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Comparison of two Bayesian Methods for Road Accident Prediction Modelling

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Abstract: In this paper two Bayesian methods for the development of accident prediction models are compared: the well acknowledged Empirical Bayes method and a recently developed method based on Bayesian Probabilistic Networks. Brief descriptions of the two methods are provided and their commonalities, differences, advantages and disadvantages are discussed. Both methods can be used to develop models for a multivariate prediction of accident events and can be included into modern road risk management systems, e.g. road safety impact assessments or road safety audits. It is shown that, although both methods result in models that can be used to predict the numbers of accident events, the models developed using the Bayesian Probabilistic Networks method have a better correlation with the actual data than the models developed using the Empirical Bayes method; measured through the higher values (approximately 10%) of the correlation coefficients for most of the model response variables and the reduced bias of the results.

1 Introduction

The influence of road design parameters and traffic properties on the occurrence of accident events and the development of road accident prediction models are important aspects of road management and are therefore research areas of considerable interest. Numerous studies have been performed to identify the most important risk indicating variables (RIVs) and to describe the relationships between these RIVs and the occurrence of road accidents. Comprehensive overviews of the different methods used for predicting accident events on roads can be found in Hauer (2009), Elvik (2011), Lord and Mannering (2010) and Savolainen *et al.* (2011). Since road accidents are relatively rare events when compared to the amount of vehicles on the roads, considerable uncertainty is often associated with the number of accident events predicted based on observed data. The capability of a methodology to deal with this uncertainty in the development of accident prediction models is of high importance to gain reliable model results.

In Hauer et al. (2002) it is shown that methods that are based only on counts of accident events may lead to inaccurate modelling results, either due to a large variance in the counts (over-dispersion) or due to a systematic bias in the predictions (regression-to-the-mean). These weaknesses can be overcome by using methods that use a combination of theoretical prediction models and real observations. In this paper two such methods are compared, the so-called Empirical Bayes (EB) method and a recently developed Bayesian Probabilistic Network (BPN) method. Brief descriptions of the methods, outlines of their main commonalities and differences and their main advantages and disadvantages, are given. The performance of the two methods is evaluated by comparing the predicted numbers of accident events with the actual observed numbers on road sections the data of which has not been used for model development.

2 Methods

2.1 Homogenous Segments and Multivariate Poisson-lognormal Regression Analysis

Both methods are based on defined sets of RIVs (model input) and model response variables (MRVs) (model output). The RIVs are observable road and traffic characteristics that are considered to influence the conditional occurrence probability of the MRVs. The RIVs and the MRVs are defined for model development, from case to case, taking into consideration the problem to be investigated and the availability of data.

With both methods the investigated road network is sub-divided into homogeneous segments, for each of which it can be assumed that the values of all RIVs to be included in the accident prediction models are constant, and therefore the occurrence probabilities of the MRVs over the length of the homogeneous segment can be considered to be uniform. When homogeneous segments are used, generic accident risk models are first developed based on data for the entire network and are only made specific when the values of the RIVs for the homogeneous segments are used as model input.

Both methods are applied by using multivariate Poisson-lognormal regression analysis (subsequently only referred to as regression analysis) to establish prior prediction models, i.e. to describe the linear causal relationships between the RIVs and the MRVs, which are often referred to as safety performance functions. The explanatory variables are the RIVs and the dependent variables are the multidimensional MRVs representing the expected accident event rates. The structural component of the proposed regression model is:

[1] $$\ln\left(E\left[\mathbf{\Lambda}\middle|\mathbf{X}\right]\right)=\mathbf{BX}+\mathbf{\Xi} \quad \hat{=} \quad E\left[\mathbf{\Lambda}\middle|\mathbf{X}\right]=\exp\left(\mathbf{BX}+\mathbf{\Xi}\right)$$

where $\mathbf{X}$ is the design-matrix of explanatory risk indicating variables, $\mathbf{\Lambda}$ is the matrix of the model response variables (accident rates),
$\mathbf{B}$ the matrix of regression coefficients and $\mathbf{\Xi}$ the matrix of the error terms.

The values of the MRVs are, however, not used directly. Instead, a gamma updating of the MRVs is performed with the assumption that the accident counts can be represented by a negative-binomial distribution. This is done:

- to deal with accident counts that are characterised by over-dispersion, i.e. the variance is larger than the mean, which is normally the case (Hauer 2001)
- to dilute the effects of individual outliers of exceedingly high counts through an embedded weighting process, and
- to avoid the preponderance of zero values;

The latter of which is something that better reflects the fact that accident frequencies are larger than zero, even if there are no observations on one homogeneous segment over the time of observation, which is often triggered by short observation periods or short segment lengths (Deublein *et al.* 2012).

