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Analysis of Prompt Payment Discounts in Construction

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Abstract: The potential use of trade credit in construction invoicing is an understudied facet of the cash flow problem. In particular, prompt payment discounts of accounting such as '2/10 net 30' are examined, which are a mechanism to incentivize faster inflows, yet at a reduced amount. A nomograph is developed from financial theory as a tool to simplify the discount decision-making process. Instructions on its use are provided from the perspectives of discount-maker and discount-taker. Furthermore, singularity functions are used to demonstrate how these discounts may be mathematically expressed. Validation employs a computer model that is built on such singularity functions.

1 Introduction

The prompt payment discount is the most typical type of trade credit (Kouvelis and Zhao 2012). A supplier may be willing to offer this contract term to entice a buyer to pay sooner in exchange for a small discount, or otherwise pay the invoice in full by the deadline. Arguably the most common prompt payment discount term is '2/10 net 30.' In the construction industry, material suppliers are sometimes willing to give prompt payment discounts to encourage timely payment from the contractors; "normally it is a 2% discount if paid in full by the 10^{th} day of each month following delivery of the materials" (Zwick and Miller 2004, p. 247).

Economic success largely relies on finding a win-win scenario for all participants of a construction project, which is not necessarily a zero-sum game. One challenge is to determine how much of a discount to give and when. Project participants therefore need a mathematical tool that can facilitate these decisions. This tool must be built with the awareness that payees should not offer trade credits whose terms will hurt their bottom line; while payers will not accept or act upon contract terms that will make them worse off either. Once validated to gain acceptance, such tool can be used by planners and researchers to perform further analyses such as net present value (NPV) and optimizing the entire cash flow of a construction project.

2. Literature Review

The cash flow problem in the construction industry is well- documented in the academic literature. Navon (1996, p. 22) noted that "[c]ash is the most important of the construction company's resources" because mismanagement of this resource leads to more company failures than any other resource type. While that study was engaged in forecasting company-level cash flows, others (e.g. El-Abbasy et al. 2012) focused on approaches for optimizing cash flows for several projects at once. However, much more research has been devoted to modeling and optimizing project-level financing (Lucko 2011, Akpan and Igwe 2001).

A recently developed method for quantifying and analyzing linear schedules using singularity functions (Lucko 2007) has been extended to schedule optimization with regard to cash flows (Terry and Lucko 2012). Lucko and Cooper (2010) demonstrated how activities can be summarized algebraically using singularity functions. Further study (Lucko 2011) demonstrated the ability of singularity functions to model analyse the entire cash flow profile of a project in a computationally efficient manner. This new approach thus represents "the first accurate model that did not have to revert to using discrete values, averaged S-curves, or envelopes" (Lucko 2011, p. 532). Further, it succeeded where previous studies largely failed, by accounting for compounding interest with financing singularity functions (Lucko and Thompson 2013).

Virtually all aforementioned studies that examined cash flows had assumed ideal payment conditions. Yet stated succinctly, "the real major [*sic*] problem is […] delay in paying for work already done" (Akpan and Igwe 2001, p. 367), which can have a significant detrimental impact. Touran et al. (2004) have examined United States Department of Transportation (DOT) provisions that require principal contractors who work for any state DOT to pay their subcontractors promptly and in full (including retainage). They have studied a sample of several transportation projects and their finance charges with and without including such a prompt retainage payment provision. This sample, albeit small, demonstrated that the early release of the retainage negatively impacts contractors' profit between 2% and 7% from the finance charges alone.

Analysis of timing with regard to an individual cash flow is one perspective that seems to be understudied within the field of construction management. Horngren et al. (2010) identified three common credit terms that companies (not specific to the construction industry) may offer; 'net 30' is where the full price is due thirty days after the invoice is delivered. '1/5, net 30' is structurally similar to '2/10, net 30' but offers that a 1% discount may be gained if payment is received within five days, otherwise the entire amount is due within thirty days. Finally, '15 EOM' is a trade credit where the full amount is due on the fifteenth day after the end of the calendar month in which the invoice is given (i.e. the fifteenth of the next calendar month).

