber reinforced polymers (CFRP) patches. Two types of CFRP were used: sheet form, with elastic tensile modulus (E_{FRP}) of 65 GPa; plate form, with E_{FRP} of 165 GPa. Three sets of relative repair patch's axial stiffness (ETR) were analysed: 0.264; 0.53 and; 0.8. Patch delamination was modeled by progressive removal of the overstressed portions of adhesive at steel interface. Results of FEA showed that the SIF decreases with increase in the ETR using both types of the FRP but patch delamination enhances the SIF for any given ETR and crack length. Without considering patch delamination, SIF of repairs with ETR 0.264 was increased by 22 % and 30 %, respectively in lower and higher E_{FRP} repairs, for the increase in crack length from 10 to 25mm. The same comparison showed the increase as 122 % and 65 %, after introduction of the patch delamination. The delamination was found to be decreased with the increase in the ETR. The SIF of the repaired plates were also evaluated analytically using tools of linear elastic fracture mechanics (LEFM), incorporating the patch delamination and these were found to be correlating well with the finite element results.

1 Introduction and Background

Application of 'Fiber Reinforced Polymers (FRPs), to improve strength and fatigue life of crack and uncrack metal structures has been successfully used since last 2-3 decades, mostly in aircraft industry, to repair cracked and corroded parts of aircrafts. Traditional fatigue repair techniques for steel structures are effective to some extent but still have many drawbacks like reduced net section capacity, increased weight and development of secondary fatigue cracks etc. Repairs involving metal welding found to be developing martensite formation (Bayraktar et al. 2004), high residual stresses and resulted in even lesser fatigue life than the un-welded plate (Alam 2005). Crack bridging using fiber reinforced composites involve adhesion of highly stiff and high strength fiber reinforced polymers (FRP) over the cracked metal plates. This method has found to be more successful in improving the fatigue behavior of cracked metal plates because it: minimizes the stress concentration at load transfer regions; relief and share stresses from the cracked plate and crack tip; has high strength-to-weight ratio; can be applied to any flat, irregular and curved surface; causes crack closure effect by restricting the crack opening.

FRP patching over the cracked steel plates reduces the crack growth rate through the effective mechanisms of stress sharing and crack bridging. Sharing of the applied stresses between steel and the bonded FRP follows the relative stiffness proportion (Duong and Wang, 2007), which means that higher the axial stiffness of the repair FRP patch lower will be stress in steel and lower will be the resulting SIF and crack growth.

Restrain to the crack opening results from straining of the fibers bridging over the crack flanks and produces crack closure stresses surrounding the crack flanks, as shown in Figure 1. Lin et al. (1991) showed that the crack growth rate enhanced by a factor of three (3) if the crack bridging mechanism is removed by cutting the fibers over the wake of crack. The crack closure stresses vary along the crack length due to decrease in the crack opening displacement (COD) and is maximum at the crack exterior edge because of maximum COD there. Crack bridging mechanism is affected by the progression of patch delamination surrounding the crack, resulting in an increase in the fiber bridging length, thus lowering the strain level in the fibers over the crack.

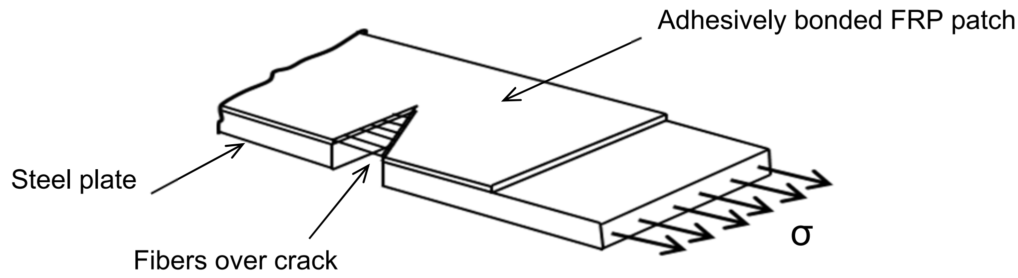

Figure 1: Fiber bridging over a Crack

Failure mode in most of the fatigue tests of cracked steel plates, repaired with adhesively bonded FRP, is the progressive delamination of the repair patch (Kennedy and Cheng 1998, Lui et al. 2009, Holden 2012, Mobeen et al. 2012, Bocciarelli et al. 2009, Papanikos et al. 2007, Colombi et al. 2003a). Bonded patch delaminates due to adhesive shear failure around the crack and patch end, resulting from non-compatibility of strains between steel and FRP patch. Delamination once starts it propagates inside the bond length of the FRP patch resulting in increased crack growth rate (Papanikos et al. 2007). The delamination shape around crack edges is found to be elliptical in several researches (Lin and Kao 1995, Colombi et al. 2003a, b).