A negative binomial distribution is a mixture of a Poisson distribution and the natural conjugate gamma distribution. The first with parameter μ_{ki} describing the probability of having a defined number of accident events on one particular homogeneous segment over a defined period of time and the latter describing the parameter μ_{ki}. μ_{ki} in this case is the mean frequency of accident events, given by:

[2] $$\mu_{ki}=\lambda''_{ki}\cdot\nu_i$$

where:

λ''_{ki} is the gamma-updated accident rates (accidents per million vehicle kilometres per year)

ν_i is the exposure (number of vehicles per kilometre and year) of the homogenous segments

k indicates the type of accident
i indicates the homogeneous segment

The gamma updating of the accident rates is done by updating the prior parameters of the gamma distribution according to (Gelman *et al.* 2004) as:

[3] $$\alpha''_{ki} = \alpha'_{ki} + \tilde{y}_{ki} \quad \text{and} \quad \beta''_i = \beta'_i + \tilde{\nu}_i$$

where:

α''_{ki} is the posterior shape (dispersion) parameter of the gamma distribution and

β''_i is the posterior inverse scale parameter of the gamma distribution.

$\tilde{y}_{ki}$ and $\tilde{\nu}_i$ are the observed accident counts and observed values of exposure, respectively.

α'_{ki} and β'_i are the prior parameters of the gamma distribution, assessed as

[4] $$\alpha'_{ki} = \dot{\lambda}_k \cdot \beta'_i \text{ with } \beta'_i = \nu_i \cdot \omega_i = \nu_i \cdot \frac{\psi}{l_i}.$$

where:

ω_i is the weight calculated as the fraction of the weighting factor ψ and the individual homogeneous segment length l_i.

ψ is a weighting factor attributed to information about the prior gamma parameter β'_i. It is used to take into account the time period of observations based on which the prior information has been gathered, experts experience and appraisal of the quality of the prior information.

$\dot{\lambda}_k$ are the averaged background accident rates of the MRVs determined based on analysis of available historical data.

2.2 Bayesian Probabilistic Network Method

Recent research on modelling the expected number accident events by means of Bayesian Probabilistic Networks includes the work done by Davis and Pei (2003), Marsh and Bearfield (2004), Ozbay and Noyan (2006) and Simoncic (2004), de Oña *et al.* (2011), Karwa *et al.* (2011), Schubert *et al.* (2011) and Hossain and Muromachi (2012). The BPN method discussed here (Deublein *et al.* 2012) is similar to those developed elsewhere in that Bayesian inference and updating algorithms are used to develop models to predict the number of accident events. It is, however, different in that it uses both: **a)** a hierarchical regression analysis and **b)** BPNs that take into account aleatory and epistemic uncertainties, as well as non-linear dependencies. This combination makes it possible to deal with a very general dependency structure (low individual correlations between RIVs and MRVs) in the data as well as non-linear causal relationships. Based on the results of the regression analysis (probability density functions of the regression coefficients and the error terms) the prior predictive distributions of the MRVs are assessed, the prior BPN is established, and parameter learning is performed by using an parameter learning algorithm (EM-algorithm) to iteratively update the internal causal interrelationships and dependencies based on observed data, to obtain the posterior BPN. In the creation of the posterior BPN, the purely empirical regression model based probabilities and linear relationships in the prior BPN are replaced by observation based posterior probabilities and non-linear relationships. By determining the posterior BPN the mean values of the posterior predictive probability density function of the MRVs λ''_{ki} are also determined. These are multiplied with the exposures of the homogeneous segments ν_i to obtain the parameter of the Poisson distribution μ_{ki} which is used to estimate the expected number of road accident events $\hat{y}_{ki}$ over a defined period of time. For more information about BPNs reference is given in Cowell (1999), Pelikan (2005), Jensen and Nielsen (2007) and Kjaerulff and Madsen (2008).