The related fields of finance and accounting offer some theoretical considerations. Hill and Reiner (1979) described three possible reasons that a company may choose to offer such discount: To receive cash sooner, to increase sales volume, or to reduce incidence of bad debt losses. In their study, they showed how a company may determine the discount at which it is indifferent between a prompt and a delayed payment. This analysis was based on the company's current accounts receivable profile (i.e. the time it typically takes for its various customers to pay their invoices) and what it assumes its profile would look like under an prompt payment discount scenario. The respective indifference discount depends on the company's marginal discount rate, which typically is the interest rate on its bank credit. Contractors and subcontractors need easy-to-use tools to help facilitate their decision-making regarding such discounts. Furthermore, the ability to model this type of trade credit is an outstanding need in the body of knowledge.

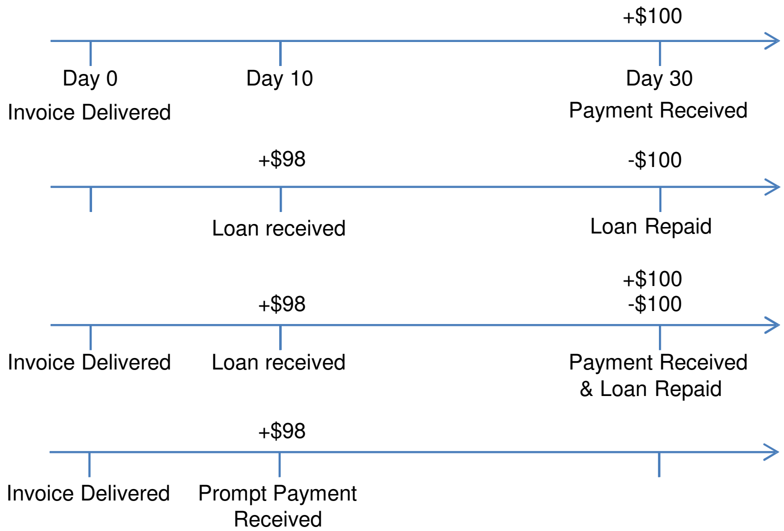
3. Prompt Payment Discounts

3.1 Definition and Theoretical Basis

A prompt payment discount is an invoice that offers a payer the option to pay early for a small discount in price. This trade credit term is commonly expressed as 'ρ / b_1 net b_2' where ρ is the discount offered (in percent), b_1 is the number of days within which one must pay the invoice to qualify to the discount, and b_2 is the number of days within which the invoice must otherwise be paid in full. For example a credit term from a contractor to an owner that says '2/10 net 30' would mean that owner will receive a 2% discount if paying within 10 days, otherwise the owner must pay in full within 30 days. For simplicity this conditional transaction can be thought of as two separate transactions. The first one is a simple invoice that is due within thirty days; the second one is a loan for 98% of the bill at 2% between day 10 and day 30. A simple illustration using cash flow timelines will clarify this representation from the perspective of the contractor.

Figure 1-a illustrates a simple invoice for $100 that is paid in thirty days. Figure 1-b is a loan taken on day 10 for $98 and $100 is repaid for it on day 30. When one combines these two events in Figure 1-c and simplifies them as Figure 1-d, one sees a prompt payment discount being taken by the owner. In effect, a

prompt payment discount is analogous to a payer extending credit to a payee for a certain rate. Here, the owner is giving the contractor a loan for 20 days at 2% interest. The importance of this conclusion is that now these discounts may be analyzed using traditional equations and methods from the field of finance.



Figures 1-a, 1-b, 1-c, 1-d: Prompt Payment Discount as a Loan

3.1 Indifference Discount

To assess the financial suitability of a potential discount, a limit needs to be established beyond which it is uneconomical for a project participant to accept or reject said discount. For a party awaiting a cash flow (payee), a ceiling (or maximum) prompt payment discount exists beyond which it would prefer receiving payment in full rather than suffering a large discount. Below this ceiling, it prefers having a smaller cash flow sooner rather than getting paid in full later. The ceiling discount is thus defined as the discount at which payee is indifferent between having its payer accept or reject a prompt payment discount. Similarly, for a party who must pay an invoice (payer) and has been offered a prompt payment discount there exists a floor (or minimum) discount below which it is financially unadvisable to accept it. Thus the floor discount is the discount at which the payer is financially indifferent between prompt payment and payment in full later. Equation [1] adapts the formula to calculate the indifference discount from Hill and Reiner (1979).