'Effective Thickness Ratio' (ETR), defined as the ratio of axial stiffness of FRP patch to the axial stiffness of steel plate, has found to be the governing repair parameter in previous researches but most of the research works involving two different E_{FRP} repair patches, with close or identical ETR values, showed higher fatigue life or the SIF of the repairs with higher E_{FRP} compared to the lower E_{FRP} repairs (Liu et al., 2009, Mobeen et al., 2012, Lam and Cheng, 2008). Analytical study by Kennedy and Cheng (1998) also showed that the stress concentration factor in steel at the patch end was different within the patches with identical ETR values but having different stiffness.

An attempt is made in the current research work to explore the impact of repair patch delamination on the stress intensity factor (SIF) and to relate it to the initial design parameters, like the ETR, E_{FRP} and adhesive shear strength etc.

2 Objective and Scope

Main objective of current research was to study, numerically and analytically, the impact of bonded patch delamination on the stress intensity factor (SIF) of cracked steel plate, repaired using high and low elastic tensile modulus (E_{FRP}) CFRP. The scope was limited to edge cracked steel plates repaired with bonded double sided repair patches made up of two forms of carbon fiber reinforced polymers (CFRP); fabric form, with elastic tensile modulus (E_{FRP}) of 65 GPa; and plate form, with elastic tensile modulus (E_{FRP}) of 165 GPa.

3 Methodology

The SIF was evaluated using finite element model (FEM) of edge cracked steel plates repaired with bonded CFRP repaired patches developed in finite element based software ABAQUS version 6.10. Delamination of the bonded patch was modeled in the FEA by progressively removing the overstressed regions of adhesive layer in shear at the steel interface and setting up new model for each delaminated step. Additionally an analytical approach was also developed using tools of linear elastic fracture mechanics (LEFM) to evaluate the SIF of edge cracked steel plates repaired with bonded CFRP patches. The details of the FEA and the analytical approach are provided in subsequent sections.

3.1 Finite Element Analysis (FEA) for SIF Evaluation

Three dimensional finite element model of cracked steel plate, with bonded CFRP repair patches, was developed using 20-node, 3D brick elements. A 300 mm long, 50 mm wide and 7.5 mm thick steel plate, with a through thickness edge crack at its mid length, was modeled. Bonded repair patches, consisted of CFRP and adhesive layers, were applied to the surface of cracked steel plate using master-slave boundary constraint. Nominal material properties of CFRP and adhesives were assigned as provided in supplier’s data sheets (Sika Canada Inc. 2007) which are reproduced in Table 1, in which SikaWrap Hex and Sika CarboDur are respectively the lower and higher E_{FRP} CFRPs while Sikadur 300 and Sikdur 30 are the adhesives for the lower and higher E_{FRP} CFRPs. The shear strength of Sikadur 300 was adopted from the research work by Stanford and Rizkalla (2009) as it was not provided in the supplier’s data. Similarly the thickness of Sikadur 300 is assumed to be 0.2 mm out of the total cured single layer thickness (1.016 mm) of the SikaWrap Hex. The E_{FRP} of Sikawrap Hex was adjusted accordingly so that its total axial stiffness remains unaltered. The CFRP and adhesive layers were 150 mm long, 50 mm wide and were centrally placed on the steel plate. Taking advantage of planes of symmetry in the analysis only one-quarter of the actual model was developed as shown in Figure 2. To achieve more accuracy in analysis a fine meshed region was defined near the crack tip. Mesh sensitivity analysis was carried out within the fine meshed region and the converged mesh was used in the development of all models. The test matrix used is shown in Table 2, in which the specimens are mainly divided into three main ETR groups; 0.264, 0.52 and 0.8 and each ETR group has specimens provided with the two different forms of CFRP; sheet form (lower E_{FRP}); and plate form (higher E_{FRP}). Due to different properties of the two CFRP types it was found that the axial stiffness of 3 layers of the lower E_{FRP} CFRP (the sheet form) was approximately equivalent to 1 layer of the higher E_{FRP} CFRP (the plate form), therefore, a double sided repaired steel plate with 3 layers of lower E_{FRP} on each face is equivalent to 1 layer of higher E_{FRP} CFRP on both faces of steel plate, both repair will have ETR of 0.264. All repair models were developed for four crack lengths; 10mm, 15mm, 20mm and 25mm, also equivalent to the non-dimensionalized crack lengths (a/b) of 0.2, 0.3, 0.4 and 0.5 respectively.