2.3 Empirical Bayes Method

The EB method can be considered as today's most common state-of-the-art method for the development of accident prediction models, and considerable research has been conducted using it, including that by Hauer (Hauer 1992, 2001, 2010), Carlin and Louis (1997), Persaud *et al.* (1999, 2010), Hauer *et al.* (2002), Carriquiry and Pawlovich (2005) and Elvik (2007, 2008). The theoretical modelling in the EB method consists of two key components:

- the determination of the safety performance functions based on the regression analysis (as mentioned in section 2.1) and
- the estimation of the values of the over-dispersion parameters as the reciprocal value of the distribution (shape-) parameter α, which are estimated by fitting a negative-binomial distribution to the observed numbers of accident events (Hauer 2001). Based on the estimates of the safety performance functions $\hat{\lambda}_{ki}$, the over-dispersion parameters ϕ_{ki} and the observed counts of accident events $\tilde{y}_{ki}$ on one specific homogeneous segment i the expected numbers $E[\cdot]$ of the accident events $\hat{\mu}_{ki}$ are calculated as

[5] $$E\left[\hat{\mu}_{ki}\left|\tilde{y}_{ki},\hat{\lambda}_{ki}\right.\right]=w_{ki}\cdot\hat{\lambda}_{ki}+\left(1-w_{ki}\right)\cdot\tilde{y}_{ki}$$

with

[6] $$w_{ki}=\frac{1}{1+\left(\hat{\lambda}_{ki}/\alpha_{ki}\right)}$$

where

the weight w_{ki} is assumed to be gamma distributed with shape parameter $\alpha_{ki}=1/\phi_{ki}$. It determines the weight given to the theoretically predicted number of accident events taking into consideration the number of observed accidents.

Using the EB method there are two types of over-dispersion parameters, individual over-dispersion parameters ϕ_{ki} of the individual homogeneous segments, and overall over-dispersion parameter ϕ_k for the entire investigated network. The latter is related to the former as a function of the length of the homogeneous segments (Hauer 2001) as:

[7] $$\phi_{ki}=\phi_k\cdot l_i^{\tau}.$$

where

τ is the weighting exponent and has a value between zero and one. It's value is selected to ensure that homogeneous segments are weighted as a function of their number of observed accident events (Hauer 2001). More information about the EB method can be found in (Elvik 2008) (Hauer 1992) (Hauer 2001) (Hauer et al. 2002) (Persaud et al. 1999) and (Persaud et al. 2010).

3 Example

3.1 Description

To compare the BPN and the EM method, both were used to develop accident prediction models for Austrian rural roads with lanes physically separated based on driving direction. The models were then used to predict the occurrence of accidents on homogeneous segments between 2004 and 2010 where it was assumed no data was available. The results were then compared with the observed number of accidents for the same period of time. The BPN and the EB methods described in sections 2.2 and 2.3 were used. The example related issues are explained in the following sections.

The cumulative length of the investigated roads and the observed numbers of the different accident events between 2004 and 2010 are given in Table 1, for the entire data set, the development dataset and the test dataset. The development dataset and the test dataset are subsets of the entire dataset where the former was used for model developments (about 75% of the entire data), and the latter was used to test the models (about 25% of the entire data). The road segments in the development and test dataset were randomly selected. The entire road network was sub-divided into *n=6'932* homogeneous segments.

Table 1: Length of investigated roads and numbers of observed accident events [2004-2010]

	length	**IAC**	**LINJ**	**SINJ**	**FAT**
	[km]	*[-]*	*[-]*	*[-]*	*[-]*
entire dataset	3'642	12'892	14'482	5'861	529
development dataset	2'952	10'282	11'460	4'677	409
test dataset	690	2'610	3'022	1'184	120

The RIVs selected include six road-specific design variables and two traffic variables (Table 2).

Table 2: Risk Indicating Variables (RIVs)

RIV	**Definition**	**Values**	**Unit**
CHAR	Different types of road sections, namely 1) exit corridors, 2) intersections, 3) tunnels and 4) normal/open roads. Exit corridors and intersections are defined over a range of one kilometre including a 500 m section before and after their centroid.	$(1,2,3,4)$	$[-]$
AADT	Annual average daily traffic pro driving direction.	$(1,2,...,10)\cdot 10^4$	$[veh/d]$
HGV	Percentage of heavy good vehicles with respect to the AADT.	$(5,10,...,30)$	$[\%]$
BEND	Horizontal curvature; integer variable having values between zero (straight road) and ten (very high curvature), determined as the fraction of the sum of the lengths of ten subsequent 50m road sections divided by the length of the straight distance between the starting point of the first and end point of the tenth section. Data of a geographical information system (GIS) was used for assessment of the bend factor.	$(0,2,...,10)$	$[-]$
SLP	Percentage of the upwards or downwards gradient (slope).	$(-6,-4,...,0,...,6)$	$[\%]$
LAN	Number of driving lanes per direction.	$(1,2,3,4)$	$[-]$
SPD	Signalized speed limit.	$(80,90,...,130)$	$[km/h]$
EML	Existence of emergency lanes, binary variable	$(0=no,1=yes)$	$[-]$