[1] $$\rho = 1-(1+i)^{b_1-b_2}$$

Where i is the daily interest rate, b_1 is the discount period, and b_2 is the number of days to receive full payment. For example, if a contractor typically collects payments on time in thirty days, and its annual interest rate is 10% (daily interest rate equivalent to 0.03%), its ceiling discount for a discount period of 10 days will be $\rho = 1-(1+0.0003)^{10-30} = 0.5\%$.

Such discounts are useful if project participants are frequently delinquent in submitting their payments. For example, a contractor may incur an owner with the poor habit of paying 30 day invoices only within 60 days. In this case, the contractor must evaluate its indifference discount based on a 60 day duration. If a positive difference exists between ceiling discount and floor discount, it makes financial sense for both the payee to offer a discount and for the payer to accept it. This beneficial scenario will occur in all cases where the payee's interest rate is greater than the payer's, which thus can be 'shared' with the payer.

Since the indifference discount relies on only few variables, it is possible to illustrate it for several possible scenarios in one concise nomograph. Figure 2 can be used to determine the indifference discount when the discount period b_1 is 10 days. A project participant can find the line that represents its annual interest rate to determine its individual indifference discount based on the estimated payment delay. As discussed above, the indifference discount functions as a floor for the discount-taker (payer) but as a ceiling for the discount-maker (payee). Each line in the nomograph intersects the *x*-axis at 10 days, because it would not make sense to offer a prompt payment discount if the payer typically pays early (less than 10 days).