Linear elastic analysis was performed with an applied far-field stress ‘σ’ of 100 MPa and the SIF was evaluated from the analysis output of each repair, which was internally evaluated in the analysis using the ‘J” integral evaluation. For each crack length and ETR, CFRP patch delamination was introduced in the analysis by progressive removal of the overstressed regions of the adhesive layer at the steel interface and setting up new model with the overstressed adhesive regions being removed. This adhesive removal and new models formation for each repair was continued until a converged level of delamination achieved, defined as when the area of delaminated adhesive region became less than 5% of the initial bonded adhesive area.

3.2 Analytical approach for SIF evaluation

An analytical approach was formulated based upon the principle of superposition of stress intensity factors corresponding to the stresses in steel resulting from two basic mechanisms involve in a bonded repaired cracked plate; the stress sharing and; the crack bridging. Refer to the section 1 for details of the two mechanisms. Similar approach had also been used in some previous researches (Marrisan 1984, Lin et al. 1995). Additionally impact of adhesive shear deformation on SIF enhancement has also been incorporated in the current approach. The formulation of the procedure is summarized here.

Table 1: Material Properties

Material	Modulus of Elasticity (GPa)	Thickness (mm)	Shear Strength (MPa)
Steel	200	7.5	
SikaWrap Hex-103C	65 (81*)	1.016 (0.816)	
Sikadur 300	1.724	0.2**	7.5**
Sika CarboDur	165	1.2	
Sikadur 30	5	2.4	15

Note: * Interpolated for 0.816 mm thickness, ** Refer section 3.1

Table 2: Test Matrix

Specimen	Description	CFRP Layers Each face	ETR
3-SkWrp	3 Layers SikaWrap Hex 103	3	0.264
6-SkWrp	6 Layers SikaWrap Hex 103	6	0.53
9-SkWrp	9 Layers SikaWrap Hex 103	9	0.8
1-CBDR	1 Layer Sika CarboDur	1	0.264
2-CBDR	2 Layers Sika CarboDur	2	0.53
3-CBDR	3 Layers Sika CarboDur	3	0.8

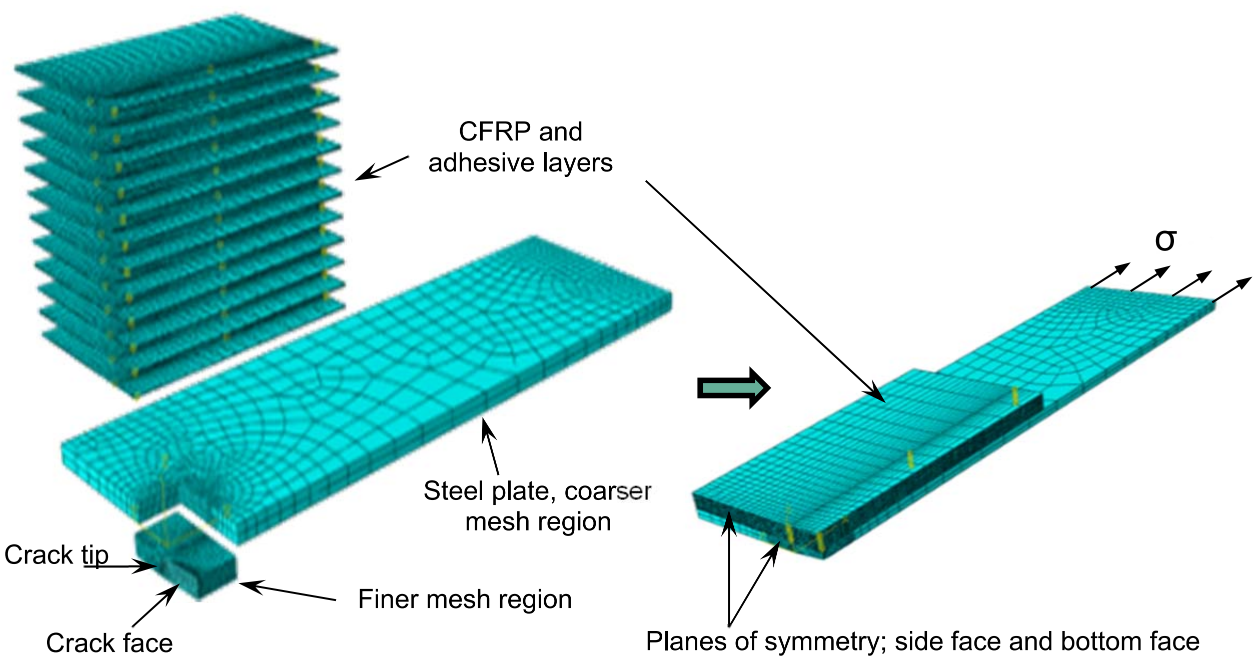

Figure 2: Finite element model of cracked steel plate with CFRP patch

The stress in steel reduced by the sharing action of CFRP, from 'σ', the far filed stress to the 'σ_s' is given by Eq. 1 (Duong and Wang 2007)