Four MRVs were selected taking into consideration the definitions of accidents and injury levels in accordance with §84 of the Austrian Penal Code (Bundesrepublik Oesterreich 2012) (Table 3). The MRVs are the occurrence rates of the accident events being the observed numbers of accident events on a homogeneous segment divided by its exposure. The exposure is the product of the number of vehicles (*veh*) with the length of the homogeneous segments (*km*) and with the observation period (*years*) having the unit of million vehicle kilometres per year ($mvkm$).

Table 3: Model Response Variables (MRVs)

MRV	**Definition**	**Values**	**Unit**
IAC	Occurrence rates of all injury accidents, i.e. accident events where at least one vehicle is involved and at least one occupant becomes at least lightly injured.	0.001 for $0 \le \lambda_{IAC} < 0.01$ 0.01 for $0.01 \le \lambda_{IAC} < 0.2$ 0.1 for $0.2 \le \lambda_{IAC} < 2$	$[IAC/mvkm]$
LINJ	Occurrence rates of light injured road users. A road user is considered to be lightly injured if the damage to his well-being lasts less than 25 days following the accident event.	0.001 for $0 \le \lambda_{LINJ} < 0.01$ 0.01 for $0.01 \le \lambda_{LINJ} < 0.2$	$[LINJ/mvkm]$
SINJ	Occurrence rates of severely injured road users. A road user is considered to be severely injured if the damage to his well-being lasts more than 24 days following the accident.	0.001 for $0 \le \lambda_{SINJ} < 0.01$ 0.01 for $0.01 \le \lambda_{SINJ} < 0.2$	$[SINJ/mvkm]$
FAT	Occurrence rates of fatally injured road users. A road user is fatally injured when he has died within 30 days following the accident event as a consequence of accident induced injuries.	0.0001 for $0 \le \lambda_{FAT} < 0.001$ 0.001 for $0.001 \le \lambda_{FAT} < 0.02$	$[FAT/mvkm]$

3.2 Multivariate Poisson-Lognormal Regression Analysis

The multivariate Poisson-Lognormal regression analysis (subsequently only referred to as regression analysis) was performed on the development dataset with the data of the homogeneous segments being weighted according to their individual exposure values. The regression coefficients were assessed by means of maximum likelihood estimation having different values for the different MRVs; simultaneously for all MRVs per road section type. Road section type is handled differently from the other MRVs as they are discrete, and fundamentally different, categories. The statistical significance of the results was tested using a Students t-test for the individual regression coefficients at a significance level of $\alpha = 0.05$.
The dependent variables used for the regression analysis were the values of the gamma updated MRVs as described in paragraph 2.1. The parameters of the gamma distribution were quantified and updated for each homogeneous segment based on the background accident rates and the weighting factor ψ as given in Table 4. The values of the background accident rates and ψ were derived by using a non-linear generalized reduced gradient optimization algorithm objecting to minimize the difference between the sum of the regression model based accident predictions and the sum of the observed average accident events in the development dataset (Deublein *et al.* 2012).

Table 4: Weighting factor and background rates for model development

ψ	$\dot{\lambda}_{IAC}$	$\dot{\lambda}_{LINJ}$	$\dot{\lambda}_{SINJ}$	$\dot{\lambda}_{FAT}$
$[-]$	$[IAC/mvkm]$	$[LINJ/mvkm]$	$[SINJ/mvkm]$	$[FAT/mvkm]$
0.3	0.08764	0.09910	0.03705	0.00315

Example safety performance functions for open roads and tunnels with respect to the RIV AADT are shown in Figure 1. It can be seen that the safety performance functions

- of the injury accidents and light injuries on open roads are similar while the safety performance functions of the severe injuries and the fatalities are different than the safety performance functions of the injury accidents and light injuries.
- vary considerably in shape and magnitude for open roads and tunnels. In terms of AADT the probability of injury accidents, light injuries and severe injuries in tunnels is considerably higher than for roads but the probability of fatalities is considerably lower (maximum occurrence rate of *0.0003 fatalities per* $mvkm$); something which may be attributed to safety enhancing measures such as reduced speed limits, increased lane width, or clear signalization.