4. Discount Nomograph

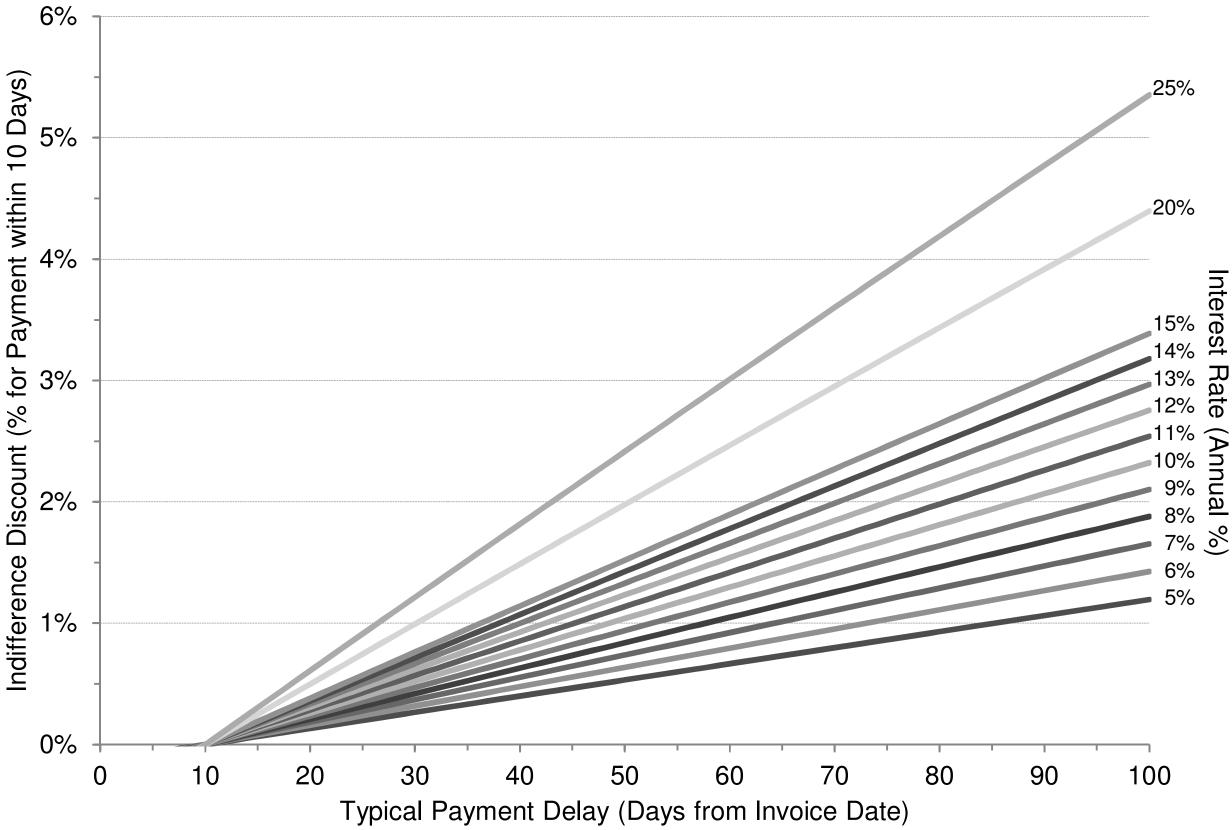


Figure 2: Discount Nomograph

4.1 Nomograph Instructions

Step 1 in using the nomograph of Figure 2 is determining whether one is a discount-taker or a discount-maker. This query ascertains whether one must establish a floor or a ceiling discount. A discount-taker is a project participant who has been offered a discount on an invoice in exchange for prompt payment and must decide whether or not to accept it; this party seeks a floor discount of how little is still acceptable. A discount-maker is a party who is prepared to offer a discount and must decide on its magnitude; this party seeks a ceiling discount of how much is still acceptable. In the case of a discount-taker looking for a floor discount, for example an owner who has been offered a prompt payment discount by a contractor, Step 2 is to determine their discount rate and typical payment time. Details on how to determine an appropriate discount rate are beyond the scope of this study, but may be reasonably assumed as the current interest rate on bank debt for the given party. The hypothetical owner in this example has a discount rate of 5%.

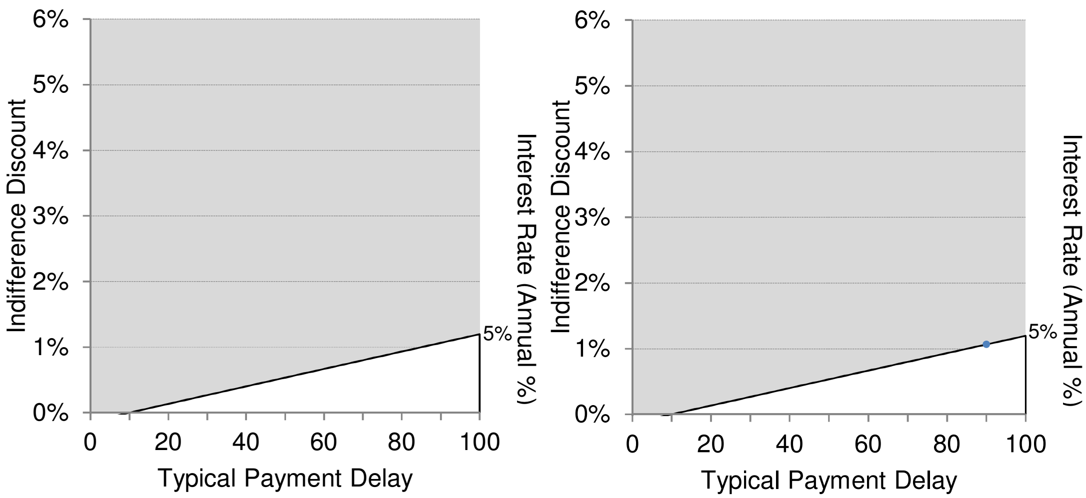


Figure 3-a, 3-b: Illustration of a floor discount

At this stage, the owner has identified the line on the chart that represents its discount rate. As illustrated in Figure 3-a, anything above this line represents a financially acceptable discount. Also, upon reviewing past invoices, the owner finds that payment is typically submitted approximately 90 days from the invoice (despite the due date of 30 days). Figure 3-b shows this spot on the line. Thus the owner's indifference discount is approximately 1%. This means that any discount greater than 1% for a prompt payment of 10 days is financially beneficial to the owner. If the owner was offered a 2% discount for payment within 10 days, it would be prudent to accept such discount. Similarly, the contractor looking for a ceiling discount must determine its discount rate and the typical delay in receipt of payment. In the present example, the contractor who invoices with a prompt payment discount knows that the owner typically delivers payment in 90 days. The contractor, being typically a smaller company, has a much greater discount rate of 10%.