$$[1] \qquad \sigma_S = \left[\frac{E_S t_S}{E_p t_p + E_S t_S}\right] \sigma$$

SIF for a finite width plate with edge crack, subjected to uniform stress (σ_S), is given by Eq. 2.

$$[2] \qquad K_S = F\, \sigma_S \sqrt{(\pi\, a)}$$

Where 'F' is the geometric factor which was taken from Tada, Paris and Irwin (1985) for a finite width edge cracked plate and is given by Eq. 3, with 'a' and 'b' be the crack length and the total plate width respectively.

$$[3] \qquad F = 1.122 - 0.231(a/b) + 10.55\,(a/b)^2 - 21.71(a/b)^3 + 30.3(a/b)^4$$

As already mentioned in section 1, the crack closure stress is related to the strain in crack bridging fibers which is caused by the COD and is to be distributed on the un-bonded fiber length, defined as the length of fiber between its two delaminated ends across the crack, so it is the sum of COD and the delaminated fiber length on each side of the crack. In current analytical approach the COD was evaluated from Tada, Paris and Irwin (1985), modified by the stress sharing and crack bridging action of the repair patch. It is reproduced as Eq. 4 below with the far field stress 'σ' replaced by the difference of stresses resulting from stress sharing and crack bridging mechanisms

$$[4] \qquad COD = \left[\frac{4\,a\,(\sigma_S - \sigma_{Cf})}{E_S}\right] V\,(a/b)$$

σ_S is given by Eq. 1 and σ_{cf} is the crack closure stress developed in steel due to straining of the crack bridging fibres. V(a/b) is the geometric factor for an edge cracked finite width plate and is given in Eq. 5.

$$[5] \qquad V\,(a/b) = [1.46 + 3.42\,(1 - (\mathrm{Cos}\,\pi\, a\,/\,2b)]\,/\,(\mathrm{Cos}\,\pi\, a\,/\,2b)^2$$

'σ_{cf}' is related to the stress in fibres bridging over the crack, which depends upon the COD and the fiber delamination length, it is given by Eq. 6.

$$[6] \qquad \sigma_{Cf} = \sigma_f \frac{t_f}{t_S} = \epsilon_f\, E_f\, \frac{t_f}{t_S} = \frac{COD}{L}\, E_f\, \frac{t_f}{t_S}$$

The subscripts 'f' and's' stands for fiber and steel respectively and 'L' is the total un-bonded length of fibers bridging over the crack. Solving Eqs. 4 and 6 for the closure stress 'σ_{cf}' in steel, gives the Eq. 7.

$$[7] \qquad \sigma_{Cf} = \frac{4\,a\,(V\,a/b)\,\sigma_S}{\left(L + 4\,a\,(V\,a/b)\,\frac{E_f}{E_S}\,\frac{t_f}{t_S}\right)} \left(\frac{E_f}{E_S}\,\frac{t_f}{t_S}\right)$$

In the current analytical approach the value of 'L' (for each crack length and ETR) was noted from the respective FEA output. σ_s and V(a/b) were evaluated from Eqs. 1 and 5 respectively and σ_{cf} was evaluated from Eq. 7, for all crack lengths and for each repair patch. Knowing σ_{cf} at the crack faces the corresponding stress intensity factor was then evaluated using standard solution given by Eq. 8, with F calculated from Eq. 3.

$$[8] \qquad K_b = F\, \sigma_{Cf} \sqrt{(\pi\, a)}$$

As already mentioned in section 1 crack closure stresses σ_{cf} is expected to be maximum at plate edge and it reduces near the crack tip because of decreasing COD from crack mouth towards the crack tip. To incorporate this variation in σ_{cf} and to simplify the analysis, the stress variation in FRP along the crack was studied in all the current FEA models and it was found that in most un-delaminated models the average fiber stress varied from 70% to 80% of the maximum σ_{cf} at crack mouth, depending upon the crack length. Therefore, 70% of the maximum σ_{cf} was adopted conservatively as the constant value of crack closure stress σ_{cf} acting as a uniform stress along the crack flanks and the same is used in Eq. 8.