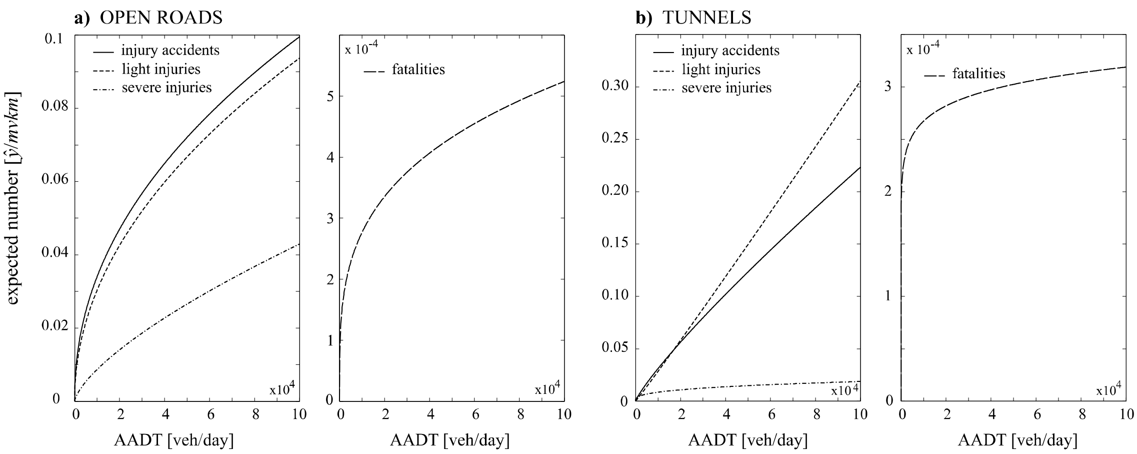

Figure 1 Safety performance functions

3.3 Bayesian Probabilistic Network Method

Both, the estimates of the regression coefficients as well as the distribution of the error term, were used to assess the prior predictive distribution of the MRVs for establishing the prior BPN. The open-source inference engine of the program GeNie 2.0 (Decision-Systems-Laboratory-Pittsburgh 2006) was applied to construct the network and to calculate the marginal probability distribution functions. The values of the continuously distributed MRVs were discretized to facilitate model development and parameter learning procedures. Structural learning was not applied in the current investigations since the causal relationships were evaluated and determined based on the outcomes of the regression analysis and based on expert judgement. In Figure 2 the structure of the established BPN is shown.
Eight RIVs were chosen as the input nodes of the BPN. When nothing was known about the considered homogeneous segments default probabilities of the states of the RIVs were used as given in the bar charts of the parent nodes in Figure 2. The values of the bar charts correspond to the relative frequencies with which the values of the RIVs were observed in the development dataset. For model application, a particular homogeneous segment was then described by putting evidence in the different nodes of the RIVs by selecting the appropriate states (e. g. *CHAR=4*, *AADT=40'000*, *HGV=12%*, etc.). The four MRVs were taken as the output nodes of the BPN. All input nodes were connected to all output nodes by directed edges.