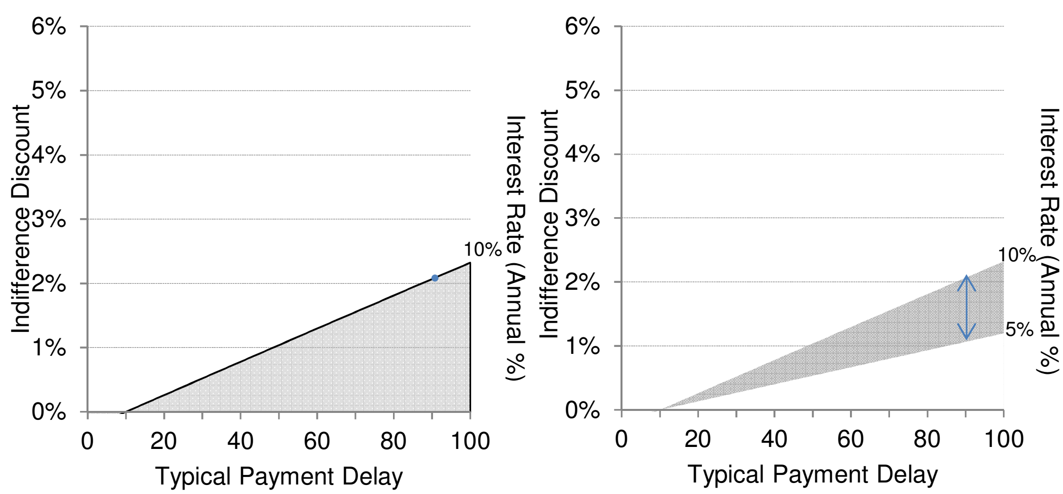


Figure 4-a, 4-b: Illustration of a ceiling discount and identification of typical payment delay

From the nomograph of Figure 4-a the contractor reads its indifference discount as approximately 2.1%. Any discount less than this 2.1% for payment within 10 days is a financially acceptable discount. Thus, if the contractor offers the owner a 2% discount for payment within 10 days, it is beneficial to the contractor. At this point, however, the contractor is not aware that the owner's indifference discount is only 1%. The contractor could indeed offer a lower discount and it would still be compelling for the owner. In this case the contractor is effectively 'leaving money on the table.' Yet if the contractor does not know the owner's indifference discount, it cannot know how much the discount could be lowered. Some owners that are publicly listed companies or government entities may have publicly available information from which one could determine their indifference discount. If the contractor could indeed determine that this owner's discount rate is 5%, then Figure 4-b illustrates the range of discounts that the contractor may offer with a mutual benefit. If the contractor chooses a 1.5% discount, it is both beneficial to itself and to the owner.

4.2 Further Considerations

For a discount-maker, this indifference discount is the ceiling – the greatest discount that a payee may offer to a payer before it would negatively affect the project economics. A discount-taker's indifference discount, on the other hand, is the floor – the discount below which a payer would prefer pay later from a financial standpoint. If the ceiling is above the floor (i.e. the contractor's interest rate is greater than the owner's interest rate), there exists an area within the nomograph that contains all discounts that are financially suitable for both parties, i.e. a win-win. Stated differently, if a payer has a relative advantage in its cost of debt, both parties may save by sharing that cost advantage. In most cases, bigger companies will have lower costs of debt than smaller companies. Typically an owner (especially governments) might be larger than a general contractor, and a general contractor might be larger than a subcontractor.

Even if a prompt payment discount is financially favourable, a discount-maker may still find that its offer is not accepted. Other factors that influence discount acceptance are often qualitative in nature and may be grouped together. Examples of such factors include a payment processing system that is not sufficiently flexible to fast-track certain invoices, a party that may hold payments longer as an alternative – if possibly non-contractual – form of retainage, a company policy against accepting prompt payment discounts, or an inability to pay early. For the purposes of this paper, these factors are identified, but it is beyond its scope to attempt to quantify them. Further research on invoicing may choose to focus on these factors, either by surveying construction companies regarding their experience with discounts or, if available, analyzing real invoicing data that include prompt payment provisions. A discount of sufficient magnitude may encourage acceptance despite these factors, but it will vary between parties exactly how large this discount must be.