The effective stress intensity factor (K_{eff}) at crack tip was then evaluated from the superposition of the stress intensity factors corresponding to stress sharing and crack bridging mechanisms (K_s and K_b) and is given by Eq. 9.

[9] $$K_{eff} = K_S - K_b$$

Additionally the effect of adhesive shear deformation was also introduced in the K_{eff} evaluated from Eq. 9 by treating it as an additional source of enhancing the COD. Knowing the steel stress from Eq. 1, adhesive and steel thicknesses and the patch bonded length, the adhesive shear deformation was calculated on the basis of average shear stress in the adhesive. Treating the adhesive shear deformation an increment in the COD and reverse calculating an equivalent hypothetical steel stress that causing that COD using equation similar to Eq. 4, an additional SIF 'K_{adh}' was calculated using Eq. 2 or 8 using only that hypothetical steel stress in these equations.

The K_{eff} from Eq. 9 was enhanced by adding this component K_{adh} and is shown in Eq.10, which can be treated as the effective stress intensity factor incorporating the impact of patch stiffness, crack length, crack closure effect cause by CFRP, patch delamination and the shear deformation of the adhesive layer at steel interface.

[10] $$K_{eff} = K_S - K_b + K_{adh}$$

4 Results and discussion

SIF variation with increasing crack length and ETR values was studied from the results of FEA together with the impact of patch delamination around the crack tip and at the patch end due to adhesive shear failure there. Figure 3 shows progressive delamination of the adhesive around 25 mm crack due to adhesive shear failure in lower and higher E_{FRP} repairs with ETR of 0.264. It is obvious that the maximum patch delamination was achieved in the first run and then the delamination reduced in successive delamination cycles until the convergence achieved. The delamination pattern is similar in all the repairs and found to be decreasing with increasing ETR values. For a given ETR the delamination was found more in longer crack lengths and in lower E_{FRP} repairs because of lower adhesive shear strength in these. Maximum delamination length in the lower E_{FRP} CFRP repair after the 9 delamination cycles was found to be 46.6 mm on each side of crack while it was only 4 mm in higher E_{FRP} repair, which was observed in the only one cycle, after which it didn't delaminate because of higher adhesive shear and bond strengths. The delamination lengths in lower E_{FRP} repairs were 37 mm and 32 mm respectively for the patch ETR of 0.53 and 0.8 respectively. For the same ETR values the delamination lengths in higher E_{FRP} repairs were 3 mm and 2.5 mm respectively. It clearly shows the impact of adhesive bond strength, which was quite low in the repairs with lower E_{FRP} repairs and resulted in large delaminated lengths.

4.1 Impact of relative patch stiffness (ETR) on SIF

The impact of patch stiffness (ETR) on the SIF was studied from the results of FEA, which showed that for any given crack length the SIF decreases with increase in the ETR value, both before and after the patch delamination, as shown in Figure 4. This finding have also validated the results of several previous researches (Lam and Cheng 2008, Holden 2012, Mobeen et al. 2012).The reduction in SIF for the increase in ETR from 0.264 to 0.8 was ranged between 25% and 45% for the two types of CFRP but its big fraction occurred in the increase in ETR from 0.264 to 0.53 and the reduction was generally observed more for larger crack lengths.