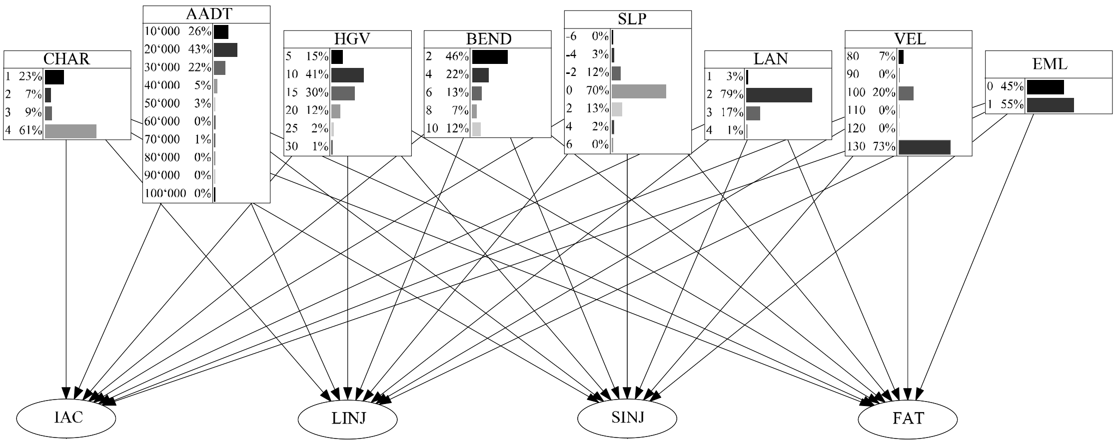

Figure 2: Developed Bayesian Probabilistic Network

Monte Carlo simulations of all regression coefficients and error terms were performed to establish the predictive probability density functions of the MRVs, and to extrapolate the information of the observed data into the entire modelling space in order to provide distributions of the MRVs also in those areas of the model space where no observations have been available. The simulations were done using the distributions of the regression coefficients and the covariance matrices of the error terms being assessed in the regression analysis.
Parameter learning was then performed using an parameter learning algorithm (EM-algorithm) assuming a rather small value for the experience factor of $e=0.1$, i.e. this value gives almost no weight to the prior information and hence the posterior distribution has high weight and becomes very similar to the observed data. During the parameter learning process only the domains of the prior BPN were updated for which there were observations in the development dataset.

3.4 Empirical Bayes Method

The overall over-dispersion parameters were assessed as described in (Elvik 2008) based on the development dataset (Table 5). The values of the overall over-dispersion parameters are similar for all MRVs for exit corridors and open roads, except for fatalities. The much higher values for fatalities for exit corridors may be due to the very small number of fatalities observed and a high statistical uncertainty associated with these values (Deublein et al, 2012).

Table 5: Overall over-dispersion parameters

	exit corridors	intersections	tunnels	open roads
injury accidents	2.4476	1.9379	4.5054	2.5312
light injuries	2.2328	1.8623	4.1462	2.3217
severe injuries	2.9240	3.2878	5.2546	2.8254
fatalities	11.1127	16.2621	9.5581	5.1938

The different overall over-dispersion parameters were taken into account for the assessment of the individual over-dispersion parameters of the homogeneous segments in the test dataset according to equation [7]. The exponent τ can have values between zero (no influence of individual section length on over-dispersion parameter) and one (over-dispersion parameter is multiplied by the length of segment). A value of $\tau = 0.8$ was chosen to give not full but considerable influence of the segment length on the over-dispersion parameter. Since it is assumed for the model evaluation that no information is available about accident events for the road segments in the test dataset, the updating was done using similar segments of the development dataset, i.e. homogeneous segments of the development dataset that have the same constant values of the RIVs as the homogeneous segments to be evaluated in the test dataset.

3.5 Model Comparison

The model predictions using the BPN and EB methods along with the number of observed accident events on the homogeneous segments of the test dataset are shown in Figure 3 for injury accidents and light injuries. The regression lines are indicated through solid lines and the regression equations as well as the coefficients of determination (R^2) and correlation (r) are provided for each. The dashed lines indicate (in average) perfect accordance between the model predictions and real observations. Regression lines above the dashed line indicate the model predictions in average to underestimate the real number of accident events, regression lines below the bisecting line indicate the model predictions in average to overestimate the real number of accident events. As can be seen both methods resulted in models that could be used to accurately predict the occurrence of injury accidents (the regression lines are very close to the dashed lines).