5. Singularity Functions

Singularity functions are a set of mathematical functions that use pointed brackets to model the behaviour of a discontinuous dependent variable *y*(*x*) over its continuous independent variable *x*. Discontinuous phenomena often materialize as rapid major changes on the vertical *y*-axis, which includes steps shapes, sloped curves, and other types. The basic notation for all singularity functions is shown in Equation [2]

[2] $$y(x)=s\cdot\langle x-a\rangle^n=\begin{cases}0 & for\ x<a\\ s\cdot(x-a)^n & for\ x\geq a\end{cases}$$

[3] $$\frac{d}{dx}\langle x-a\rangle^n=n\cdot\langle x-a\rangle^{n-1}$$

[4] $$\int\langle x-a\rangle^n dx=\frac{1}{n+1}\cdot\langle x-a\rangle^{n+1}+C$$

Where *x* is the independent variable, *y* is the dependent variable defined as a function of *x*. The exponent *n* describes the order of the polynomial equation that is scaled by *s*. The variable *a* defines the boundary of the singularity function, in other words where the singularity function is 'switched on.' They can also be differentiated and integrated normally per Equations [3] and [4]. Singularity functions are cumulative over their terms and right-continuous, and terms may be added if they describe the same order of behaviour.

In structures, this unique notation was used for loads, shears and moments on beams. More recently, it has been used in linear scheduling analysis (Lucko 2007) and cash flow optimization (Lucko 2011). This expression is a compelling choice to model and validate prompt payment discounts as discussed above.

6. Derivation of a Decision Function

6.1 Foundational Equations

Assume that direct cost of an activity grows linearly. Its start and finish time can be at any time in periods, here months, and shifts or delays may occur in timing. A contractor will bill the owner at the end of each month for these costs plus a markup. Further, one may assume that the billing-to-payment-delay is one month. These periodical phenomena can use the floor operator $\lfloor \; \rfloor$ to be represented in a singularity function. The floor operator rounds a real number down to an integer. Similarly, the ceiling operator $\lceil \; \rceil$ rounds a real number up to the next integer. The singularity functions of costs, bills, payments, and balances are computed per the foundational Equation [5] through [10]:

[5] $$z(y)_{cost} = \frac{C}{D+d_2} \cdot \left[\left\langle y - a_S^* \right\rangle^1 - \left\langle y - a_F^* \right\rangle^1 \right]$$

[6] $$z(y)_{bill} = \frac{C \cdot (1+\mu)}{D+d_2} \cdot \left[\left\langle \lfloor y \rfloor - a_S^* \right\rangle^1 - \left\langle \lfloor y \rfloor - a_F^* \right\rangle^1 \right]$$

[7] $$z(y)_{pay} = \frac{C \cdot (1+\mu)}{D+d_2} \cdot \left[\left\langle \lfloor y \rfloor - (a_S^* + b) \right\rangle^1 - \left\langle \lfloor y \rfloor - (a_F^* + b) \right\rangle^1 \right]$$

[8] $$z(y)_{balance} = z(y)_{pay} - z(y)_{cost}$$

[9] $$z(y)_{disc-pay} = \frac{C \cdot (1-\rho) \cdot (1+\mu)}{D+d_2} \cdot \left[\left\langle \lfloor y + 1 - l \rfloor - a_S^* - b \right\rangle^1 - \left\langle \lfloor y + 1 - l \rfloor - a_F^* - b \right\rangle^1 \right]$$

[10] $$z(y)_{disc-balance} = z(y)_{disc-pay} - z(y)_{cost}$$

Where y is the independent variable representing time, z is the dependent variable that measures cash flow, C is the cost of an activity; D is its duration, μ is the markup rate for profit, a_S is the start and a_F is the finish, d_1 is a shift, d_2 is a delay, $a_S^* = a_S + d_1$, $a_F^* = a_F + d_1 + d_2$, and b is the billing-to-payment-delay. Consider the prompt payment discount case: The discount-taker will pay sooner if he or she decides to take the discount. The singularity functions representing payment and cash flow balance are computed by Equations [9] and [10], where ρ is the prompt payment discount and l is the early payment duration, i.e. the time period within which a discount offer is available before expiring, which is 10 days for 2/10 net 30.