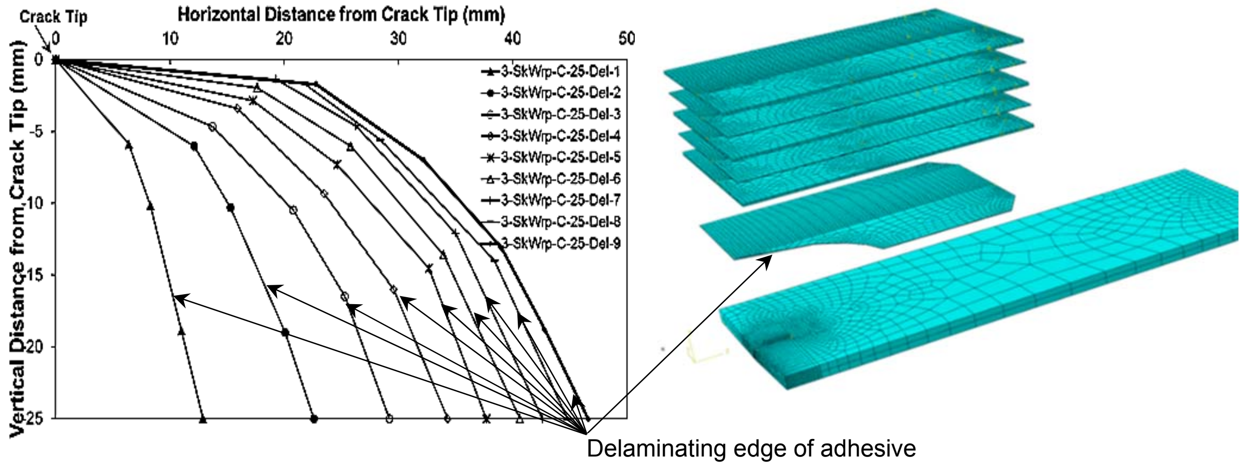

Figure 3: Shear failure and successive reduction of adhesive layer at steel interface

4.2 Impact of Patch Delamination on SIF

Delamination of bonded patch resulted in the enhancement of the SIF of any given repair and the crack length, as shown in Figure 4 (for ETR of 0.264, 0.53 and 0.8). It implies that for a constant applied stress range and given crack length the SIF will increase due to the patch delamination (caused by the adhesive shear failure) and, as a result of which, the crack length is expected to increase. The effect of delamination in increasing SIF reduced with the increase in repair ETR values for both types of CFRP repairs, which implies that for any given crack length the delamination and the SIF decrease by increasing the ETR or the number of bonded CFRP layers. For a given ETR, the SIF increased more in repairs with lower E_{FRP} values because of more patch delamination in these. In repairs with lower E_{FRP} and ETR of 0.264 the increase in SIF, after patch delamination, was found to be 128% and 25% respectively for non-dimensionalized crack lengths of 0.5 and 0.2. The same comparison in higher E_{FRP} repairs showed the SIF enhancement of 48% and 16%, which implies that if considering two repairs with identical ETR of 0.264, one having 3 layers of lower E_{FRP} CFRP and other having 1 layer of higher E_{FRP} CFRP, with an initial non-dimensionalized crack length of 0.5 and under a 100 MPa applied stress, the SIF of the repair with lower E_{FRP} CFRP will increase by 128% after the adhesive shear failure around the crack, compared to a lesser increase of 48% in the repair with higher E_{FRP} CFRP. But similar comparison of the delamination effects at higher ETR values showed lesser impact on the SIF enhancement because of lesser patch delamination in these repairs. It is important to mention here that in the repairs with E_{FRP} of 0.53 and 0.8, with non-dimensionalized crack lengths of 0.2, the maximum shear stress in the adhesive layer was less than its shear strength all over, which implied that no patch delamination expected in these repairs and hence no delaminated models were developed for these repairs.

The increase in SIF due to patch delamination in repairs with ETR of 0.53 and initial non-dimensionalized crack length of 0.5 was found to be 95% and 38%, respectively for lower and higher E_{FRP} repairs, while for the ETR of 0.8 it was found to be 80% and 36% respectively. Conclusively, the maximum increase in SIF in lower E_{FRP} repairs caused by the delamination was 5 times more than the higher E_{FRP} repairs but for higher ETR values it reduced to 2.5 times. Patch delamination was observed more in lesser ETR repairs because of least number of FRP layers to carry the steel stresses over the crack resulted in higher FRP stress and higher corresponding adhesive shear stresses surrounding the crack, thus the repair suffered with maximum adhesive failure and patch delamination.