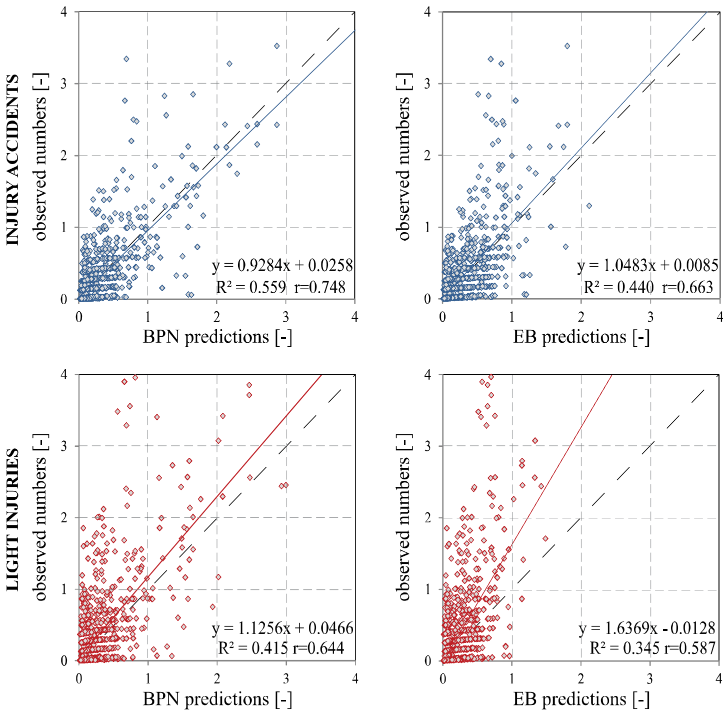

Figure 3: Scatter plots of real observations and model predictions

The EB method, however, resulted in models that considerably underestimate the number of light injured road users (the regression line is much steeper than the dashed line). The BPN and EB methods resulted in models with values of the correlation coefficients of $r=0.748$ and $r=0.663$ for the prediction of injury accidents, respectively, i.e. an improvement of approximately 10%. A similar difference can also be observed for the prediction of lightly injured road users. Since the correlation coefficients for the prediction of severe injuries and fatalities are relatively low, i.e. $r<0.50$, only limited conclusions for the prediction of severe and fatal injury events can be made. The poor correlation is expected to be due the low occurrence frequencies of these two MRVs.

One way to overcome this problem could be seen in the implementation of theoretical prediction models which are available in literature as results from other investigations on different (probably larger) datasets. For the BPN method it would be straightforward to implement a combination of data-based interrelationships between the RIVs and the MRVs and literature-based theoretical models and improvements in the predictive capabilities of severe injuries and fatalities are expected. The implementation of theoretical prediction models was beyond the scope of this work.
Comparison of the predictions for injury accidents on the homogeneous segments of the test dataset resulting from the EB and BPN models (Figure 4) show that BPN models tend to provide more accurate (and higher) estimates than EB models (the regression line is below the dashed line). The value of the correlation coefficient between the predictions made using the BPN and the EB models is $r=0.883$.

4 Conclusions

In this paper, the ability of the using the Bayesian Probabilistic Networks method and the Empirical Bayes method to develop models that result in accurate prediction of accident events were compared. In both methods Bayesian inference and updating algorithms are used and both combine theoretical safety performance functions with real observations of accident events. This is done by using a multivariate Poisson-lognormal regression analysis for the assessment of prior inferences that are then used to update the theoretical relationships with information of real observations.
The model response variables used for the comparison were the numbers of injury accident events and the number of injured road users having no more than light injuries, severe injuries and fatal injuries. The risk indicating variables were selected taking into consideration both road design and traffic parameters. The comparison was made by comparing the predicted number of accident related events with the observed number of accident related events and by comparing the predicted numbers of accident related events using the two models.

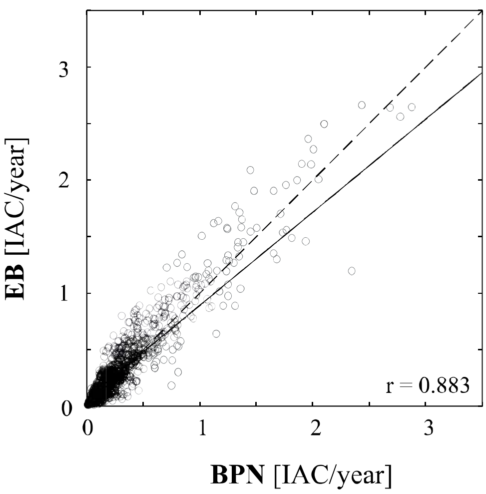

Figure 4: Comparison of EB and BPN predictions for the expected number of injury accidents (IAC) per year

It was found that:

- the models developed using both methods showed good agreement between the predicted and observed numbers of accident related events, i.e. both models resulted in a good correlation between the predicted and observed numbers of the values of the model response variables and there was a good correlation between the predicted values of both models.
- the models developed using the Bayesian Probabilistic Network method gave slightly more accurate estimates of the number of accident related events, i.e. the values of the correlation coefficients of the Bayesian Probabilistic Network models were approximately 10% higher than those of the Empirical Bayes models.

Bayesian Probabilistic Network models, once developed, can readily be implemented into Geographic Information Systems (GIS) for geo-referenced road safety assessment and accident risk based decision making.

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