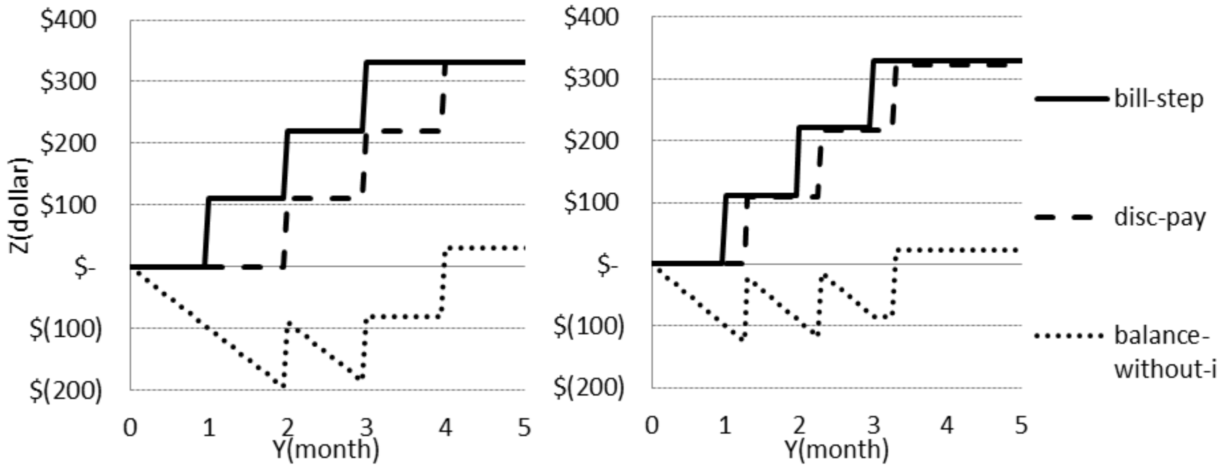


Figure 5-a, 5-b: Normal and Early Payment Scenarios

Figures 5-a and 5-b show the difference between the normal payment and early payment scenarios. In the normal payment case, the billing-to-payment-delay *b* is one month and the payee's funding deficit almost reaches $200,000 at the end of months 2 and 3. After month 4, the initial deficit finally becomes a surplus and the project has a positive cash flow. In the early payment case the payer took the prompt payment discount to pay the bill early and receive a discount. In Figure 5-b it can be clearly seen that the payee's negative balance is smaller with prompt payments than under the normal scenario. In this case, the contractor must only finance approximately $130,000 and will realize savings on finance charges.

6.2 Decision Function

If the contractor offers a trade credit of the form 'ρ / b_1 net b_2' to the owner, the following equations apply. The following is a comparison of project cost without and with a prompt payment discount that takes the owner's view. It takes time value of money into account and it is separated into three parts (start time to the end of the first month, the end of the first month to the start of the last month, the start of the last month to finish time). Therefore, the following billing equation is the combination of three terms. Equation [11] is derived from the foundational equations above. Equation [12] determines the floor discount d_{floor}.

[11]

$$z_b(y)_{bill} = \Big\{ \left(\lceil a \rceil_S - a_S\right) (1+i_O)^{\lceil a_F \rceil - \lceil a_S \rceil} \cdot \left[\langle y - a_S \rangle^0 - \langle y - \lceil a_S \rceil \rangle^0 \right] + \frac{(1+i_O)^{\lfloor y \rfloor - \lceil a_S \rceil - b} - 1}{i_O} \cdot \left[\langle y - \lceil a_S \rceil \rangle^0 - \langle y - \lfloor a_F \rfloor \rangle^0 \right]$$

$$+ \left(a_F - \lfloor a_F \rfloor\right) \cdot \left[\langle y - \lfloor a_F \rfloor \rangle^0 - \langle y - a_F \rangle^0 \right] \Big\} \cdot \frac{C}{D} \cdot (1+i_O)^b \cdot \frac{\left[\langle \lfloor y \rfloor - a_S - b \rangle^1 - \langle \lfloor y \rfloor - a_F - b \rangle^1 \right] \cdot (1+\mu)}{\lfloor y \rfloor \cdot \langle y - a_S \rangle^0 - \left(\lfloor y \rfloor - a_F\right) \cdot \langle y - \lceil a_F \rceil \rangle^0 - \lceil a_S \rceil - b}$$

[12] $$(1-\rho) \cdot (1+i_O)^{b_2 - b_1} \cdot z_{b_1}(a_F + b_1) \le z_{b_2}(a_F + b_2) \quad \Rightarrow \quad \rho \ge 1 - (1+i_O)^{b_1 - b_2}$$

Where y is the independent variable (here time), $z(y)_{bill}$ is the billing function, C is the cost of an activity; D is its duration, μ is the markup rate for profit, a_S is the start, a_F is the finish, i_O is the interest rate of the owner, b_1 is the early billing-to-payment-delay and b_2 is the normal billing-to-payment-delay that can each be inserted in place of the generic variable b in Equation [12] as needed to compare the normal versus the discount scenario, and ρ is the discount rate. For the 'ρ / b_1 net b_2' scenario the following comparison of payments takes the contractor's view. Equations [13] and [14] represent the contractor's profit and are derived from the foundational equations above, with which Equation [15] determined the ceiling discount.