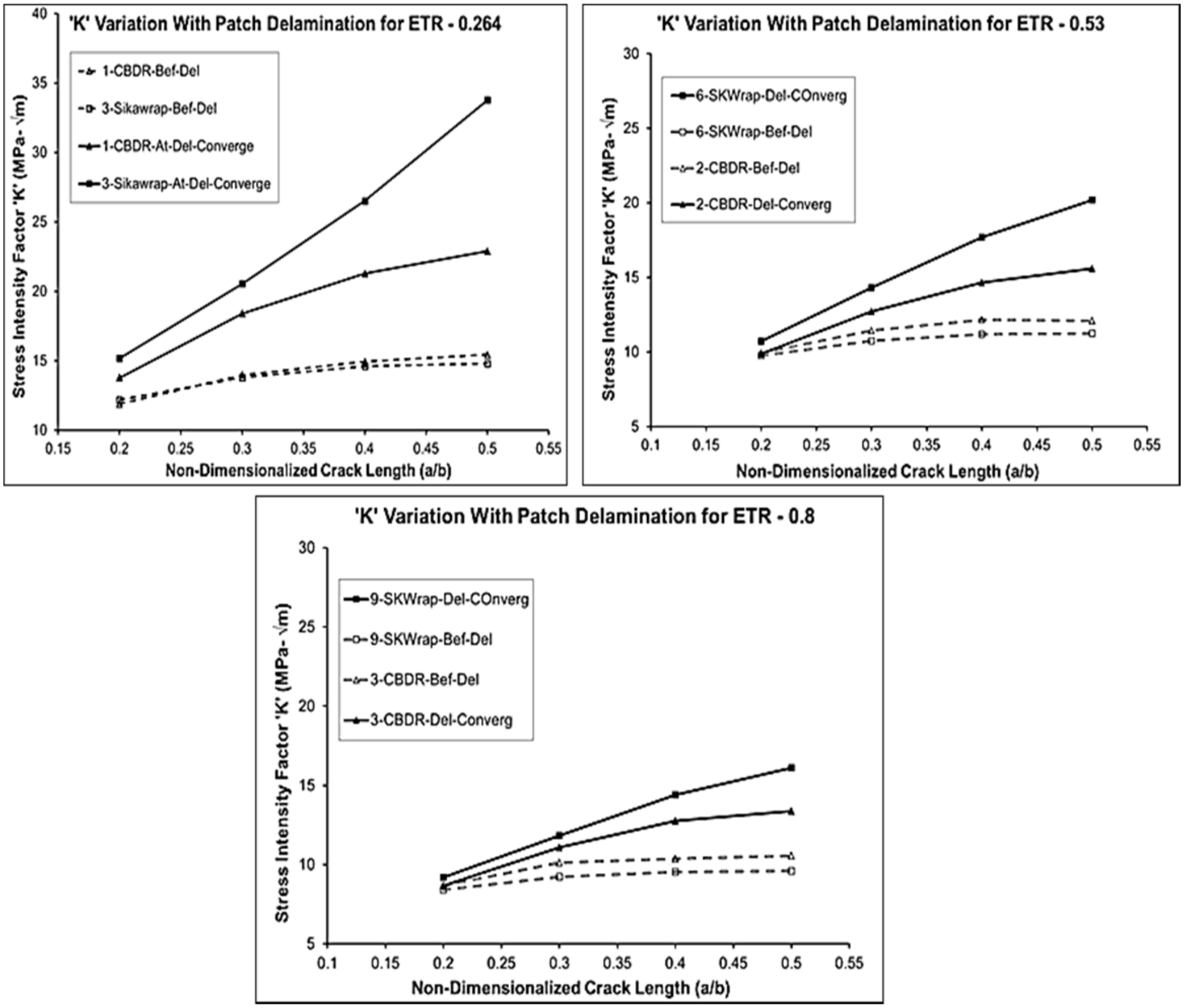

Figure 4: Effect of patch delamination on SIF from FEA results

4.3 Results of Analytical Approach

The SIF of all repairs were also evaluated using the analytical approach (Eq. 10) described in section 3.2 for each delaminated model and its results are shown in the Figure 5 for ETRs of 0.264, 0.53 and 0.8, together with the FEA analyses results. It is obvious that the SIF obtained using the analytical approach showed acceptable agreement with those obtained using the FEA. It is also important to mention here that the SIF values shown in these figures correspond to the maximum delaminated level of the adhesive. It is obvious from these figures that better agreement between the two found in repairs with ETR of 0.264 and 0.53, with maximum difference of 15% but for ETR of 0.8 the SIF values deviated more from the FEA results with maximum difference of 22%. It is also obvious in Figure 5 that the trends of SIF variation were quite different within the FEM and the analytical approaches except for the repair with ETR of 0.264 with lower E_{FRP} CFRP. The reasons being least patch sharing stress in this repair (because of lowest ETR value) and maximum patch delamination (because of maximum adhesive failure) which turned the behavior of the patched plate towards an un-patched plate.

A common difference in the trends of non-delaminating FEMs and the analytical solutions is the reverse in curvature of the trends which is mainly because of the geometric correction factors used in the analytical approach (F and V), as these may not be valid for the behavior of a patched steel plate.