[13] $$p_{b_1}(y) = (1-\rho)\cdot z_{b_1}(y)_{bill} - z(y)_{cost} - 0.5\cdot i_C \cdot z(y)_{cost}$$

[14] $$p_{b_2}(y) = z_{b_2}(y)_{bill} - z(y)_{cost} - 0.5\cdot i_C \cdot z(y)_{cost}$$

[15] $$p_{b_1}(a_F + b_1) \geq p_{b_2}(a_F + b_2) \quad \Rightarrow \quad \rho \leq 1-(1+i_C)^{b_1-b_2}$$

Where y is the independent variable (here time), $p(y)$ is the profit function, $z(y)_{\text{bill}}$ is the billing function, $z(y)_{\text{cost}}$ is the cost function, a_F is the finish, i_C is the interest rate of the contractor, b is the billing-to-payment-delay, again specified as b_1 or b_2 for the normal or discount scenario, and ρ is the discount rate.

6.3 Discount Function

Based on the concept of floor and ceiling discount, the contractor should accept to provide a discount between the two. Any discount between d_{floor} and d_{ceiling} will lead to a win-win situation for both parties. The value of such discount depends on the interest rates of both owner and contractor and the billing-to-payment-delay, but per Equation [16] it is not related to the activity start or finish, the cost, or the markup. It yields the discount rate, where y is the independent variable and b is the billing-to-payment-delay.

[16] $$Discount = \rho(y) = \left[1-\left(1+\frac{i}{365}\right)^{b_2-b_1}\right]\cdot\langle y-b_1+b_2\rangle^0 + \left[\left(1+\frac{i}{365}\right)^{b_2-b_1+1} - \left(1+\frac{i}{365}\right)^{b_2-b_1}\right]\cdot\langle y-b_1+b_2\rangle^1$$

This equation yields the same results as the nomograph provided above. If the value of $b_1 - b_2$ is equal to 20, then the actual payment duration might be much longer than b_2 because of a delinquent payment. Therefore the discount decision function could be obtained using d_{floor} and d_{ceiling}. It can use the actual or average payment duration. Equation [17] is the decision function for the two discounts d_{floor} and $d_{\text{ceiling.}}$

[17] $$Discount \in \left[1-(1+i_O)^{b_1-b_2}\,;1-(1+i_C)^{b_1-b_2}\right] \Rightarrow d_{floor} = 1-(1+i_O)^{b_1-b_2}\,;\, d_{ceiling} = 1-(1+i_C)^{b_1-b_2}$$

6.4 Validation of Decision Function

The decision function is derived from the foundational Equations [5] to [10]. To validate it they have been coded dynamically in a computer model. If inputs (discount rate, billing-to-payment-delay, markup, etc.) are changed, the payment timing and profit change accordingly. To see if results of the decision function agree with the theoretical framework discussed above, the value of the discount has been varied to find the point where the NPV without and with the prompt payment discount is the same for different payment delays. All findings from these computations conformed to the nomograph provided in Figure 2. Further, the fact that the model is flexible with regards to many variables offers researchers and practitioners alike the opportunity to understand impacts in scenarios beyond those that have been described in this paper.

7. Conclusions

This paper provides a framework for project participants in the construction industry to assess the options that a prompt payment discount provides them. Contractors who bill owners or subcontractors who bill general contractors can use the nomograph to find their ceiling discount. Owners and general contractors who have been offered a prompt payment discount can use this tool to determine if the discount is above their floor. The decision function facilitates these analyses toward a better understanding of the impacts of discounts. Moreover, the equations provide a theoretical framework for further work on the subject. Future research may focus on factors that determine the discount acceptance by analyzing real project data.

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