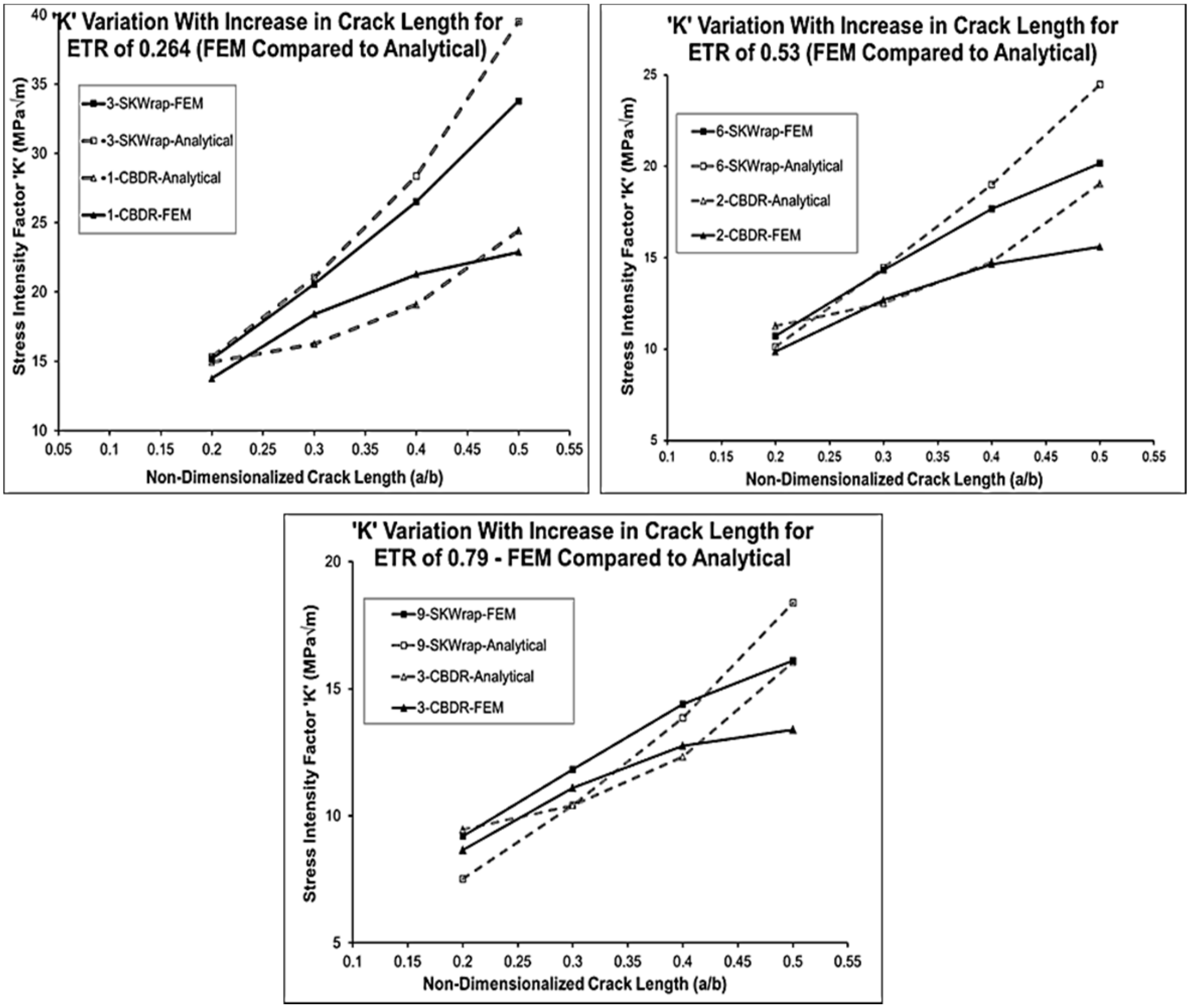

Figure 5: Comparison of SIF obtained from FEA and the analytical approach

5 Summary of Results and Conclusions

Following conclusions can be drawn from the results:

- CFRP patches were effective in reduction of SIF.
- Higher ETR repair patches were more effective in reducing the SIF.
- Patch delamination significantly affect the SIF in repairs with lower E_{FRP} CFRP – 5 times more than the repairs with higher E_{FRP} CFRP (Repair with non-dimensionalized crack length of 0.5).
- Lower E_{FRP} repairs delaminated more because of lower adhesive shear strength, so, adhesive shear strength was found to be an important factor in reducing patch delamination and SIF.
- Higher ETR patches suffered with lesser delamination for both higher and lower E_{FRP} repairs.
- SIF values predicted by the analytical approach were in better agreement with those obtained from the FEM for lower ETR and E_{FRP} patches as these shared lesser plate stress and were more delaminated. SIF values in higher ETR and E_{FRP} patches deviated more while compared with those obtained from FEM, with a maximum difference of 21%, which might be because of lesser delamination and the use of the correction factors for finite width plate ('F' and 'V') in the analytical model, as these were developed for an un-patched plate.

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